Sensitivity Analysis of Compact Antenna Arrays in Correlated Nakagami Fading Channels

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Abstract

This paper presents the average error probability performance of a compact space diversity receiver for the reception of coherent binary PSK signals through a correlated Nakagami fading channel. Analytical expression of the average BER is derived as a function of the covariance matrix of the multipath component signals at the antenna elements. The sensitivity of the performance to the operating environment (fading, angular spread, mean angle-of-arrival and input branch SNR) and the antenna array configuration (diversity order, antenna separation and electromagnetic coupling) is investigated.

1. Introduction

Antenna arrays have been intensively employed in the base stations for increased capacity and enhanced performance by mitigating multipath fading and co-channel interference in mobile wireless communication systems. To make full use of both the up-link and down-link, it is desired to implement antenna array in the mobile units as well. Due to the compact nature of the mobile receiver, the received signals will be at least partially correlated. The performance of these compact receiver arrays thus requires an analysis that addresses the effects of correlated multipath arrivals at the elements of the receiver array.

Many of works [3]-[7] used the Rayleigh distribution to model the fading statistics of the channel, while the Nakagami-m distribution[1], [2] can represent more general fading conditions. Previous studies of the diversity receiver performance in the correlated Nakagami environment all assume that the channel covariance matrix is known [9]-[11]. Those results are useful for the analysis of experimental data where the covariance matrix is explicitly measured from data, but it is not readily applied to the theoretical evaluation of the performance sensitivity to the antenna array configuration and its operating environment.

In this paper, the analytical expressions for the average BER of an arbitrary order space diversity receiver is obtained in Section 2, as a function of the covariance matrix of the multipath component signals at the antenna elements. In this way, we can directly include various physical and environmental parameters in the performance assessment, since only the correlation between Gaussian component waves is required in the average BER expression. Section 3 provides the close form expression of the spatial component correlation under a Gaussian angular power profile assumption, taking account of the mutual coupling between antenna elements. In Section 4, the sensitivity of the BER performance to the array characteristics (antenna spacing, diversity order and electromagnetic coupling) and the operating environment (fading, angular spread, mean angle-of-arrival and input branch SNR) is addressed.

2. Correlated Nakagami Channels

2.1. Channel model

Considering an M-branch diversity receiver, the received baseband signal on kth branch can be written as

\[ r_k(t) = s(t) \cdot A_k e^{j \phi_k} + n_k(t), \quad k = 1, 2, ..., M \]  \hspace{1cm} (1)

where \( s(t) \) is the transmitted signal, and \( n_k(t) \) is i.i.d. white Gaussian noise with zero mean and one-side spectral density \( N_0 \). The phase \( \phi_k \) is uniformly distributed over the range \([0,2\pi)\). \( A_k \) is Nakagami distributed signal envelope with pdf given in [1]
\[ p_{\tilde{\kappa}}(A_k) = \frac{2}{\Gamma(m_k)} \left( \frac{m_k}{\Omega_k} \right)^{m_k/2} \cdot A_k^{2m_k-1} \cdot e^{-\frac{m_k A_k}{\Omega_k}} \]  

(2)

where \( \Gamma(\cdot) \) is the Gamma function, \( \Omega_k = A_k^2 \) is the average power on \( k \)th branch, and \( m_k \geq 1/2 \) is the fading parameter, with the special case \( m_k = 1 \) corresponding to the Rayleigh distribution.

In the mobile radio environment, it is reasonable to suppose that the received signal on the \( k \)th branch is superimposed by a large number of multipath signals. All component signals can be treated as \( m \) independent groups: in each group, there are \( n_j \) (\( j = 1, 2, \ldots, m_k \)) unresolvable "subpath" signals \( r_{k,j}^{(n_j)} \), which have almost identical amplitude and phase; and the sum of these "subpath" signals in each group forms the \( j \)-th "resolved" multipath component signal, i.e.

\[ r_{k}^{(j)} = \sum_{n_j} r_{k,j}^{(n_j)} = R_{k}^{(j)} \cdot e^{i\Phi_k} = x_{k}^{(j)} + iy_{k}^{(j)} \]  

(3)

Assuming the set of \( n_j \) is large, then by invoking the Central Limit Theorem, both \( x_{k}^{(j)} \) and \( y_{k}^{(j)} \) can be well approximated by independent Gaussian distributed random variables, thus the amplitude \( R_{k}^{(j)} \) is Rayleigh distributed. The received signal power on \( k \)th branch can be expressed as

\[ A_k^2 = \sum_{j=1}^{m_k} |x_{k}^{(j)} + iy_{k}^{(j)}|^2 = \|r_k\|^2 \geq 0 \]  

(4)

where \( r_k = (x_k^{(1)}, x_k^{(2)}, \ldots , x_k^{(m_k)})^T \), \( \| \cdot \| \) represents the Euclidean norm.

Assuming flat fading and perfect knowledge of the channel, the instantaneous SNR \( \gamma \) at the output of the MRC combiner is given by [14]

\[ \gamma = \frac{E}{N_0} \sum_{k=1}^{M} A_k^2 = \frac{E}{N_0} \sum_{k=1}^{M} \gamma_k \]  

(5)

where \( \gamma_k = (E_{s}/N_0) \cdot A_k^2 \) is the instantaneous input SNR per symbol for each branch.

Further assuming each branch experiences the same extent of fading, i.e. \( m_k = m \) for \( k = 1, 2, \ldots, M \), the general expression for the characteristic function of \( \gamma \) is obtained from [12] in term of the covariance matrix, \( \Lambda \), of Gaussian component correlation coefficients, i.e.

\[ \Phi_{\gamma}(t) = \left| I_d - \frac{it}{m} \cdot H \cdot \Lambda \right|^{-m} \]  

(6)

where \( H = \text{diag} \{ \bar{\gamma}_1, \bar{\gamma}_2, \ldots, \bar{\gamma}_M \} \), with \( \bar{\gamma}_k = \frac{E_s}{N_0} \cdot \bar{A}_k^2 \) being the average input SNR per symbol for the \( k \)th branch, and

\[ \Lambda = \begin{bmatrix} 1 & B_{12} & B_{13} & \cdots & B_{1M} \\ B_{21} & 1 & B_{23} & \cdots & B_{2M} \\ B_{31} & B_{32} & 1 & \cdots & B_{3M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B_{M1} & B_{M2} & B_{M3} & \cdots & 1 \end{bmatrix} \]  

(7)

where \( B_{ij} = b_{ij} + ib_{ij} \) and

\[ b_{ij} = \frac{E(x_i x_j)}{\sqrt{E(x_i x_i) \cdot E(x_j x_j)}}, \quad b_{ii} = b_i \]  

(8a)

\[ \beta_{ij} = \frac{E(x_i y_j)}{\sqrt{E(x_i x_i) \cdot E(y_j y_j)}}, \quad \beta_{ii} = -\beta_i \]  

(8b)

are referred as "Component Correlation Coefficients", and the asterisk indicates the complex conjugate.

2.2. BER performance

The average error probability in the presence of fading is obtained by averaging the conditional error probability over the pdf of \( \gamma \), i.e.,

\[ P_e = \int_0^{\infty} P(e | \gamma) \cdot p(\gamma) \cdot d\gamma \]  

(9)

The conditional error probability for coherent binary phase-shift-keying (CBPSK) is given by [14], i.e.

\[ P(e | \gamma) = Q(\sqrt{2\gamma}) \]  

where \( Q(x) \) is the Gaussian Q-function. Using the alternative representation of \( Q(x) \) given in [8], the average BER can then be written as

\[ P_e = \frac{1}{\pi^2} \int_0^{\infty} \int_0^{\infty} \exp \left( -\frac{\gamma}{\sin^2 \vartheta} \right) d\theta \cdot p(\gamma) d\gamma \]  

(10)

The above expression only requires an integral with finite limits, and it is easy and accurate to use numerical integration tools in Maple or Matlab software package to evaluate the results. By setting \( M=2 \), the average BER for the dual diversity receiver is obtained as

\[ P_e = \frac{1}{\pi} \int_0^{\pi^2} \int_0^{\pi^2} \left[ 1 + \frac{\gamma_{1} \gamma_{2}}{m \cdot \sin^2 \vartheta} + \frac{\gamma_{1} \gamma_{2}}{m \cdot \sin^2 \vartheta} \right]^{-m} d\theta \]  

(11)

3. Spatial Correlation Coefficients

The compact receiver antenna is modeled as a linear array of \( M \) vertical omni-directional antennas, e.g. dipoles or monopoles, with the horizontal separation \( d \) between antenna elements. Fig. 1 shows the geometrical model of antenna array, \( q_0 \) is the mean direction of arrival, \( \sigma_0 \) is the angular spread.
Fig. 1 Geometry Model of Linear Antenna Array

The azimuthal angle-of-arrival is assumed to be Gaussian distributed, i.e.

\[ p(\varphi) = \frac{K}{\sqrt{2\pi} \sigma_{\varphi}} e^{-\frac{(\varphi - \varphi_0)^2}{2\sigma_{\varphi}^2}}, \varphi \in [-\varphi_0, \varphi_0 + \varphi_0] \] (12)

where \( K \) is the normalization factor to make \( p(\varphi) \) a physical density function.

Assuming the fading of the amplitude and phase are statistically independent, and plane-wave arrival, we have the normalized component cross-correlation [4]

\[ B_{kl} = \frac{1}{PK} \int_{0}^{\frac{\pi}{2}} \int_{0}^{\frac{\pi}{2}} R_k R_l \cdot p(\varphi) d\varphi = \int_{0}^{\frac{\pi}{2}} e^{i(2\pi \lambda \frac{d}{\lambda} \sin \theta) \cdot \varphi} \cdot p(\varphi) d\varphi \] (13)

where \( B_{kl} \) is defined in (8), and \( P_k \) is the average power of a component signal received on the \( k^{th} \) branch

\[ P_k = \frac{\rho_0}{d_k} \int_{0}^{\frac{\pi}{2}} |R_k(\varphi)|^2 \cdot p(\varphi) d\varphi, \quad k = 1,2,\ldots,M \] (14)

Let \( z = 2\pi \frac{d}{\lambda} \), the closed-form expressions for the component correlation coefficients are given in [12]:

\[ b_{kl} = J_0((k-l)z) + 2\sum_{n=1}^{\infty} J_{2n}(k-l)z \cdot \cos(2n\varphi_0) \cdot e^{-z^2 \sigma_{\varphi}^2} \cdot Re \left[ \text{erf} \left( \frac{\pi + i(2n+1)\varphi_0}{2 \sigma_{\varphi}} \right) \right] \] (15)

\[ \beta_{kl} = 2\sum_{n=1}^{\infty} J_{2n}(k-l)z \cdot \sin((2n+1)\varphi_0) \cdot e^{-z^2 \sigma_{\varphi}^2/2} \cdot Re \left[ \text{erf} \left( \frac{\pi + i(2n+1)\varphi_0}{2 \sigma_{\varphi}} \right) \right] \] (16)

where \( \text{erf}(a+ib) \) is the complex-input error function.

In above analysis, it was assumed that the receiving antenna array only passively sampled the incident fields spatially. But previous results in [16] and [7] indicate that for small antenna spacing \((d < 0.5\lambda)\), the coupling effect is significant due to antenna elements’ re-radiating the incident fields, so mutual coupling (MC) should be taken into account in the performance analysis of a compact receiver.

Considering the \( M \)-element array as an \( M+1 \) terminal linear bilateral network responding to an outside source, and assuming each antenna is terminated with a known load impedance \( Z_L \) [15] gives the following relationship between two sets of received voltage \( V \) and \( S \), with and without mutual coupling, respectively

\[ Y = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_M \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \ldots & \frac{Z_{1M}}{Z_L} \\ \frac{Z_{21}}{Z_L} & \frac{Z_{22}}{Z_L} & \ldots & \frac{Z_{2M}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{M1}}{Z_L} & \frac{Z_{M2}}{Z_L} & \ldots & \frac{Z_{MM}}{Z_L} \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_M \end{bmatrix} \]

where \( Y \) is referred as the array admittance matrix, depending only on the self-impedance \( Z_{kk} \) and mutual impedance \( Z_{kl} \) normalized by the load impedance. It acts like a transforming matrix, transforming the open circuit element voltages to the terminal voltages.

Applying the pattern multiplication method to a dual-branch Uniform Linear Array of half-wave dipole or monopole, for which [15] provides the closed-form expressions of the self-impedance and mutual impedance, the source vector obtained with coupling can be written as

\[ V(\varphi) = Y \cdot S(\varphi) = \begin{bmatrix} a \\ b \end{bmatrix} \begin{bmatrix} g_1(\varphi, \theta) + 1 \\ g_2(\varphi, \theta) \cdot e^{i(2\pi \lambda \sin \theta) / \lambda} \end{bmatrix} = \begin{bmatrix} a + b \cdot e^{i\varphi} \\ b + a \cdot e^{i\varphi} \end{bmatrix} \] (18)

where \( Y = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \), \( \varphi \) and \( \theta \) are the azimuthal and elevation angle, respectively; \( g_1(\varphi, \theta) \) is a vector function proportional to the electrical field and represents the radiation pattern of the identical antenna element. We restrict our interest to the horizontal plane \( \theta = \pi/2 \) only, and for omni-directional antennas \( g_1(\varphi, \pi/2) = 1 \).

Substituting the expression of \( V_k(\varphi) \) into (14), and noticing that integral terms are the same as the one in (13) with \( k-l = 1 \), we get

\[ P_1 = |a|^2 + |b|^2 + 2b_1 \cdot \Re e(ab^*) + 2\beta_{12} \cdot \Im e(ab^*) \] (19)

Similarly, we have

\[ P_2 = |a|^2 + |b|^2 + 2b_2 \cdot \Re e(ab^*) - 2\beta_{12} \cdot \Im e(ab^*) \] (20)
and the normalized power correlation can be written as

\[
\rho_m = \frac{1}{P_1 P_2} \left| \int_{\pi} r_1(\phi) \cdot r_2(\phi) \cdot p(\phi) d\phi \right|^2
\]

\[
= \frac{1}{P_1 P_2} \left| \langle |a|^2 + |b|^2 \rangle \cdot \beta_{12} + 2 \cdot \text{Re}(ab^*) \cdot j \cdot \langle |a|^2 - |b|^2 \rangle \cdot \beta_{12} \right|^2
\]

(21)

The results in next section show that this effect reduces the spatial correlation, and improves BER performance.

4. Sensitivity Analysis

In this section, we present some numerical examples to illustrate the impacts of the antenna configuration and the operating environment (i.e., fading, the nominal AOA, angular spread and mutual coupling) on the BER performance of the compact receiver in different Nakagami-m fading channels.

4.1. Influence of the fading parameter

Fig. 2 plots the average BER of dual diversity (M=2) MRC receiver versus the average input branch SNR, broadside (ϕ₀ = 0°) receiving, the antenna spacing 1/4λ, and fixed angular spread σₐ=60°, in different fading channels. The cases for no diversity (M=1) are shown for comparison and to calculate the diversity gain. As expected, the average BER performance improves with increased fading parameter m, and in more severe fading channel, a larger diversity gain is obtained. For example, at Pₛ = 10⁻², the diversity gain is 16dB for m=0.5, the diversity gain decreases to 7.4dB for m=1.

4.2. Diversity order

Fig. 3 shows the average BER for M=1, 2, 3 branches, broadside receiving, with the antenna spacing 1/4λ, fixed angular spread σₐ=60°, in m=1 & m=3 fading channels. We can see that the diversity gain obtained by increasing from M=1 to M=2 is dominant. It also shows that at large values of SNR, the BER curves become linear with the input branch SNR, and the asymptotic slope of \( \log_{10} P_r \) versus \( \log_{10} (\text{input branch SNR}) \) is \(-mM\) [17]. For example, the curve for m=1, M=3 has the same slope as that for m=3, M=1. This means the BER performance improvement for a low diversity order receiver in the severe fading channels is the most significant.

4.3. Sensitivity to the angular spread

Fig. 4 shows the BER versus the angular spread at fixed SNR and two extreme mean AOA cases in m=1 fading channel. For broadside, the average BER has a noticeable degradation when the angular spread is decreased below 25°; while for end-fire (ϕ₀ = 90°), there is a fast degradation as the angular spread decreasing from 60°.

4.4. Sensitivity to the mean AOA

Fig. 5 illustrates the sensitivity of BER performance to the mean AOA in different fading channels, with fixed branch SNR = 20dB. We can see the gradual degradation when the mean AOA deviates from the broadside direction, but the degradation is relatively small in the whole range [0, π/2]. Regardless of the angular spread, less severe fading channels create a large variation when the mean AOA changing from broadside to end-fire. For severe fading channel, the large angular spread case is less sensitive to AOA, while for less severe fading channel (m ≥ 3), small angular spread cases are less sensitive to AOA.

4.5. Effect of unbalanced branch SNR

Fig. 6 shows that unequal branch SNR has a small effect on the average BER performance as long as the sum of average input SNR is equal, i.e. \( \gamma_1 + \gamma_2 = 42dB \) for M=2, and \( \gamma_1 + \gamma_2 + \gamma_3 = 42dB \) for M=3. However, there is a significant difference when the total SNR is different. For example, the BER performance greatly improves when \( \gamma_1 + \gamma_2 = 44dB \) for M=2, and \( \gamma_1 + \gamma_2 + \gamma_3 = 45dB \) for M=3.

4.6. Mutual coupling’s effect

Figs. 7, 8 plot the power correlation versus the angular spread at different mean AOAs, for the antenna spacing of 0.1λ and 0.5λ, by numerically evaluating (21). It is evident that the effect of coupling can be neglected for the antenna separation larger than a half wavelength. And when the mutual coupling is taken into account, the spatial correlation is greatly reduced for applications with large angular spreads.

Fig. 9 shows the BER versus the antenna spacing, with fixed input branch SNR=20dB, broadside receiving, angular spread σₐ=60°, in m=0.8, m=1 and m=1.5 Nakagami fading channels, respectively. When the antenna spacing is decreased from 0.5λ to 0.1λ, the BER increases by approximately five-fold and 10-fold, for m=0.8 and m=1.5, respectively. However, if the effect of mutual coupling is included, even for antenna separation \( d = 0.1λ - 0.25λ \), the BER performance degradation is very small. Therefore, applying spatial diversity on mobile terminals is feasible, and can act as an important complement to adaptive arrays at the base stations, where the reverse link currently constitutes the limit to potential capacity gains.
5. Conclusion

This work extends previous work [5]-[7] investigating the effect of the spatial correlation on the average BER performance of a compact antenna array in Rayleigh fading to the Nakagami fading environment. The formulation does not require the knowledge of the spatial covariance matrices as in [10] and [11]. The numerical results clearly illustrate the sensitivity of the performance of the compact array receiver to the fading parameter, diversity order, unbalanced branch SNR, antenna spacing, mean AOA, angular spread and mutual coupling. For $d=1/4\lambda$, broadside reception, there is negligible degradation as long as angular spread $\sigma_p > 25^\circ$, while for the end-fire case, a larger angular spread ($\sigma_p > 60^\circ$) is needed. In addition, when mutual coupling was taken into account and using omni-directional antenna as array elements, the correlation coefficients decreased for $d=0.1\lambda$, which is a favorable result for applying spatial diversity receiver in handheld terminals.

References


Fig. 2 BER comparison of dual-diversity and no diversity, in different $m$ fading channels, with $d=1/4\lambda$, broadside, angular spread $\sigma_p=60^\circ$.

Fig. 3 Effect of diversity order in $m=1$ and $m=3$ fading channel, broadside, antenna spacing $d=1/4\lambda$, angular spread $\sigma_p=60^\circ$. 
Fig. 4 BER versus angular spread with $1/4\lambda$, antenna separation in $m=1$ Nakagami fading channel (Rayleigh).

Fig. 5 Effect of AOA on average BER for different angular spread $\sigma_\theta = 10^\circ$ & $60^\circ$, and in different fading channels.

Fig. 6 Effect of unbalanced branch SNR of dual and triple diversity receiver in $m=0.8$ fading channel.

Fig. 7 Spatial correlation vs. angular spread, antenna spacing $d=0.1\lambda$, with and without the presence of mutual coupling.

Fig. 8 Spatial correlation vs. angular spread, antenna spacing $d=0.5\lambda$, with and without the presence of mutual coupling.

Fig. 9 BER versus antenna spacing in different fading channels, angular spread $\sigma_\theta = 60^\circ$, with and without the presence of mutual coupling.