ADAPTIVE INTERFERENCE SUPPRESSION FOR CDMA OVERLAY SYSTEMS*

Paul Wei†, James R. Zeidler††, and Walter H. Ku††
†Department of Electrical and Computer Engineering, University of California at San Diego, La Jolla, CA 92093-0407.
††Naval Command, Control and Ocean Surveillance Center RDT&E Division, Code 804, San Diego, CA 92152-5000

ABSTRACT

It has been proposed that CDMA systems can be assigned to spectral bands which are presently occupied by narrowband users to further increase spectral capacity. Such CDMA overlay systems could provide new options for efficient utilization of the spectrum with minimal disruption to existing narrowband users, especially if adaptive interference suppression techniques are utilized in the spread spectrum receiver.

In this paper the bit-error-rate (BER) performance of the mobile-to-base link of a CDMA system for a single narrowband user which occupies a significant portion of the CDMA bandwidth is considered to evaluate the performance of the adaptive filtering. It is shown that adaptive filtering provides a significant improvement when the CDMA and narrowband powers are of the same order. The dependence of the BER on the filter order, the bandwidth of the interference, and its center frequency relative to the CDMA carrier frequency are demonstrated.

1. INTRODUCTION

Spread spectrum communications are currently under development for mobile communication applications due to their efficient utilization of channel bandwidth, the relative insensitivity to multipath interference, and the potential for improved privacy [1, 2]. In addition to providing multiple accessing capabilities and multipath rejection, spread spectrum communications also offer the possibility of further increasing overall spectrum capacity by overlaying a Code Division Multiple Access (CDMA) network over existing narrowband users [2](Fig. 1). As discussed by Pickholtz, Milstein, and Shilling [2], such CDMA overlay systems would provide new options for the most efficient utilization of the spectrum with minimal disruption to existing narrowband users, especially if signal processing techniques which suppress the narrowband interference are utilized in the spread spectrum receiver. Such narrowband suppression techniques will not only provide improved performance of the CDMA system, but will also allow the performance impact on the existing users to be minimized, since the CDMA power level can then be correspondingly decreased [2].

A number of authors have explored the performance of such narrowband interference suppression filters for spread spectrum communications signals [3–8]. These studies have concentrated on quantifying a SNR improvement ratio at the filter output and have also obtained the bit error rate (BER) performance using Gaussian approximations for the filter output statistics. The SNR improvement ratio has been found for tone interferers [3] and for order one autoregressive (AR1) interferers [4]. In [4] it is assumed that the center frequency of the interferer is at the carrier frequency of the CDMA signal. In commercial applications the interferers are likely to occupy a significant portion of the CDMA bandwidth and to have center frequency which is offset from the carrier frequency of the CDMA signal. In this paper we examine the performance of the mobile to base link of this system where a single interferer occupies a significant portion of the direct sequence (DS) CDMA bandwidth and may also be placed at any position in the CDMA bandwidth. Effects of the digital implementation of the adaptive suppression filter are characterized as a function of the adaptive time constant and quantization errors. Approximate closed form solutions will be provided for the SNR improvement and BER.

In Section 2 the CDMA overlay problem is described and the BER performance examined. Section 3 provides a summary of the derivation of the optimum suppression filter and the resulting SNR improvement when an interference suppression filter is used.

2. PRELIMINARIES

2.1. CDMA Overlay

First, the basic system model and notation of the CDMA overlay model is introduced and the performance criterion described. The CDMA system is assumed to be a BPSK DS CDMA system which overlays a narrowband BPSK signal (Fig. 1). The transmitted signal from the kth user in the CDMA sys-

---

*This content is a excerpt from a technical document and does not include all the original context and details. It is intended to provide a summary of the concepts discussed in the document. ---
tem takes the form

$$S_k(t) = \sqrt{2P_k}a_k(t)\cos[\omega_0 t + \phi_k]$$  \hspace{1cm} (1)

where $P_k$, $\omega_0$ are the transmitter power and carrier frequency; $\phi_k$ is the phase angle introduced by the $k$th PSK modulator.

We assume that the channel between the CDMA mobile user and the base is a frequency-nonselective Rayleigh fading channel, and is characterized by the parameters $\beta_k$, $\tau_k$, and $\gamma_k$, which are defined as the gain, delay, and phase of the $k$th signal at the receiver. The gain $\beta_k$ is an independent Rayleigh variable with parameter $\rho = E[\beta^2]/2$; while the delay $\tau_k$, also independent, has a uniform distribution in $[0,T_b]$. The phase $\gamma_k$ is absorbed into the phase of the CDMA modulator under the assumption that the ensemble of CDMA modulator phases are uniformly distributed. The rate of the CDMA information sequence $b_k(t)$ ($1/T_b$) and the spreading sequence $a_k(t)$ ($1/T_c$) are related by the processing gain $N$ via $T_b = NT_c$, and the bandwidth (BW) of the CDMA signal is given by $T_c = 2/T_b$.

The received narrowband BPSK interference is assumed to be nonfading, given by

$$J(t) = \sqrt{2J}j(t)\cos[(\omega_0 + 2\sigma_0 t) + \theta]$$  \hspace{1cm} (2)

where $\sigma_0$ is the offset of the BPSK carrier frequency from the CDMA carrier frequency. $J$ and $\theta$ denote the received interference power and phase, respectively. The BPSK information sequence $j(t)$ has bit rate $1/T_c$, where $T_c$ is the duration of one bit.

For $K$ CDMA users, the received signal consists of the independently fading CDMA signals, the interfering narrowband BPSK, and thermal noise, i.e.,

$$r(t) = \sum_{k=1}^K [\beta_k S_k(t - \tau_k)] + J(t) + n(t)$$  \hspace{1cm} (3)

where $n(t)$ is additive white Gaussian noise with a two sided power density $N_0/2$. Two important quantities of this system are the ratio of the interference BW to CDMA signal BW, $p$, which is given by $p = T_b/T_c$, and the ratio of the interfering carrier offset to half of the spread spectrum bandwidth, $q$, where $q = 2\sigma_0 T_c/2\pi$.

2.2. System Performance

At the receiver, as shown in Fig. 2, the CDMA signal is 1) coherently demodulated, 2) sampled at the chip rate $1/T_c$, 3) filtered, and 4) despread, to produce the decision statistic. Without loss of generality, we assume that user 1 is the reference user with ($\phi_1 = 0$), and the decision statistic $\lambda(m)$, over $N$ time samples, is given by

$$\lambda(m) = \sum_{n=1}^N r_f(mN + n)a_1(n)$$  \hspace{1cm} (4)

where $r_f(t)$ is the output of the suppression filter, with coefficients $\{C_i\}$, given by

$$r_f(n) = \sum_{i} r(n - i)C_i$$  \hspace{1cm} (5)

and

$$r(n) = \int_{-\infty}^{\infty} r(t)2 \cos(\omega_0 t)dt,$$  \hspace{1cm} (6)

is the demodulated signal for user 1 (Fig. 2).

To find the BER we use a recent result described by Wang and Milstein in [9] and derived in [10] which gives the approximate BER for this system employing a suppression filter, with coefficients $\{C_i\}$. For a chip synchronized system and large number of CDMA users ($K \gg 1$), Wang and Milstein proved that for the BER of this system is approximately given by

$$BER = \frac{1}{2} \left[ 1 - \sqrt{\frac{T_b}{1 + \tau_b}} \right] \approx \frac{1}{4\tau_b}$$  \hspace{1cm} (7)

where

$$\frac{1}{\tau_b} = \left( \frac{E_b}{N_0} \right)^{-1} + \frac{K - 1}{N} \sum_i C_i^2 + \frac{1}{NS} \sum_{i,m} C_i C_m \phi_j(l - m)$$

is the noise to signal ratio at the output of the suppression filter divided by the spreading gain $N$. The interfering narrowband BPSK correlation function is $\phi_j()$, $S = 2P\eta$ is the average CDMA signal power, and $E_b = ST_b$ is the average energy per bit of the CDMA user.

The three terms which appear in the noise to signal ratio at the output of the suppression filter ($1/\tau_b$) are the (1) thermal noise, (2) interfering BPSK power, and (3) multi-access interference power at the output of the suppression filter normalized by the spreading gain $N$ and the signal power $S$. Thus

$$\frac{1}{\tau_b} \approx \xi \frac{E_b}{NS}.$$  \hspace{1cm} (8)

is proportional to the prediction error power ($\xi$) at the output of the suppression filter. From the BER expression (7) the system performance is seen to be highly dependent on the suppression output power and can determined once the coefficients of the
Figure 3: Transversal LMS filter in ALE configuration

suppression filter are found.

2.3. Suppression Filter

The suppression filter which minimizes the BER is the Wiener filter [3]. This filter may be constructed either by determining, a priori, the input interference and solving for the Wiener filter coefficients, or using an adaptive algorithm. Due to the often unknown or changing characteristics of the interference, an adaptive algorithm may be more useful in actual implementations. In this paper we will examine a fixed-point implementation of the adaptive line enhancer (ALE) for the suppression filter. This filter employs the Widrow-Hopf least mean square (LMS) algorithm to predict the narrowband interference component and cancel it from the signal. The structure of the ALE filter is shown in Fig. 3. In this configuration the suppression filter coefficients are given by

\[ C_0 = 1 \quad C_{(\alpha+k)} = -w_k \quad k = 0, \ldots, L - 1 \quad (9) \]

where the \( w_k \)'s are the coefficients of the \( L \) tap prediction filter.

The steady state performance of this filter may be analyzed as a sum of three independent terms [11] consisting of the fixed Wiener filter component, the misadjustment component due to the noise in the gradient estimation process, and the component due to the quantization errors [12]. In this paper we shall describe the Wiener filter component as the adaptation and quantization noise can be made negligible [13].

3. PERFORMANCE ANALYSIS

An detailed analysis of the performance of the system is given in [13]. We summarize the analysis here. Normalizing by the total CDMA power, the input autocorrelation is

\[ \phi_x(l) = \phi_j(l) + (\sigma_n + 1)\delta(l) \quad (10) \]

The interfering BPSK signal with autocorrelation

\[ \phi_j(l) = \sigma_j^2 (1 - \frac{l}{T}) \cos(\omega_0 l), \quad |l| < T \quad (11) \]

is first modeled as an autoregressive process with autocorrelation \(^1\)

\[ \phi_{ar2}(l) = \sigma_j^2 e^{-\alpha|l|} \cos(\omega l). \quad (12) \]

\(^1\)Note that the interference power \( \sigma_j^2 \) is also a function of the carrier offset due to the demodulation. In this case as this is \( \sigma_j^2 = 2J(1 - \cos(\omega_0))/\omega_0^2 \) from the demodulation process.

\[ z_{1,2} = \pm \alpha + j\omega \quad p_{1,2} = \pm \beta + j\psi \quad (15) \]

are the zeros and poles and \( G \) is a normalizing constant. Previous work using the approximation that the zeros and poles lie on the same radius is not valid here due to the large interference bandwidth. The zeros are given by

\[ \beta = \text{acosh}(\sqrt{u}) = \text{acos} \left( \frac{A}{\sqrt{u}} \right) \quad (16) \]

where

\[ u = B + \sqrt{B^2 - A^2} \quad (17) \]

\[ A = (\cosh \alpha + \frac{S\text{NR}_j}{\sinh \alpha}) \cos \omega \]

\[ B = (\cosh 2\alpha + S\text{NR}_j \sinh 2\alpha + \cos 2\omega + 2)/4 \]

As can be seen Fig. 4 the location of the zeros can deviate significantly from the radial line, especially for large bandwidths \( (\alpha \approx 1/T) \).

The Wiener solutions is found by solving the Wiener-Hopf equation [14] for \( \psi^* \)

\[ \sum_{l=1}^{L} \phi_x(l)\psi^*(l) = \phi_j(l + 1) \quad (18) \]

For the ARMA process this can be found analytically [15] and in this case, where the number of poles and zeros are the same,
the Wiener solutions consists of damped exponentials terms with the zeros of the spectrum as the damping constants. The method of undetermined of coefficients is used whereby the form of the solution is assumed and substituted in (18) to solve for the multiplying constants. A simplification results when the filter length \( L \gg 1 \) and terms with a positive damping constant can be neglected. This gives

\[
w^*(k) = 2|B|^2 e^{-\rho k} \cos(\psi k + \theta), \quad k = 0, \ldots, L - 1
\]  

(19)

for the Wiener filter coefficients and from this the performance gain with use of a filter can be determined.

A useful figure is the SNR improvement factor for the CDMA given by

\[
\eta = \frac{\sigma_n^2 + \sigma_i^2}{\xi_{\text{min}} - 1}
\]

(20)
The number '1' in the denominator arises from the normalization by the total CDMA power.

For large filter lengths \( L \rightarrow \infty \) this reaches a maximum at \( \omega = 0 \)

\[
\eta_0 = \frac{\sigma_n^2 + \sigma_i^2}{\sigma_n^2 + \sigma_i^2 \frac{1 + q}{1 + q_0}}
\]

(21)

and a minimum at \( \omega = \pi/2 \)

\[
\eta(\pi/2) = \frac{\sigma_n^2 + \sigma_i^2}{\sigma_n^2 + \sigma_i^2 \frac{1 - q}{1 + q_0}}
\]

(22)
at \( \omega = \pi/2 \), where \( q_1 = e^{-\rho \omega (k_0 + \omega)} \) and \( q = e^{-\alpha} \). The difference between SNR improvement results from the offset of the interference and reaches a minimum when the cross-correlation is zero for all odd delays, thus the similarity between \( \eta_0 \) and \( \eta_{\pi/2} \). The SNR improvement is close to the actual for lengths \( L \gg (\text{correlation length of signal}) \). These are plotted in Figs. 5-6. Fig. 5 plots the maximum SNR improvement, and Fig. 6 shows the maximum decrease in that SNR improvement when the carrier is offset. It is interesting to note that there is a maximum decrease in improvement of 1.5dB for very narrow-band signals versus the 3dB expected for sinusoids. As further illustrated in Fig. 7 for \( (\alpha = 0.1, L = 10) \) there is decrease of 1.5dB. However as the BW is decreased to \( \alpha = 10^{-9} \), a tap length of \( L = 10 \) does not cover the correlation length of the interferer, and there the decrease is 3dB between \( \omega = 0 \) and \( \pi/2 \). Finally as when \( L \) is increased this difference is reduced to 1.5dB. Note that on the bound to the maximum and minimum only hold for \( L \rightarrow \infty \), and when \( L < (\text{correlation length of signal}) \).

An interesting plot is the input SNR vs. the output SNR (Fig. 8) for large \( L \), which illustrates the SNR levels seen by the detector on a per chip basis. Here it can be seen that the suppression filter provides a significant increase in the SNR and, in this case the, same factor of improvement in the system BER(7).

4. CONCLUSION

In this paper we have studied the performance of an adaptive interference suppression system for a CDMA system overlaid on a single BPSK system which occupies a significant portion of the CDMA bandwidth. Adaptive interference suppression filters have been shown to provide significant improvements for this CDMA overlay system where large interference levels are expected.

The BER was shown to be related to the power at the output of the suppression filter by a scaling constant. The system performance is characterized by the output power of the suppression filter and the performance of the suppression filter characterized by a SNR improvement ratio. The optimum linear suppression filter, given by the Wiener filter, is solved by modeling the BPSK interference by a second order process. It was shown that the narrowband model used in previous studies does not apply for this system, especially at large interference SNR's. The regions where the narrowband model is applicable was characterized. Expressions are derived to provide the exact solution for the zeros of input spectrum.

The difference between the SNR improvement when the interference center frequency is centered on the CDMA carrier and when it is offset is characterized. When the BPSK interference is centered on the CDMA carrier, it was shown that the


