

# DIFFERENTIAL SPACE-TIME CODING WITH OFFSET QUADRATURE PHASE-SHIFT KEYING

Adam L. Anderson, James R. Zeidler\*

University of California, San Diego  
Department of Electrical and Computer  
Engineering  
La Jolla, CA 92093-0407

Michael A. Jensen†

Brigham Young University  
Department of Electrical and Computer  
Engineering  
Provo, UT 84602

## ABSTRACT

Differential space-time coding (DSTC) has been well established as a precoding technique that allows maximum likelihood detection of the received symbols without requiring knowledge of channel state information. This type of detection is useful for highly time-varying channels where the resulting cost of channel estimation is high. However, to date, this approach has not been developed for the commonly-used offset modulations that offer improved spectral performance in the presence of nonlinear transmit power amplifiers. This paper illustrates modifications to the DSTC detection strategy for offset quadrature phase shift keying. Furthermore, it is shown that while traditional DSTC achieves a reduced transmission rate, the combination with offset modulation allows full rate transmission if desired.

## 1. INTRODUCTION

Space-time block codes, such as the code proposed by Alamouti [1], have received considerable attention as practical and efficient communication strategies for multiple-input multiple-output (MIMO) systems. While these approaches provide excellent performance in multipath fading environments, they require estimation of the channel state information (CSI), a process that becomes impractical for rapidly varying channels. New space-time codes have therefore been developed that allow MIMO transmission over fading channels and do not require receiver CSI for detection. For example, [2] presents a scheme for creating large orthogonal code matrices that enable block detection at the receiver without CSI. In [3] and [4] a differential approach to unitary block transmission is taken that allows for smaller, simpler code creation with the same absence of CSI at the receiver.

\*This work is supported by, or in part by, the U. S. Army Research Office under the Multi-University Research Initiative (MURI) grant #W911NF-04-1-0224.

†Dr. Jensen's work is also supported, in part, by the abovementioned MURI grant.

Though attractive in principle, such differential space-time coding (DSTC) schemes suffer from a lower data rate [3], [5] than coherent codes. For example, Alamouti's code will transmit roughly 1.7 times the amount of data as the  $SL_2(F_5)$  differential code shown in [6] when using the same modulation symbols. This suggests that training (or other methods of CSI acquisition) would need to occupy a significant amount of transmission time before the differential code would achieve higher throughput than Alamouti's code.

Another interesting point regarding these techniques is that most of them, differential or otherwise, have assumed non-offset modulation as the underlying basis for code transmission. Many wireless systems, however, use offset modulation as it provides improved performance in the presence of nonlinear components (such as power amplifiers). Therefore, examination of space-time codes with offset modulation is an important topic.

In this paper, we examine the DSTC proposed in [3] in the context of offset quadrature phase shift keying (OQPSK). This modification necessitates development of new detection strategies to accommodate the complexities associated with offset modulation. We propose one such detection strategy, and demonstrate that it performs nearly as well as traditional DSTC detection for non-offset modulation. We also show that use of DSTC with offset modulation allows transmission at the full rate enabled by the modulation constellation.

## 2. DIFFERENTIAL SPACE-TIME CODING

DSTC has been presented in various forms in the literature. To adequately judge between different techniques it is useful to first define quantitative metrics that are shared between most approaches. With differential codes, data bits are mapped to elements in a group  $\mathcal{G}$  which contains  $L$  matrices each with dimension  $M$  equal to the number of transmit antennas. We can therefore define the *rate* of the DSTC

$$R = \frac{1}{M} \log_2(L) \text{ b/s/Hz}, \quad (1)$$

which quantifies the amount of transferred data per given amount of time. Further, the probability of incorrectly detecting a transmitted block is directly related to the *diversity product* of  $\mathcal{G}$  given as

$$\zeta_{\mathcal{G}} = \frac{1}{2} \min_{0 \leq l < l' < L} |\det(\mathbf{G}_l - \mathbf{G}_{l'})|^{\frac{1}{M}}, \quad (2)$$

where any group with  $\zeta_{\mathcal{G}} > 0$  is said to have full diversity. Being able to maximize both  $R$  and  $\zeta_{\mathcal{G}}$  for fixed  $M$  is ideal for any differential code.

In [3] Hughes introduces a systematic approach to differential group creation through algebraic cyclic and dicyclic group construction. This technique produces fully diverse groups as well as good data rates for  $M = 2$  antenna systems. In [6] Shokrollahi et al. show the construction of *all* possible fully diverse, unitary differential groups with any number of transmit and receive antennas. They were able to produce high rate, highly diverse codes with the slight disadvantage of atypical modulation schemes (for example, the  $SL_2(F_5)$  code contains symbol values that can best be described as coming from a 60QAM-like constellation which may be unrealistic for pre-existing systems).

This paper will focus on the general approach that Hughes suggests of using cyclic group codes, and more specifically the group created with matrix elements containing QPSK symbols. Hughes' approach commences by defining the group

$$\mathcal{G} = \left\{ \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \pm \begin{bmatrix} j & 0 \\ 0 & -j \end{bmatrix}, \dots, \right. \\ \left. \pm \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \pm \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \right\}, \quad (3)$$

which is an  $L = 8$  group taking values in the QPSK constellation. This group is analogous to the algebraic quaternion group and will be called the quaternion code when associated with DSTC. To send a block  $\mathbf{G}_k$ , we use the *standard encoding equation*

$$\mathbf{C}_k = \mathbf{C}_{k-1} \mathbf{G}_k, \quad k = 1, \dots, K, \quad (4)$$

where  $\mathbf{G}_k$  represents a matrix taken from group  $\mathcal{G}$  and  $\mathbf{C}_k$  is the encoded block that is actually transmitted at block time  $k$ .  $\mathbf{C}_0$  is an initialization matrix that is not used to transmit data and can be any  $M \times M$  matrix that satisfies  $\mathbf{C}_0 \mathbf{C}_0^\dagger = M \mathbf{I}_M$ , where  $\mathbf{I}_M$  represents the  $M \times M$  identity matrix and  $\{\cdot\}^\dagger$  is a conjugate transpose. For our analysis we will place a further constraint that  $\mathbf{C}_0$  must be chosen such that  $\mathbf{C}_k \in \mathbf{C}_0 \mathcal{G}$  contains matrix elements in the QPSK constellation. Also, for notational simplicity we define the group  $\mathcal{C} = \mathbf{C}_0 \mathcal{G}$  that contains all possible transmit blocks.

After differential encoding and transmission, received blocks can be expressed as

$$\mathbf{X}_k = \sqrt{\rho} \mathbf{H} \mathbf{C}_k + \mathbf{N}_k, \quad k = 1, \dots, K, \quad (5)$$

where  $\mathbf{H}$  is the complex channel coefficient matrix and  $\mathbf{N}_k$  is additive white Gaussian noise (AWGN), both normalized such that  $\rho$  is the signal-to-noise ratio (SNR) per receive antenna. When no CSI is available at the receiver, the maximum likelihood detector for  $\mathbf{G}_k$ , based only on the two most recent blocks  $\mathbf{X}_k$  and  $\mathbf{X}_{k-1}$ , is

$$\hat{\mathbf{G}}_k = \arg \min_{\mathbf{G} \in \mathcal{G}} \|\mathbf{X}_{k-1} \mathbf{G} - \mathbf{X}_k\|, \quad (6)$$

which represents the *standard decoding equation* for DSTC.

To assist in the description of DSTC with offset modulation in Section 3, it is worthwhile to rewrite (6) in an equivalent but slightly more complex form. Consider the matrix block estimate,  $\hat{\mathbf{G}}_k$ , when used with (4)

$$\hat{\mathbf{G}}_k = \hat{\mathbf{C}}_{k-1}^\dagger \hat{\mathbf{C}}_k, \quad (7)$$

which suggests that an estimate for  $\mathbf{G}_k$  is possible given estimates for  $\mathbf{C}_{k-1}$  and  $\mathbf{C}_k$ . Now define a cost function

$$\lambda_k(l, m) = \|\mathbf{X}_{k-1} \mathbf{C}_l^\dagger \mathbf{C}_m - \mathbf{X}_k\|, \quad (8)$$

where  $l, m = 1 \dots L$  are indices into the matrix group  $\mathcal{C}$  of size  $L$  and should not be confused with the time index  $k$ . Minimizing (8) over  $l$  and  $m$  will produce estimates for  $\hat{\mathbf{C}}_{k-1}$  and  $\hat{\mathbf{C}}_k$  which ultimately result in the desired estimate  $\hat{\mathbf{G}}_k$  from (7). While both detection schemes will produce the same error rate, (6) requires on the order of  $L$  matrix operations per block detected while (8) requires  $L^2$  operations.

### 3. DSTC WITH OFFSET MODULATION

Offset modulation such as OQPSK is often used wherein the transitions of the in-phase and quadrature symbol components are offset by a half-symbol time. This approach never allows transitions directly through the origin in the complex constellation and therefore reduces out of band radiation when used with nonlinear power amplifiers. In this section we examine DSTC with OQPSK. To simplify notation, we will use DSTC and ODSTC to indicate differential space-time coding with QPSK and OQPSK modulation, respectively.

To allow detailed examination of the matrices involved, we will simplify the problem by restricting the system to two transmit and one receive antenna. The extension of this simplified scenario to larger arrays will not be addressed in this paper, although it should be possible to extend the results to this case with additional research. Consider first the transmit block represented as complex-baseband samples which has element values

$$\mathbf{C}_k = \begin{bmatrix} a_k^{(1)} + jb_k^{(1)} & a_{k+\frac{1}{2}}^{(1)} + jb_{k+\frac{1}{2}}^{(2)} \\ a_k^{(1)} + jb_k^{(2)} & a_{k+\frac{1}{2}}^{(2)} + jb_{k+\frac{1}{2}}^{(1)} \end{bmatrix}, \quad (9)$$

$$\mathbf{C}'_k = \begin{bmatrix} a_k^{(1)} + \frac{j}{2}(b_{k-\frac{1}{2}}^{(1)} + b_k^{(1)}) & \frac{1}{2}(a_k^{(1)} + a_{k+\frac{1}{2}}^{(1)}) + jb_k^{(1)} & a_{k+\frac{1}{2}}^{(1)} + \frac{j}{2}(b_k^{(1)} + b_{k+\frac{1}{2}}^{(1)}) & \frac{1}{2}(a_{k+\frac{1}{2}}^{(1)} + a_{k+1}^{(1)}) + jb_{k+\frac{1}{2}}^{(1)} \\ a_k^{(2)} + \frac{j}{2}(b_{k-\frac{1}{2}}^{(2)} + b_k^{(2)}) & \frac{1}{2}(a_k^{(2)} + a_{k+\frac{1}{2}}^{(2)}) + jb_k^{(2)} & a_{k+\frac{1}{2}}^{(2)} + \frac{j}{2}(b_k^{(2)} + b_{k+\frac{1}{2}}^{(2)}) & \frac{1}{2}(a_{k+\frac{1}{2}}^{(2)} + a_{k+1}^{(2)}) + jb_{k+\frac{1}{2}}^{(2)} \end{bmatrix} \quad (10)$$

where  $k$  denotes matrix time index,  $a_k$  and  $b_k$  are the in-phase and quadrature parts of the transmitted symbol respectively, and the superscript denotes from which antenna the symbol is transmitted. For DSTC, this form is retained after transmission through the channel. For ODSTC, however, the offset modulation rearranges the transmission. Using two samples per symbol (to properly capture the offset modulation), the received symbol block can be represented as shown in (10), where the mf correlation function is assumed to be  $R_p(\tau) = 0$  for  $|\tau| > T_s$  and  $R_p(\tau) = \frac{1}{2}$  for  $|\tau| = \frac{T_s}{2}$  as discussed in [7].

The form in (10) reveals the difficulty associated with ODSTC detection. In a SISO system the detector will simply use alternating samples to extract the in-phase and quadrature components to reconstruct the original signals. However, in our  $2 \times 1$  system, the transmissions from each antenna experience a different channel phase, and therefore a simple phase rotation cannot be performed that will allow extraction of the in-phase and quadrature components for *both* transmit antennas. Since the matrix in (10) therefore must be detected as a *non-square* block, the detection algorithm must change.

To overcome these difficulties with ODSTC we will first separate (10) into two  $2 \times 2$  matrices. While several permutations are possible, we recognize that the second and third columns of the matrix are dependent only on the current transmit block  $\mathbf{C}_k$ , while the first and fourth columns have inter-block interference (IBI) from previous and future blocks  $\mathbf{C}_{k-1}$  and  $\mathbf{C}_{k+1}$ , respectively. As a received set with no IBI most closely mimics the behavior of DSTC received matrices, we will use this grouping for formulating the detector.

Notationally, we can perform the above arrangement of mf outputs by defining the two matrix valued functions

$$\begin{aligned} \mathbf{P}(\mathbf{C}_{k-1}, \mathbf{C}_k, \mathbf{C}_{k+1}) &= \\ & \begin{bmatrix} a_k^{(1)} + \frac{j}{2}(b_{k-\frac{1}{2}}^{(1)} + b_k^{(1)}) & \frac{1}{2}(a_{k+\frac{1}{2}}^{(1)} + a_{k+1}^{(1)}) + jb_{k+\frac{1}{2}}^{(1)} \\ a_k^{(2)} + \frac{j}{2}(b_{k-\frac{1}{2}}^{(2)} + b_k^{(2)}) & \frac{1}{2}(a_{k+\frac{1}{2}}^{(2)} + a_{k+1}^{(2)}) + jb_{k+\frac{1}{2}}^{(2)} \end{bmatrix} \\ \mathbf{Q}(\mathbf{C}_k) &= \\ & \begin{bmatrix} \frac{1}{2}(a_k^{(1)} + a_{k+\frac{1}{2}}^{(1)}) + jb_k^{(1)} & a_{k+\frac{1}{2}}^{(1)} + \frac{j}{2}(b_k^{(1)} + b_{k+\frac{1}{2}}^{(1)}) \\ \frac{1}{2}(a_k^{(2)} + a_{k+\frac{1}{2}}^{(2)}) + jb_k^{(2)} & a_{k+\frac{1}{2}}^{(2)} + \frac{j}{2}(b_k^{(2)} + b_{k+\frac{1}{2}}^{(2)}) \end{bmatrix}. \end{aligned} \quad (11)$$

We further define the two sets  $\mathcal{P}$  and  $\mathcal{Q}$  as containing all possible outcomes of  $\mathbf{P}(\cdot)$  and  $\mathbf{Q}(\cdot)$ . It can easily be demon-

strated that the cardinality for  $\mathcal{P}$  and  $\mathcal{Q}$  will be  $L = 128$  and  $L = 8$ , respectively, for the quaternion code. The mf outputs at the receiver can then be divided into the forms

$$\begin{aligned} \mathbf{Y}_k &= \sqrt{\rho} \mathbf{H} \mathbf{P}(\mathbf{C}_{k-1}, \mathbf{C}_k, \mathbf{C}_{k+1}) + \mathbf{N}_{y_k} \\ \mathbf{Z}_k &= \sqrt{\rho} \mathbf{H} \mathbf{Q}(\mathbf{C}_k) + \mathbf{N}_{z_k}, \end{aligned} \quad (12)$$

with  $\mathbf{N}_{y_k}$  and  $\mathbf{N}_{z_k}$  representing appropriately correlated noise samples [8]. Because this rearrangement leads to a form similar to that encountered in DSTC, we can modify the DSTC detector to accommodate this structure.

Some comments are in order regarding the prearranged sets  $\mathcal{P}$  and  $\mathcal{Q}$ . Both sets will ultimately contribute to detection of the encoded block  $\mathbf{G}_k$  which can take on eight different values for the quaternion group. The set  $\mathcal{P}$  is much larger than needed, due to IBI, while  $\mathcal{Q}$  is of the desired size - a direct result of the set creation. An unexpected result of this particular set creation is that  $\mathcal{P}$  has zero diversity from (2) but  $\mathcal{Q}$  is fully diverse. This is satisfying as it suggests that ODSTC enjoys a similar diversity advantage as DSTC.

To motivate the structure of the detector, consider first discarding the entire set  $\mathcal{P}$  and using only  $\mathcal{Q}$  to find estimates for  $\mathbf{G}_k$ . Since  $\mathcal{Q}$  is not a group in the algebraic sense, it is useful to use the alternate detector suggested in (8), or

$$\lambda_k^{(q)}(l, m) = \|\mathbf{Z}_{k-1} \mathbf{Q}(\mathbf{C}_l)^{-1} \mathbf{Q}(\mathbf{C}_m) - \mathbf{Z}_k\|, \quad (13)$$

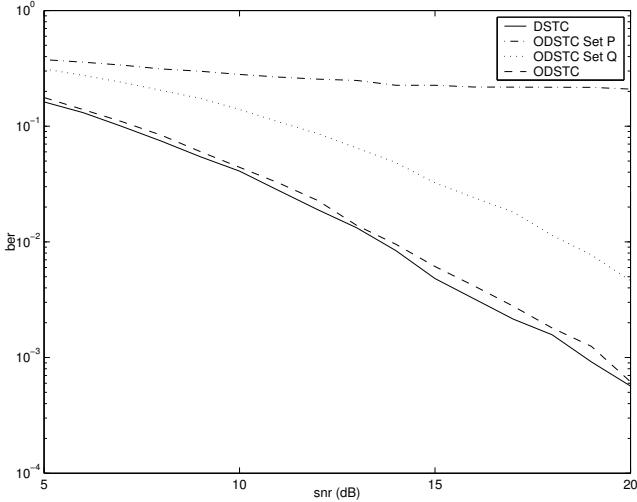
where  $l, m = 0 \dots L-1$ , the superscript in  $\lambda_k^{(q)}$  indicates the set over which the cost function is defined, and the conjugate transpose operator has been changed to an inverse because the elements of  $\mathcal{Q}$  are no longer unitary. For completeness, we could alternately discard  $\mathcal{Q}$  and write a detector using only set  $\mathcal{P}$

$$\lambda_k^{(p)}(l, m) = \min_{l', m'} \|\mathbf{Y}_k \mathbf{P}^{-1}(\mathbf{C}_{l'}, \mathbf{C}_l, \mathbf{C}_m) \mathbf{P}(\mathbf{C}_l, \mathbf{C}_m, \mathbf{C}_{m'}) - \mathbf{Y}_k\|, \quad (14)$$

where any information not specifically coming from the current and previous blocks is removed by minimization.

With these cost functions established, we recognize that using either cost function independently would discard information. It is therefore more intuitive to jointly minimize both cost functions by minimizing their sum

$$\lambda_k(l, m) = \lambda_k^{(q)}(l, m) + \lambda_k^{(p)}(l, m) + \min_{l'} \lambda_{k-1}(l', l), \quad (15)$$



**Fig. 1.** Bit-error performance for ODSTC using either or both of the created sets  $\mathcal{P}$  or  $\mathcal{Q}$ . Also shown is the BER for DSTC.

where  $\lambda_{k-1}$  is included as a residual term as it contains information for the current block. The desired estimate  $\hat{\mathbf{G}}_k$  from (7) is realized by minimizing (15) over  $l$  and  $m$  which produces estimates for  $\hat{\mathbf{C}}_{k-1}$  and  $\hat{\mathbf{C}}_k$ .

Since an analytical approach to arriving at (15) as a possible maximum likelihood detector may prove intractable, we resort to simulation to prove that only a small degradation in performance is observed by using ODSTC versus DSTC. For this simulation, the quaternion group is encoded and transmitted exactly as has been explained with in-phase and quadrature components offset by half a symbol period. The mf outputs at the receiver are arranged as suggested and detection is performed by minimization of the cost functions (13), (14), and (15) respectively, with the latter termed simply ODSTC in the following. Figure 1 shows the performance of these detectors compared to that of standard DSTC. Because the cost function (14) uses on the set  $\mathcal{P}$  which has zero diversity product, this detection approach performs poorly. Detection using (13) alone exhibits improved error performance, although the loss over standard DSTC is significant. However, the combined detection based on (15) produces near optimal results since it exploits the full diversity of  $\mathcal{Q}$  while drawing on the additional information in  $\mathcal{P}$ . The major consequence of using (15) as a detector is the immense search space required for minimization, since an ODSTC transmit group with  $L$  matrices has on the order of  $L^4$  states. We also mention that other simulations were run with different groupings of the mf outputs. However, these groupings did not perform well, arguably due to the fact that the proposed arrangement is the only one that provides one group with full diversity.

#### 4. INCREASED RATE WITH ODSTC

The analysis of Section 3 reveals that under ODSTC, the received blocks lose most of the characteristics of the original transmit group, but retain full diversity. This relaxed constraint at the receiver implies additional freedom in implementation relative to traditional DSTC. In fact, careful examination of (10) reveals that the received ODSTC matrix elements have 8 possible phases, similar to what would be achieved using DSTC with 8PSK symbols. This immediately suggests the possibility of increasing the data rate since the ability to use 8PSK symbols will allow the creation of larger, fully diverse sets [3]. For example, if we relax the constraint of requiring a fully diverse group at the transmitter, we can define an  $L = 16$  element group to replace the quaternion code

$$\mathcal{G} = \left\{ \begin{bmatrix} \pm 1, \pm j & 0 \\ 0 & \pm 1, \pm j \end{bmatrix} \right\}, \quad (16)$$

which consists of all possible diagonal unitary matrices within the QPSK constellation. From (1) it can be seen that this new group will transmit at a rate of 2.0 b/s/Hz (rate 2.0) versus 1.5 b/s/Hz (rate 1.5) for the quaternion code. It should be noted that this group is not fully diverse and would perform poorly in a fading channel if used with DSTC.

However, when used with ODSTC combined with the detection scheme suggested in this paper, the received set  $\mathcal{Q}$  is fully diverse. From Figure 1 and discussion at the end of the last section, we concluded that for ODSTC it is sufficient for either  $\mathcal{P}$  or  $\mathcal{Q}$  to be fully diverse, but not necessarily both. ODSTC, therefore, can exploit a non-diverse group at the transmitter in order to receive differentially encoded blocks while using the offset nature of OQPSK to produce diversity. In this manner a higher rate code is created using the same modulation constellation.

We will now present another possible detector in order to simplify algorithm implementation. Rather than splitting mf outputs into two different sets of  $2 \times 2$  matrices, we create one group  $\mathcal{C}$  of all possible  $2 \times 4$  matrices. For this case, the receive block will take the form

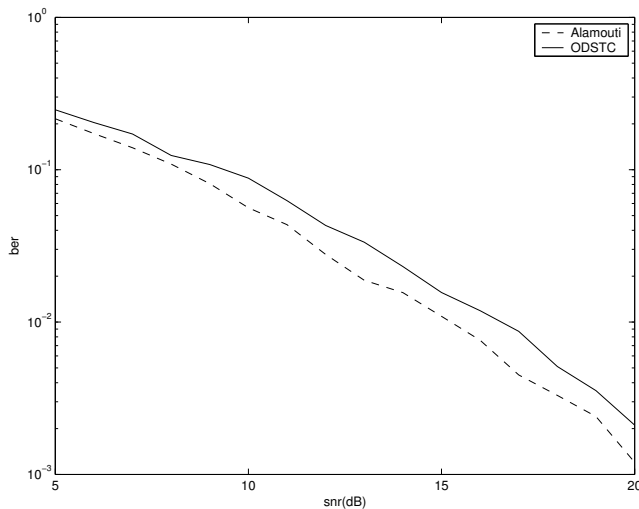
$$\mathbf{X}_k = \sqrt{\rho} \mathbf{H} \mathbf{C}'_k + \mathbf{N}_k \quad (17)$$

where  $\mathbf{C}'_k$  is defined in (10) and  $\mathbf{N}_k$  is the correlated noise. The receiver cost function can now be written as

$$\lambda_k(l, m) = \|\mathbf{X}_{k-1} \mathbf{C}'_l + \mathbf{C}'_m - \mathbf{X}_k\| + \min_{l'} \lambda_{k-1}(l', l), \quad (18)$$

where  $[\cdot]^+$  denotes pseudo-inverse and is used since members of  $\mathcal{C}$  are no longer square.

To demonstrate the performance of this approach, the simulation scenario from Section 3 was used using the transmit group (16) with the detector (18). Figure 2 shows the performance of the rate 2.0 ODSTC group when using (15)



**Fig. 2.** Bit-error performance for ODSTC transmitting at a rate of 2.0 b/s/Hz. Also shown is the rate 2.0 Alamouti code with coherent detection.

as a detector as well as the equivalent rate 2.0 Alamouti code. The rate 2.0 ODSTC code performs well in the simulated fading channel with a slight coding loss over Alamouti's coherent code. A closer comparison of Figures 1 and 2 demonstrates that with ODSTC we can lower the coding advantage of the system in order to increase the overall transfer rate at the cost of even greater complexity at the receiver.

## 5. CONCLUSION

It has been shown that differential space-time codes can be used in systems that also employ offset modulators. The implementation requires modification of the detection algorithm, and while the proposed method was not proven to be optimal, simulations demonstrate that it performs very close to the optimum solution. The major consequence of the approach is significant increase in the detection computational complexity, which could have impact in practical application. Further research is needed to find a cost efficient solution and to demonstrate its optimality. We have also shown that, due to the offset nature of the transmission, a larger, zero diverse group can be differentially encoded at the transmitter that results in a fully diverse set of the same cardinality at the receiver. This adds complexity at the receiver but increases the data rate significantly. The extension of ODSTC to higher order constellations as well as systems using more antennas would be of interest for further research.

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