A Dual-Mode Algorithm for Blind Equalization of QAM Signals: CADAMA

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Abstract
The misadjustment exhibited by the constant modulus algorithm (CMA) can be excessive when it is used for blind equalization of QAM signals, thus degrading the performance of decision-directed carrier recovery algorithms. This paper introduces a hybrid blind equalization algorithm, the CMA-Assisted Decision Adjusted Modulus Algorithm (CADAMA), that exhibits reductions in misadjustment on the order of 18 to 25 dB for higher order QAM constellations. CADAMA employs CMA to perform initial adjustment of the equalizer, and switches to Decision Adjusted Modulus Algorithm (DAMA) mode when the amplitude equalization has been sufficiently accomplished. DAMA is based on a phase-blind cost function that goes to zero at each ring of the QAM constellation.

1. Introduction
Because its cost function does not go to zero at any of the signal points of most standard QAM constellations, the misadjustment exhibited by CMA [1], [2] can be excessive when it is used with QAM signals. This is a concern because excessive misadjustment can retard the convergence of decision-directed carrier recovery algorithms that must operate in conjunction with the blind equalization algorithm [3]. This paper introduces a hybrid blind equalization algorithm, the CMA-Assisted Decision Adjusted Modulus Algorithm (CADAMA), that provides dramatic reductions in misadjustment for QAM signals. CADAMA employs CMA to perform initial adjustment of the equalizer, and switches to Decision Adjusted Modulus Algorithm (DAMA) mode when the amplitude equalization has been sufficiently accomplished. Because DAMA is based on a cost function that goes to zero at each ring of the QAM constellation, it exhibits reductions in misadjustment on the order of 18 to 25 dB for many standard QAM constellations. The criterion for switching between the CMA and DAMA modes is based on comparing the histogram of radius decisions over an observation interval to the template distribution of QAM constellation radii using a goodness-of-fit test.

2. The Decision Adjusted Modulus Algorithm
DAMA was proposed in [4] for real-valued PAM signals and reintroduced in [3] as Radius Decision Equalization (RDE) for QAM signals. Generalized for complex-valued signals, the phase-blind DAMA cost function for QAM is

\[ J_{DAMA} \triangleq \frac{1}{4} \sum_{i=1}^{\rho} \left[ \min_j \left( |\hat{a}(n)|^2 - r_i^2 \right) \right]^2, \quad 1 \leq i \leq \rho, \tag{1} \]

where \( \hat{a}(n) \) is the output of the equalizer and \( \{ r_i \}_{i=1}^{\rho} \) are the signal point radii of the QAM constellation. Since each signal point lies on one of the \( r_i \), eq. (1) clearly goes to zero as a function of \( \hat{a}(n) \) at all of the constellation’s signal points. However, with each symbol estimate \( \hat{a}(n) \) produced by the DAMA-adjusted equalizer, a decision must be made as to which radius of the constellation is the closest to the estimate. The corresponding term of the overall DAMA cost function then applies to the next cycle of the equalizer’s tap-weight update, which is given by

\[ w(n+1) = w(n) - \mu \hat{a}(n) \left( |\hat{a}(n)|^2 - r_i^2 \right) u(n) \tag{2} \]

where

\[ \left| \hat{a}(n) \right|^2 - r_i^2 \triangleq \min_j \left| \hat{a}(n) \right|^2 - r_j^2, \quad 1 \leq i \leq \rho. \tag{3} \]

DAMA can only be applied if the amplitude distortion has been sufficiently equalized. DAMA will not converge if the number of incorrect radius decisions associated with the minimum operation in (3) is too large, and therefore, it cannot be used reliably for the initial stage of blind equalization of the amplitude distortion. Consequently, DAMA needs to be assisted by another algorithm to perform the initial adjustment of the blind equalizer. CADAMA employs CMA for this purpose.
3. The CMA-Assisted Decision Adjusted Modulus Algorithm

CADAMA begins operation in CMA using the all-pass initialization [1]. CADAMA switches from CMA mode to DAMA mode when the amplitude equalization has been sufficiently accomplished. The criterion for switching between the modes is based on comparing the histogram of radius decisions over an observation interval ($N$ iterations) to the template distribution of radii in the constellation. The template distribution is determined by the number of signal points on each radius. Let $p_i$ be the template probability of signal point radius $i$. Then $p_i = (\text{number of signal points on radius } i)/M$ where $M$ is the order of the modulation; $\sum p_i = 1$. The comparison can be made using Pearson's chi-square or the Kolmogorov-Smirnov goodness-of-fit test [5]. To maintain consistency across the various constellations, the observation interval $N$ used in CADAMA is scaled with $M$: $N = \kappa M$. An observation interval of $\kappa M$ yields an expected $K$ observations per symbol $a_n$. It has been determined empirically that $\kappa = 20$ is required to provide adequate resolution in the test statistic [6]. The appropriate levels of significance for goodness-of-fit tests in the CADAMA application have also been determined empirically, over a range of amplitude distortion levels, and vary from 0.05% to 0.5% as a function of the number $\rho$ of constellation radii [6]. Higher order constellations require that the goodness-of-fit tests be conducted at lower levels of significance because the averaged equalizer gain error associated with making radius decision errors in (3) decreases as $\rho$ increases [6].

In order to inhibit noise induced switching out of the DAMA mode back into the CMA mode, the switching between the two modes in CADAMA is accomplished using some degree of hysteresis as follows. At the end of each observation interval $N$, the test statistic $X$ is calculated and compared to a threshold $X_0$. If CADAMA is in CMA mode and $X(mN) > X_0$, $m$ integer, then it stays in CMA mode; if $X(mN) \leq X_0$ then it switches into DAMA mode. If CADAMA is in DAMA mode and $X(mN) \leq X_0$, then it stays in DAMA mode. If $X(mN) > X_0$ while in DAMA mode, CADAMA will stay in DAMA mode for an additional observation interval. Then, if and only if $X((m+1)N) > X(mN) > X_0$, CADAMA will switch back into CMA mode until the test statistic again falls below the threshold level after a subsequent observation interval. In other words, once CADAMA is in DAMA mode, it stays in DAMA mode unless the test statistic exceeds the threshold in two consecutive observation intervals and the test statistic is increasing.

4. Misadjustment in CMA and CADAMA

It can be shown [6] that the variance of the steady-state CMA tap-weight update contains components that depend on the mismatch between Godard's cost function [1] and the QAM constellation. When the communications channel can be modeled as an FIR filter, these components are given by [6]

$$\gamma_1 \triangleq E \left[ |a_n|^4 \left( |a_n|^3 - R_1 \right)^2 \right], \quad (4)$$

and

$$\gamma_2 \triangleq E \left[ |a_n|^2 \right] E \left[ |a_n|^2 \left( |a_n|^2 - R_2 \right)^2 \right], \quad (5)$$

where $R_1 \triangleq E \left[ |a_n|^4 / E \left[ |a_n|^2 \right] \right]$ is Godard's modulus constant [1] and all expected values are evaluated over the QAM constellation $\{a_n\}_{n=0}^{M-1}$ under the assumption that all $M$ symbols are equally likely. Eqs. (4) and (5) are plotted vs. the number of signal point radii in Figure 1 for several standard QAM constellations. It can also be shown [6], that (4) and (5) are zero for the steady-state DAMA tap-weight update. This follows from viewing DAMA as $\rho$ CMAs operating in parallel, each one matched to one of the $\rho$ radii of the QAM constellation. Consequently, the misadjustment exhibited by CADAMA (in DAMA mode) is dramatically less than that exhibited by CMA for QAM.

![Figure 1 Components of steady-state CMA tap-weight update variance.](image-url)
5. Simulation Experiments

In the following simulations, the channel impulse response is
\[ h(n) = \sum_{k=1}^{3} h_k \delta(n-k), \]
where
\[ h_k = \frac{1}{2\sqrt{h^2}} \left[ 1 + \cos \left( \frac{2\pi}{W} (k-2) \right) \right] , \quad k = 1, 2, 3 \]
and \[ h = [h_1, h_2, h_3]^\top. \] Here, \( W = 3.5 \) which yields an
eigenvalue disparity of 16.8 dB. All learning curves shown were obtained by averaging 10 individual trials.

Figure 2 plots CADAMA and CMA learning curves for the 64-QAM constellation defined in Figure 5.22 of
[7]. Here \( S/N = 40.2 \) dB for an ideal channel. The step-
size used in each algorithm is \( \mu = 2 \times 10^{-6}. \) In this example, CADAMA converges to a solution that achieves
an asymptotic MSE that is 13.2 dB lower than that achieved by CMA. The Wiener MSE \( \epsilon_{\min}^2 = 0.0194 \) is shown by the horizontal dashed line. The misadjustment
achieved by CADAMA is \( \mathcal{M} = 0.13 \) whereas for CMA \( \mathcal{M} = 22.66. \) The reduction in misadjustment is therefore
22.5 dB. Figure 3 plots the chi-square test statistic \( \chi^2(n) \) averaged over the same 10 runs. The observation interval
was \( \kappa M = 20M = 1280 \) iterations. The threshold of
27.87 corresponds to a chi-square goodness-of-fit test
with 9 degrees of freedom at the 0.05% level of significance [6].
Figure 4 plots the CADAMA mode indicator function (0 implies CMA mode, 1 implies
DAMA mode) averaged over the same 10 runs. In this
set of runs, the earliest switching from CMA to DAMA
occurred at iteration 15,360, whereas by iteration 26,880
CADAMA was solidly in DAMA mode for all runs.

![Figure 2](image1.png)

**Figure 2** CMA and CADAMA learning curves for 64-
QAM, \( \mu = 2 \times 10^{-6}. \)

![Figure 3](image2.png)

**Figure 3** Averaged chi-square test statistic corresponding
to Figure 2.

![Figure 4](image3.png)

**Figure 4** Averaged CADAMA mode indicator function
corresponding to Figure 2.

Figure 5 plots CADAMA and CMA learning curves for the 128-QAM constellation defined in Figure 5.22 of
[7]. Here \( S/N = 43.1 \) dB for an ideal channel. The step-
size used in each algorithm is \( \mu = 5 \times 10^{-7}. \) In this example, CADAMA converges to a solution that achieves
an asymptotic MSE that is 13.1 dB lower than that achieved by CMA. The Wiener MSE \( \epsilon_{\min}^2 = 0.0281 \) is shown by the horizontal dashed line. The misadjustment
achieved by CADAMA is \( \mathcal{M} = 0.10 \) whereas for CMA \( \mathcal{M} = 21.37. \) The reduction in \( \mathcal{M} \) is therefore 23.3 dB. Figure 6 plots the Kolmogorov-Smirnov test statistic
\( D(n) \) averaged over the same 10 runs. The observation interval was \( \kappa M = 20M = 2560 \) iterations. The threshold
of 0.04 corresponds to a level of significance equal to
0.055%. Figure 7 plots the CADAMA mode indicator function averaged over the same 10 runs. In this set of runs, the earliest switching from CMA to DAMA occurred at iteration 17,920, whereas by iteration 51,200, CADAMA was solidly in DAMA mode for all runs.

6. Conclusions

When a CMA equalizer is applied to a QAM signal, the variances of the converged equalizer tap-weights have components that depend on the mismatch between Godard's cost function and the QAM constellation. The magnitudes of these components increase with the number of radii in the QAM constellation and contribute to excessive levels of misadjustment in the converged solution of the CMA equalizer. This paper has presented CADAMA which is one of the two viable alternatives to CMA for blind equalization of QAM signals that are developed in [6]. The other alternative, the Multiple Modulus Algorithm (MMA), is also presented in [8]. Both CADAMA and MMA exhibit dramatic reductions in misadjustment when compared to CMA. However, the practical application of MMA appears to be limited to standard QAM constellations with a maximum of three signal point radii [6], [8]. CADAMA, on the other hand, has been demonstrated in this paper to be applicable to 128-QAM which has 16 signal point radii. Also, MMA exhibits an exaggerated sensitivity to eigenvalue disparity for three signal point radii (e.g., 16-QAM) [6], [8]. CADAMA exhibits much less sensitivity than MMA to eigenvalue disparity for three signal point radii [6]. Thus CADAMA is probably a better choice than MMA for 16-QAM and other QAM constellations with three signal point radii [6]. In addition to [6] and [8], further details on the development of and the results of additional simulation experiments with MMA and CADAMA can be found in [9] and [10].

Notice

This technology may be covered by an invention disclosure assignable to the U.S. Government. Parties interested in licensing this technology may direct inquiries to COMMANDING OFFICER, (ATTN: HARVEY FENDELMAN, LEGAL COUNSEL FOR PATENTS, CODE 0012), NCCOSC RDT&E DIV, 53560 HULL ST, SAN DIEGO, CA, 92152-5001, telephone (619) 553-3001.

Figure 5 CMA and CADAMA learning curves for 128-QAM, $\mu = 5 \times 10^{-7}$.

Figure 6 Averaged Kolmogorov-Smirnov test statistic corresponding to Figure 5.

Figure 7 Averaged CADAMA mode indicator function corresponding to Figure 5.
References


