Adaptive Interference Suppression for CDMA Overlay Systems

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Abstract—It has been proposed that CDMA systems can be assigned to spectral bands which are presently occupied by narrowband users to further increase spectral capacity. Such CDMA overlay systems could provide new options for efficient utilization of the spectrum with minimal disruption to existing narrowband users, especially if adaptive interference suppression techniques are utilized in the spread spectrum receiver. Previous studies have defined the SNR improvement ratio which can be achieved for tone interferers and for narrowband interferers for which the center frequency of the interference is at the carrier frequency of the CDMA signal. In this paper the bit-error-rate (BER) performance of the mobile-to-base link of a CDMA system for a single narrowband user which occupies a significant portion of the CDMA bandwidth is evaluated. It is shown that the narrowband model used in previous studies does not apply in this case, especially for the large, effective, bandwidths which are characteristic of the interferers in the overlay system. The dependence of the BER on the filter order, the bandwidth of the interference, and its center frequency relative to the CDMA carrier frequency are defined. Additionally the increase in BER for a digital implementation of the adaptive suppression filter relative to the optimal Wiener filter is characterized with respect to the adaptive time constant and the quantization errors due to finite wordlength. It is shown that these implementation errors can be made negligible compared to the errors which are characteristic of the optimal Wiener filter. Analytic results are validated by simulation for typical system parameters.

I. INTRODUCTION

Spread spectrum communications are currently under development for mobile communication applications due to their efficient utilization of channel bandwidth, the relative insensitivity to multipath interference, and the potential for improved privacy [1], [2]. In addition to providing multiple accessing capabilities and multipath rejection, spread spectrum communications also offer the possibility of further increasing overall spectrum capacity by overlaying a Code Division Multiple Access (CDMA) network over existing narrowband users [2]. As discussed by Pickholtz, Milstein, and Shilling [2], such CDMA overlay systems would provide new options for the most efficient utilization of the spectrum with minimal disruption to existing narrowband users, especially if signal processing techniques which suppress the narrowband interference are utilized in the spread spectrum receiver. Such narrowband suppression techniques will not only provide improved performance of the CDMA system, but will also allow the performance impact on the existing users to be minimized since the CDMA power level can then be correspondingly decreased [2].

A number of authors have explored the performance of such narrowband interference suppression filters for spread spectrum communications signals [3]–[8]. These studies have concentrated on quantifying a SNR improvement ratio at the filter output and have also obtained the bit-error-rate (BER) performance using Gaussian approximations for the filter output statistics. The SNR improvement ratio has been found for tone interferers [3] and for order one autoregressive (AR1) interferers [4]. In [4] it is assumed that the center frequency of the interferer is at the carrier frequency of the CDMA signal. In commercial applications the interferers are likely to occupy a significant portion of the CDMA bandwidth and to have center frequency which is offset from the carrier frequency of the CDMA signal (Fig. 1). In this paper we examine the performance of the mobile to base link of this system where a single interferer occupies a significant portion of the direct sequence (DS) CDMA bandwidth and may also be placed at any position in the CDMA bandwidth. Effects of the digital implementation of the adaptive suppression filter are characterized as a function of the adaptive time constant and quantization errors. Approximate closed form solutions will be provided for the SNR improvement and BER.

In Section II the CDMA overlay problem is described and the BER performance examined. Section III derives the optimum suppression filter and describes the SNR improvement factor as a function of the interference parameters. Section IV describes the effects of the filter adaptation and quantization effects. Analytical results are validated by simulations for typical system parameters in Section V.

II. PRELIMINARIES

A. CDMA Overlay

First, the basic system model and notation of the CDMA overlay model is introduced and the performance criterion described. The CDMA system is assumed to be a BPSK DS CDMA system which overlays a narrowband BPSK signal (Fig. 2). The transmitted signal from the $k$th user in the CDMA system takes the form:

$$S_k(t) = \sqrt{2P_b}b_k(t)a_k(t)\cos[\nu_0 t + \phi_k]$$

(1)
where $P_c$, $\omega_0$ are the transmitter power and carrier frequency; $\phi_k$ is the phase angle introduced by the $k$th PSK modulator.

We assume that the channel between the CDMA mobile user and the base is a frequency-nonselective Rayleigh fading channel, and is characterized by the parameters $\beta_k$, $\gamma_k$, and $\gamma_k$, which are defined as the gain, delay, and phase of the $k$th signal at the receiver. The gain $\beta_k$ is an independent Rayleigh variable with parameter $\rho = E[|\beta|^2]/2$, while the delay $\gamma_k$ is independent, has a uniform distribution in $[0, T_s]$. The phase $\gamma_k$ is absorbed into the phase of the CDMA modulator under the assumption that the ensemble of CDMA modulator phases is uniformly distributed. The rate of the CDMA information sequence $b_k(t)(1/T_k)$ and the spreading sequence $a_k(t)(1/T_c)$ are related by the processing gain $N$ via $T_k = NT_c$, and the bandwidth (BW) of the CDMA signal is given by $B_k = 2/T_c$.

The received narrowband BPSK interference is assumed to be nonfading, given by

$$J(t) = \sqrt{2} j(t) \cos[(\omega_0 + \delta\omega_0) t + \theta]$$ (2)

where $\delta\omega_0$ is the offset of the BPSK carrier frequency from the CDMA carrier frequency. $J$ and $\theta$ denote the received interference power and phase, respectively. The BPSK information sequence $j(t)$ has bit rate $1/T_j$, where $T_j$ is the duration of one bit.

For $K$ CDMA users, the received signal consists of the independently fading CDMA signals, the interfering narrowband BPSK, and thermal noise, i.e.,

$$r(t) = \sum_{k=1}^{K} [\beta_k S_k(t - \tau_k)] + J(t) + n(t)$$ (3)

where $n(t)$ is additive white Gaussian noise with a two-sided power density $N_0/2$.

Two important quantities of this system are the ratio of the interference BW to CDMA signal BW, $p$, which is given by

$$p = T_c/T_j,$$ (4)

and the ratio of the interfering carrier offset to half of the spread spectrum bandwidth, $q$, where

$$q = \delta\omega_0 T_c/2\pi.$$ (5)

### B. System Performance

At the receiver, as shown in Fig. 3, the CDMA signal is 1) coherently demodulated, 2) sampled at the chip rate $1/T_c$, 3) filtered, and 4) despread, to produce the decision statistic. Without loss of generality, we assume that user 1 is the reference user with $(\phi_1 = 0)$, and the decision statistic $\lambda(m)$, over $N$ time samples, is given by

$$\lambda(m) = \sum_{n=1}^{N} r_f(mN + n)a_1(n)$$ (6)

where $r_f(i)$ is the output of the suppression filter, with coefficients $\{c_1\}$, given by

$$r_f(n) = \sum_{l=1}^{l} r(n-l)c_1$$ (7)

and

$$r(n) = \int_{nT_c}^{(n+1)T_c} r(t)2\cos(\omega_0 t)dt$$ (8)

is the demodulated signal for user 1.

To find the BER we use a recent result described by Wang and Milstein in [9] and derived in [10] which gives the approximate BER for this system employing a suppression filter, with coefficients $\{c_1\}$. For a large number of CDMA users ($K \gg 1$), Wang and Milstein [10] proved that the BER of this system is approximately given by

$$\text{BER} = \frac{1}{2} \left[ 1 - \sqrt{\frac{r_b}{1 + r_b}} \right] \approx \frac{1}{4r_b}$$ (9)

where

$$\frac{1}{r_b} = \left[ \frac{E_b}{N_0} \right]^{-1} \sum_{l} c_l^2 + \left[ \frac{1}{NS} \right] \sum_{l,m} c_l c_m \phi_j (l - m) + \frac{(K - 1)}{N} \left[ \frac{2}{3} \sum_{l} c_l^2 + \frac{1}{3} \sum_{l} c_l c_{l+1} \right]$$ (10)

is the noise-to-signal ratio at the output of the suppression filter divided by the spreading gain $N$. The interfering narrowband
BPSK correlation function is \( \phi_2(l) = 2P_2 \) is the average CDMA signal power, and \( E_b = S T_0 \) is the average energy per bit of the CDMA user.

The three terms which appear in the noise to signal ratio at the output of the suppression filter \( 1/\tau_b \) are the (1) thermal noise, (2) interfering BPSK power, and (3) multiaccess interference power at the output of the suppression filter normalized by the spreading gain \( N \) and the signal power \( S \). It is seen that under the assumption \( K \gg 1 \), the noise to signal ratio above is approximately the output power of the suppression filter \( \xi \) normalized by the energy per bit \( NS \).

Thus

\[
1/\tau_b \approx \frac{\xi}{NS}.
\]

(11)

The approximation being that the self-interference can be neglected for \( K \ll 1 \).

The bracketed portion of the multi-access interference is a result of the differences in the delay \( \tau_b \) introduced by the channel. In a chip-synchronous system when \( \tau_b = 0 \), the multi-access interference is white and the bracketed term is \( \sum C_i \). To simplify our analysis, we will assume a chip-synchronized CDMA system \( \tau_b = 0 \).

From the BER expression (9) the system performance is seen to be highly dependent on the suppression output power and can be determined once the coefficients of the suppression filter are found.

C. Suppression Filter

The suppression filter which minimizes the BER is the Wiener filter [3]. This filter attempts to remove the distortion due to the narrowband interference before despread of the CDMA by making use of the great disparity in the bandwidth of the two signals and notching out the narrowband interference.

The filter may be constructed either by determining, a priori, the input interference and solving for the Wiener filter coefficients, or by using an adaptive algorithm. Due to the often unknown or changing characteristics of the interference, an adaptive algorithm may be more useful in actual implementations. In this paper we will examine a fixed-point implementation of the adaptive line enhancer (ALE) for the suppression filter. This filter employs the Widrow–Hoff least mean square (LMS) algorithm to predict the narrowband interference component and cancel it from the signal. The structure of the ALE filter is shown in Fig. 4. In this configuration the suppression filter coefficients are given by

\[
C_0 = 1; C_{(k+1)} = -w_k, k = 0, \cdots, L - 1
\]

(12)

where the \( w_k \)s are the coefficients of the \( L \) tap prediction filter.

The steady state performance of this filter may be analyzed as a sum of three independent terms [11], [12] consisting of the fixed Wiener filter component, the misadjustment component due to the noise in the gradient estimation process, and the component due to the quantization errors. In Section III we will define the characteristics of the optimal Wiener filter. The misadjustment and quantization characteristics will be defined in Section IV.

III. PERFORMANCE WITH WIENER FILTER

In this section the performance of the fixed Wiener filter is examined. While it is possible to find the optimum filter coefficients for BPSK input, it produces very lengthy results and is not generalizable to arbitrary filter lengths. It is, however, possible to obtain closed form analytical results for arbitrary filter lengths by using Wiener filter solutions which have been previously derived in [13], [14] for signals with rational spectral densities.

A. Second Order Approximation of BPSK Spectrum

In this section the BPSK is modeled as a second order process to account for the two parameters of the BPSK signal; namely the bandwidth \( 1/T \) and the center frequency \( \omega_0 \). The BPSK autocorrelation function is given by [15]

\[
\phi_2(l) = \sigma_2^2 \left( 1 - \frac{|l|}{T} \right) \cos(\omega_0 l), \quad |l|<T
\]

(13)

where \( T = T_2/T_0 \) is the correlation length and \( \omega_0 = 2\pi \Gamma \) is the normalized offset carrier frequency of the narrowband BPSK interferer. This will be approximated by the second-order process with the autocorrelation function given by [13]

\[
\phi_{a2}(l) = \sigma_2^2 e^{-\alpha |l|} \cos(\omega l)
\]

(14)

where \( \alpha \) is the bandwidth parameter and \( \omega \) specifies the location of the interference similar to the \( 1/T \) and \( \omega_0 \) parameters of the BPSK signal. This second order ARMA (2, 1) model has been used in [14] to model narrowband processes in broadband noise.

This approximation is made by equating the autocorrelation function of the BPSK and second-order process in the lag 0, 1, and 2 positions, and solving for \( \omega \) and \( \alpha \):

\[
\phi_{a2}(l) = \phi_2(l), \quad l = 0, 1, 2
\]

(15)

gives

\[
\omega = \arctan \left\{ \sqrt{T_2^2(\omega_0)} \right\}
\]

\[
\alpha = -\log \left\{ \left( 1 - \frac{1}{T} \right) \cos(\omega_0) \sqrt{1 + T_2^2(\omega_0)} \right\}
\]

where

\[
T_2^2(\omega_0) = \tan^2 (\omega_0) \left[ \frac{T(T - 2)}{(T - 1)^2} \right] + 1/(T - 1)^2.
\]

(16)

This essentially approximates the triangular envelope in the BPSK autocorrelation with an exponential envelope.
For many cases of interest, the bandwidth disparity between the BPSK and the CDMA is large (i.e., T large), and the parameters are well approximated by \( T^2(\omega_0) \approx \tan^2(\omega_0) \), and

\[
\omega \approx \omega_0 \quad \alpha \approx 1/T.
\] (17)

These relationships between the parameters of the BPSK and second order process are illustrated in Figs. 5 and 6. In Fig. 5 the bandwidth parameter \( \alpha \) is seen to approximate \( 1/T \) closely for \( T > 10 \). Similarly in Fig. 6 the frequency parameter \( \omega \) closely approximates \( \omega_0 \) of the BPSK signal for large values of \( T \). A sample spectrum of a BPSK signal and a spectrum of the second order process which approximates its spectrum is shown in Fig. 7. Here it is seen that the second order process essentially models the main lobe of the BPSK spectrum, and is normalized to provide the same power over the input spectrum. A comparison of results obtained by use of numerical solution for the BPSK input and the analytical results obtained using the above second order approximation is provided in Appendix B-A in terms of the output power ratios.

B. Narrowband Interference in White Noise

With the second order model of the BPSK, the overlay system can now be expressed with a rational power spectral density. The Wiener filter coefficients are derived by first finding the power spectral density of the input process and then solving a set of matrix equations, as described in Appendix B-A.

\[
\phi_{xx}(l) = \sigma^2_0 e^{-\alpha |l|} \cos(\omega l) + (1 + \sigma^2_0) \delta(l)
\] (18)

where, without loss of generality, we have normalized the input power by the total CDMA power. To relate the normalized interference power \( \sigma^2_0 \) and the normalized white noise power \( \sigma^2_0 \) to the actual system parameters, the input correlation of the demodulated signal at the filter input is derived in Appendix A. From this it is found that \( \sigma^2_0 = [J\phi_F(\omega)]/(S(K + 1)) \), and

\[
\sigma^2_0 = NE_0/(N_0(K + 1)), \quad \text{where } \phi_F(\omega) \text{ is a scaling factor introduced by the demodulation of a frequency offset BPSK defined in (A.6)}.
\]

The power spectrum of this model takes the form

\[
S_{xx}(z) = G \frac{(z - e^{-z_1})(z - e^{-z_1})(z - e^{-z_1})(z - e^{-z_1})}{(z - e^{-p_1})(z - e^{-p_1})(z - e^{-p_2})(z - e^{-p_2})}
\] (19)

where \( G \) is a normalizing constant and

\[
p_{1,2} = \pm \alpha + j\omega \quad z_{1,2} = \pm \beta + j\psi
\]
denote the location of the poles and zeros of the spectrum. Here the poles are determined by the bandwidth and frequency of the interferer, and the zeros can be found by taking the \( Z \)-transform of the input autocorrelation (18) and factoring the expression into the form defined by (19) to determine the poles.
and zeros. Denoting the signal-to-noise ratio at the input to the prediction filter as

$$SNR_j = \frac{\sigma_j^2}{1 + \sigma_i^2}.$$  \hspace{1cm} (20)

the zeros of the spectrum are related to the input second-order process parameters \((SNR_j, \alpha, \omega)\) by

$$\beta = \cosh \left( \sqrt{u} \right) \ln \left( \sqrt{u} + \sqrt{u - 1} \right)$$

$$\psi = \cosh \left( \frac{A}{\sqrt{u}} \right).$$ \hspace{1cm} (21)

where

$$u = B + \sqrt{B^2 - A^2}$$

$$A = \left( \cosh \alpha + \cosh \alpha \frac{SNR_j}{2} \sinh \alpha \right) \cos \omega$$

$$B = \left( \cosh 2\alpha + SNR_j \sinh 2\alpha + \cos 2\omega + 2 \right)/4.$$  \hspace{1cm} (22)

For very narrow bandwidths \((\alpha \ll 1, \sinh \alpha \approx 0, \cosh \alpha \approx 1)\) this reduces to

$$\beta = \cosh \left( \cosh \alpha + \frac{SNR_j}{2} \sinh \alpha \right)$$

$$\psi = \omega.$$ \hspace{1cm} (23)

which agrees with the expressions for the filter bandwidth and center frequency of the optimum filter with a single narrowband input described in [13]. The parameters \(\beta\) and \(\psi\) are the bandwidth and notch frequency of the suppression filter. The filter bandwidth \(\beta\) is plotted in Fig. 8 for both the actual values and the narrowband approximation. It is seen that the narrowband approximation produces a \(\beta\) larger than the actual, and the filter bandwidth \(\beta\) associated with a particular interference bandwidth \((\alpha)\) is always larger, and increases with the SNR. This dependence on the SNR can be intuitively seen as the filter’s determination of the effective bandwidth of the signal. This is illustrated in terms of the filter transfer function in Fig. 4 of [13]. As a function of the carrier offset \((\omega)\) the \(\beta\), in Fig. 9, is seen to reach minimum at \(\omega = 0\) and a maximum at \(\omega = \pi/2\). A pole and zero plot of the first quadrant for various input frequencies as the bandwidth is increased from \(\alpha = 0.001\) to 0.4 is shown in Fig. 10 to illustrate the breakdown of the narrowband approximation \((\psi \approx \omega)\) especially at small \(\omega\)’s.\(^1\) This shows that for small values of \(\omega\) the zeros deviate from the radial line of the pole and enter the real axis. An upper bound on the region for which the narrowband approximation is valid can be determined by plotting the value of \(\omega\) for which the zero is real for a given bandwidth and SNR (Fig. 11). In Fig. 11 the zeros are complex in the region to the left of the line. In this region the narrowband approximation may be valid. For the CDMA overlay system, where both the CDMA and narrowband BPSK must coexist \((SNR_j \gg 1)\), the narrowband approximation generally does not hold for the larger bandwidths \((\alpha \gg 0)\).

\(^1\)The poles and zeros were calculated for 260 values of \(\omega\) between \(\alpha = 10^{-3}\) and \(\alpha = 0.4\) for each value of \(\omega\). The pole and the zero at \(\alpha = 10^{-3}\) and \(\alpha = 0.4\) are shown for each value of \(\omega\). The locations of the remaining 258 values of \(\omega\) are used to generate the plot.

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**Fig. 8.** Bandwidth parameter \(\beta\).

**Fig. 9.** Dependence of \(\beta\) on \(\omega\).

Therefore the locations of the zeros must be calculated from (21).

Since the case of interest is when the BPSK occupy a significant fraction of the CDMA spectrum, the assumption \(e^{-\Delta L} \ll 1\) is made to simplify the expressions for the optimum filter coefficients. This is justified by the fact that \(\beta \gg \alpha\), and the approximation can be easily satisfied by moderately sized \(L\). From Appendix B-B the optimum filter coefficients are found to be

$$w^*(k) = 2|B^*|e^{-\beta k}\cos(\psi k + \theta)$$ \hspace{1cm} (24)

where for \(\Delta = 1\) the \((\theta, |B^*|)\) simplify to

$$\theta = \arctan \left( \frac{\sinh (\beta - \alpha)}{\sin \psi \left( \cos \omega - e^{-(\beta - \alpha)} \cos \psi \right) - \cot \psi} \right)$$

$$|B^*| = \frac{e^{-\alpha} \sinh (\beta - \alpha)}{\cos (\psi - \theta)}. \hspace{1cm} (25)$$

---

\(^1\)
Similar expressions of $(r, \theta)$ for a general $\Delta$ found in Appendix B-A. However, for the suppression of narrowband interference in white noise, $\Delta = 1$ is the desired delay since increasing $\Delta$ only decorrelates the signal and reduces the SNR improvement [13].

With the optimum filter coefficients the minimum BER (9) can now be expressed analytically as a function of the filter output power.

C. SNR Improvement Factor

In this section the reduction in the interference power is examined. Since the BER is related to the suppression filter output by a scaling factor, this SNR improvement factor is directly related to the system BER performance. The SNR improvement factor is the ratio of the interference power at the filter input versus the non-CDMA power at the output of the suppression filter. In the CDMA system modeled by (18), this is given by the ratio

$$\eta = \frac{\sigma_n^2 + \sigma_r^2}{\xi_{\min} - 1}$$

(26)

where $\xi_{\min}$ is the minimum prediction error [4]

$$\xi_{\min} = \sigma_n^2 + \left(1 + \sigma_n^2\right) \sum_{l=0}^{L-1} w^*(l) w(l)$$

(27)

$$\quad + \sum_{l=0}^{L-1} \sum_{m=0}^{L-1} w^*(l) w(m) \phi_2(l-m).$$

Equivalently $\xi_{\min}$ is the output power of the filter when configured as an ALE.

Using the Wiener filter weights developed in Section III-B, we examine the SNR improvement of a system employing a suppression filter. Without loss of generality, $\sigma_n^2 = 0$ is assumed to emphasize the suppression characteristics of the filter. When $\sigma_n^2 \neq 0$ this simply translates into more users in the system.

The SNR improvement factor as a function of the interference carrier offset $\omega$ is plotted in Fig. 12 for a wide range of interference bandwidths. It is seen that for $\beta L$ large the SNR improvement factor reaches a minimum at $\omega = \pi/2$ (as well as $3\pi/2$). This is reasonable since, as seen by the filter, odd lags in the correlation $\phi_{xx}(\text{odd}) = 0$ at these offsets, which in turn degrades the ability of the filter to predict the interference.

We can now define a loss in SNR improvement when the interference carrier coincides with the CDMA carrier and when it is offset by $\omega = \pi/2$ (or $3\pi/2$). The critical difference in these cases is that at $\omega = \pi/2$ all the odd lag correlations $\phi_{xx}(\text{odd}) = 0$. For $L \rightarrow \infty$ the SNR improvement factor at

2 The $r = 10^{-5}$, $L = 10$ is plotted numerically as the approximations used in the Wiener filter derivation do not hold. The apparent oscillatory behavior is similar to the Gibbs phenomenon where the $\eta$ here consists of a finite sum of near sinusoidal terms due to $\omega L \ll 1$. Approximate Wiener coefficients in this case are found in [13].
\( \omega = 0 \) simplifies to the form found in [4] for the AR1 process

\[
\eta_0 = \frac{\sigma_n^2 + \sigma_\eta^2}{\sigma_n^2 + \sigma_\eta^2 \left( 1 - a^2 \right)}
\]  
(28)

and from Appendix C

\[
\eta(\pi/2) = \frac{\sigma_n^2 + \sigma_\eta^2}{\sigma_n^2 + \sigma_\eta^2 \left( 1 - a^2 \right)}
\]  
(29)

at \( \omega = \pi/2 \) where \( b_1 = e^{-\alpha \sigma_n^2} \) and \( a = e^{-\alpha} \). The \( b_0 \) and \( b_{\pi/2} \) are found from (21) with \( \omega = 0 \) and \( \omega = \pi/2 \), respectively. The similarity between (28) and (29) is not surprising since at \( \omega = \pi/2 \) the autocorrelation of the second order process is essentially the same as an AR1, with the odd terms \( \phi_{xx}(odd) = 0 \).

The baseline SNR improvement \( \eta_0 \) when the carrier of the interference and CDMA coincide, \( \omega = 0 \), is shown in Fig. 13 for an infinitely long filter. This is plotted as a function of 1/SNR, since the improvement is of interest in the CDMA system, and 1/SNR is proportional to the SNR/chip for the CDMA system. It is seen that as the interference bandwidth decreases or as the interference SNR increases, a larger SNR improvement is possible since the interference is becoming more deterministic and predictable. Note also that the SNR gain is a nonlinear function of the input SNR and \( \alpha \), since a portion of the signal energy is removed by the variable bandwidth suppression filter. The decrease in SNR improvement, when the interference carrier is offset by \( \omega = \pi/2 \), is plotted in Fig. 14. There are three notable aspects in this plot when the interference carrier offset is moved from \( \omega = 0 \) to \( \omega = \pi/2 \).

1) For very narrowband interferences the loss of the odd correlations \( \phi_{xx}(odd) \) reduces the SNR improvement by 1.5 dB regardless of the SNR. Note that this applies only to inputs with nonzero bandwidths. For a tone interference the reduction is 3 dB as found from results in [3]. This apparent discontinuity is due to the fact that nonzero bandwidth interferers the filter tap weights are damped and \( \sigma_n^2(k) \) converges to 0 as \( k \to \infty \). However for the tone interferer the tap weights are sinusoidal [16]. This discrepancy can be explained by considering a very long filter with \( L \) fixed taps. If the filter length does not span the correlation length of the input (i.e., \( 1/\alpha \gg L \)) then the input appears to be sinusoidal and the difference is approximately 3 dB. However, if the filter length does span the correlation length of the input (i.e., \( 1/\alpha \ll L \)) the loss is less than 3 dB as illustrated in Fig. 14. This effect can be seen from \( \alpha = 10^{-5} \) of Fig. 12 as \( L \) is increased from 10 to 500. Also note that for short filter lengths the difference may be much greater than 3 dB as shown for \( \alpha = 10^{-5}, L = 10 \) in Fig. 12.

2) For 1/SNR > 0 dB, increasing the interference bandwidth \( \alpha \) reduces the difference in SNR improvement. This is because the power of the predictable portion of the input \( \sigma_n^2 \) is less than the power of the uncorrelated CDMA signals. Thus when the bandwidth is increased the predictability of both the \( \omega = 0 \) and \( \omega = \pi/2 \) cases approaches 0.

3) For 1/SNR < 0 dB, decrease of the bandwidth \( \alpha \) first increases the SNR improvement by as much as 3 dB. Then as \( \alpha \) is reduced further, the difference converges to 1.5 dB. This can be seen from Fig. 12 as \( \alpha \) is reduced from \( 10^{-1} \) to \( 10^{-5} \).

The observations noted above are also evident in Fig. 12. As seen from Fig. 12 the loss for 1/SNR, \( \alpha = 10^{-6} \) dB at \( \alpha = 0.1, L = 10 \) is 2.02 dB which agrees with the predicted loss seen in Fig. 14. At \( \alpha = 10^{-5} \), this loss is 2.9 dB for \( L = 10 \) which is close to the 3 dB for a sinusoid [3]. Lastly for \( \alpha = 10^{-5} \), when \( L \) is increased to 500 the loss reduces to 1.5 dB as predicted in Fig. 14.

Thus the expected SNR improvement for a general BPSK interferer can be determined by noting its SNR improvement at \( \omega = 0 \) where the exact \( \eta_0 \) is available (Appendix C). Fig. 15 plots the ratio of the input versus output SNR for large filter lengths at \( \omega = 0 \). When the interference carrier is offset the reduction in \( \eta \) is on the order of 3 dB when \( L \) is large. It is seen here that the suppression filter can make
an overlay system viable. For example, with input SNR at 0 dB and interference BW of 1%, the SNR at the output of the suppression filter is between 7 to 9 dB depending on the location of the interference. The nonlinear relationship between the input and output SNR discussed for Fig. 13 is also evident in this plot.

IV. ADAPTATION AND FINITE PRECISION EFFECTS WITH LMS ALGORITHM

In this section we analyze the contributions of the misadjustment noise due to gradient estimation error and quantization errors to the output power. We will show that for large interferences where the filter output is dominated by the interferer, the errors introduced by the adaptive filter do not contribute significantly to the output power or the BER, provided the additional noise introduced by the filter is small. In this section the fixed point LMS is considered to illustrate the effects of misadjustment noise and finite wordlength effects on the resulting BER. The fixed-point LMS algorithm is given by

\[
\begin{align*}
    e_n &= Q_d[d_n] - Q_d[\bar{W}_n X_n] \\
    \bar{W}_{n+1} &= \bar{W}_n + Q_e[\mu \bar{X}_n e_n]
\end{align*}
\]

where \( \bar{W}_n \) is the filter weight vector, \( \bar{X}_n \) is a vector whose components consist of the last \( L \) input samples, and \( d_n \) is the \( (L+1) \)th previous input in the ALE configuration. The algorithm is assumed to be implemented as having: a) input scaled to \( \pm 1 \), b) the data quantized to \( B_d \) bits plus sign bit, c) the filter coefficient quantized to \( B_c \) bits plus sign bit; and d) no overflows in additions. The function \( Q_e[\ ] \) denotes the quantization of the data and coefficients to \( B_d \) and \( B_c \), respectively.

The output power from the fixed-point LMS is given by \[12\]

\[
\xi = \left( 1 + \frac{1}{2} \mu L \sigma_x^2 \right) \xi_{\text{min}} + \frac{L \sigma_x^2}{2 \mu} + (\bar{W}^T \bar{W} + c) \sigma_x^2
\]

where \( \sigma_x^2 = 2^{-2B_d}/12 \), \( \sigma_x^2 = 2^{-2B_c}/12 \) for rounding, \( \bar{W} \) is the vector of optimal filter weights, and \( c \) is a constant depending on the way the inner product \( \bar{W}^T \bar{X} \) is computed (\( \text{var}[\bar{W}^T \bar{X}] = \sigma_x^2 \)). The first term consists of the Wiener optimal filter output and the misadjustment noise. The remaining two terms represent the effects of the finite precision effects of the fixed-point representation. Similar expression for the floating point arithmetic is also found in [12].

Another consideration is the adaptation constant \( \mu \) which scales the size of the weight update. This must satisfy two constraints, both from convergence considerations.

\[
\sqrt{\xi_{\text{min}} \sigma_x^2} < \mu < \frac{2 \sqrt{\xi_{\text{min}} \sigma_x^2}}{\mu_{\text{max}}}
\]

The lower bound (\( \mu_{\text{min}} \)) ensures that the error signal generated is large enough to allow the weights be updated\(^4\) while the upper bound ensures that update is not too large, to allow the algorithm to converge [18]. Additionally, by differentiation of \( \xi \), it can be shown that the \( \mu \) which minimizes \( \xi \) is given by the lower limit (\( \mu_{\text{min}} \)).

By examination of (31) it can be seen the misadjustment is reduced by choosing a small \( \mu \). This can be set to any desirable value by choosing the \( B_c \) properly so as to ensure that \( \mu_{\text{min}} \) is smaller than the desired \( \mu \). Assuming that \( \mu = C \mu_{\text{min}} \), the misadjustment noise is

\[
C \frac{L \sigma_x}{2} \sqrt{\xi_{\text{min}} \sigma_x^2}
\]

which decreases as \( 2^{-B_c} \). Similarly the first term of the quantization noise is

\[
\frac{1}{C} \frac{L \sigma_x}{2} \sqrt{\xi_{\text{min}} \sigma_x^2}
\]

which also decreases as \( 2^{-B_c} \). Thus, to a scale factor, \( B_c \) should be chosen as a function of \( \log_2(L) \).

The last quantization noise term in (31) is seen to depend on the filter weights. In Table I, we tabulate the maximum value of \( W^T W = \sum_i |w^*(i)|^2 \) over the range 10^{-5} < \alpha < 0.1 and 0 < \omega < 2 \pi for various \( \text{SNR}_j \) by use of the filter weights described by (24). From Table I it is seen that \( W^T W \) increases as the \( \text{SNR}_j \) increases, but is very insensitive to the filter length \( L \). Thus, the last quantization noise term is reduced as a function of \( 2^{-2B_i} \). This suggests that this term is easier to reduce than the first quantization term or the misadjustment. This analysis is in agreement with the observation that in the implementation of the LMS adaptive algorithm, the choice of wordlengths should satisfy (\( B_c > B_d \)) [19].

\(^4\)For example, if the inner product is performed using \( B_c \) bits multiply/accumulate, then \( c = L \) from the \( L \) finite-precision multiplications performed. However if the multiply/accumulate is performed using a double precision accumulator this will be reduced to \( c = 1 \) (\( L-1 \) bits).

\(^5\)A detailed discussion on finite precision implementation of gradient algorithms is discussed in [17].
TABLE 1

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\omega \in [0, 2\pi]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 10$</td>
<td>$L \to \infty$</td>
</tr>
<tr>
<td>0.01</td>
<td>$5.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0135</td>
</tr>
<tr>
<td>1</td>
<td>0.142</td>
</tr>
<tr>
<td>10</td>
<td>0.573</td>
</tr>
<tr>
<td>100</td>
<td>0.981</td>
</tr>
<tr>
<td>1000</td>
<td>1.32</td>
</tr>
</tbody>
</table>

V. SIMULATION RESULTS: BER

In this section simulation results are obtained in order to demonstrate the validity of the assumptions made and to illustrate the effects of the filter parameters on system performance. The CDMA overlay channel is simulated using both double-precision floating numbers and fixed-point numbers. Double precision is assumed to approximate the infinite precision results. To concentrate on the CDMA vs. the BPSK interference, $\sigma_n^2 = 0$ is assumed. The input thus consists of a large BPSK interference in a CDMA background. Since the BPSK is sinusoidal, a normalization by the BPSK amplitude plus the CDMA amplitude is performed. For this simulation this was set at 50. Scaling and other implementation considerations are discussed in [19, ch. 7].

The simulation parameters were chosen to correspond with those selected by Wang and Milstein in [10]. For the CDMA system the parameters are power $P = 0.5$, spreading gain $N = 255$, $K = 10$ users, and fading parameter $\rho = 1$. For the narrowband interference the parameters are $J = 100$ and bandwidth $p = 0.1$.

The parameter $K = 10$ users was chosen to minimize the effects of neglecting the self-interference introduced by the suppression filter; and $N = 255$ was chosen to ensure that the central limit theorem is applicable for the decision statistic $\chi(n)$. The narrowband BPSK power is 9.6 dB above the CDMA powers. The carrier offset $q$ is chosen to be $q = 0.0111$ so that ensemble averaging over the random BPSK carrier phases does not have to be performed since most initial phases are seen.

The fixed point ALE is implemented using $B_d$ data bits and $B_c$ bits for the filter coefficients. The adaptation constant is chosen to be $4 \mu_{\text{min}}$, to ensure adaptation. Finally, to minimize the quantization noise the multiply/accumulate operator is assumed to be double precision with $2B_c$ bits, which is available in most DSP chips.

The experiment was performed using 20 runs of 200 data bits each, with coefficients initially set to the optimum Wiener coefficients. The averaging over the other CDMA carrier phases was performed by changing the carrier phases of the other users at the end of each respective data bit. This produces the desired uniform distribution of carrier phases. Also as the CDMA signal level was much lower than the BPSK signal level this did not affect the filter adaptation. The averaging over the BPSK carrier phase is obtained by choosing a $q$ which does not produce an integral period in the sampling of the BPSK signal; in this way all initial phases are seen.

Three filters were implemented: 1) a fixed Wiener filter, 2) a double precision ALE, and a 3) fixed-point ALE. The experimental minimum mean square error ($\sigma_{\text{min}}$), misadjustment, and quantization noise were measured from the power at the output of filter 1, the variance of the difference between filters 1 and 2, and the difference, between filters 2 and 3, respectively.

Fig. 16 plots the three components of the output power for both $B_c = 8$ and $B_c = 12$ bits. It is seen that as the filter length increases, the misadjustment and the quantization noise increase. As expected when $B_c$ is increased, both the quantization noise and the misadjustment noise are reduced. There is a notable difference between the theoretical results and the simulation results for the $B_c = 8$. This is because as $L$ increases, the upper bound on $\mu$ decreases. In this case the point when the filter no longer converges is at $L = 254$ taps, and the deviations are due to the nonconvergence of the filter coefficients as $L$ approaches this limit. For $B_c = 12$, there is no noticeable difference between the theoretical and simulation results.

Fig. 17 plots the associated simulated and theoretical BER. As expected, when the output is dominated by the interference the BER expression holds very well. However, when the misadjustment is no longer insignificant, the actual BER is seen to be significantly greater than the BER predicted from the optimal Wiener filter output noise.

VI. CONCLUSION

In this paper we have studied the performance of an adaptive interference suppression system for a CDMA system overlaying a single BPSK system which occupies a significant portion of the CDMA bandwidth. Adaptive interference suppression filters have been shown to provide significant improvements for this CDMA overlay system where large interference levels are expected.

The BER was shown to be related to the power at the output of the suppression filter by a scaling constant. The
system performance is characterized by the output power of the suppression filter and the performance of the suppression filter characterized by a SNR improvement ratio. The optimum linear suppression filter, given by the Wiener filter, is solved by modeling the BPSK interference by a second order process. It was shown that the broadband model used in previous studies does not apply for this system, especially at large interference SNR's. The regions where the broadband model is applicable was characterized. Expressions are derived to provide the exact solution for the zeros of input spectrum.

The difference between the SNR improvement when the interference center frequency is centered on the CDMA carrier and when it is offset is characterized. When the BPSK interference is centered on the CDMA carrier, it was shown that the second-order process reduces to the AR1 model in [4], where closed form solutions are available for the SNR improvement. When the BPSK interference is offset, it was shown that for large filter lengths this loss is at most 3 dB. However, for small filter lengths it was shown that the loss can be significantly more.

It was shown that it is generally possible to define a range of wordlengths and adaptation time constants for which the misadjustment noise and quantization noise are negligible compared to the noise at the output of the optimal Wiener filter. In these cases simulation results were obtained to illustrate that the BER can be accurately predicted by the BER results for an optimal Wiener filter.

For the digital implementation of the adaptive filter studied here the time dispersion of the CDMA signal due to filtering is not a consideration, as the CDMA signal is assumed to have been correctly demodulated before signal processing is performed. However, for analog implementations, where filtering is performed before the acquisition of the CDMA signal, the demodulation will be more difficult due to the filtering. This, however, is ameliorated by the fact that the interference has been removed and the dispersion is due to a known filter. Additional BER reduction may be obtained by despreading the self-interference induced by the suppression filter. However, this improvement will be small since the self-interference is small when compared with the multi-access interference from the other users for $K \gg 1$.

### Appendix A

#### Autocorrelation Function of the Received Signal

Neglecting the double frequency terms, the demodulated/sampled received signal is given by

$$
 r(n) = \int_{nT_c}^{(n+1)T_c} r(t) 2 \cos (\omega_0 t) \, dt 
$$

$$
 = 2 \beta_1 P h_1(n) a_1(n) + 2P \sum_{k=2}^{K} \beta_k b_k(n) a_k(n) \cos (\phi_k) 
 + J_j(n) \cos (\omega_0 n) F(n; \omega_0, \theta) + n(n) 
$$

(A.1)

where $r(t)$ is the received signal at the antenna defined in (3), $n(n)$ is white noise with variance $N_0 T_c$ and

$$
 F(n; \omega_0, \theta) = 2 \left[ \frac{\sin (\omega_0 (n + 1) + \theta)}{\omega_0} - \frac{\sin (\omega_0 n + \theta)}{\omega_0} \right] 
$$

(A.2)

is the scaling factor introduced by the demodulation of the BPSK with an offset carrier. The frequency of the demodulated BPSK signal at the input to the suppression filter is $\omega_0 = \delta \omega_0 T_c$.

The correlation function is easily obtained using the independence of the signal and interference and averaging over the phases $(\phi_k, \theta)$ and time index $n$. The correlation function of the chip synchronized $(\tau_k = 0)$ CDMA signal is:

$$
 \phi_{\text{CDMA}}(l) = 2P \rho(K+1) \delta(l) 
$$

(A.3)

where the factor $K+1$ arises because the signal is demodulated with respect to the phase $\phi_1$ of the reference user.

The correlation function of the thermal noise is

$$
 \phi_n(l) = N_0 \theta(l) \delta(l). 
$$

(A.4)

The correlation function of the BPSK interference is

$$
 \phi_j(l) = \frac{[J \phi_F(\omega_0) \left( 1 - \frac{|l|}{T} \right) \cos (\omega_0 l)]}{\sigma_j^2}, \quad |l| < T 
$$

(A.5)

where

$$
 \phi_F(\omega_0) = 2 \left[ 1 - \frac{\cos \omega_0}{\omega_0^2} \right] 
$$

(A.6)

is the correlation of the scaling factor $F(n; \omega_0, \theta)$ with respect to the phase $\theta$ and time index $n$. Fig. 17 plots this scaling factor $\phi_F(\omega_0)$ as a function of the BPSK carrier offset $\omega_0$. It is seen that the effective BPSK interference power is reduced as the BPSK is moved from the center of the CDMA bandwidth.

Thus the autocorrelation of the input process is given by

$$
 \phi_e(l) = \phi_n(l) + \phi_{\text{CDMA}}(l) + \phi_j(l) 
$$

(A.7)
**APPENDIX B**

**OPTIMUM PREDICTION FILTER WEIGHTS**

A. Generalized Wiener–Hopf Solutions for Inputs with Rational Spectral Densities

When the input power spectrum can be written as a rational power spectral density, the filter coefficients which minimize the interference can be found analytically. The optimum filter weights are found by solving the Wiener-Hopf equation \[ (R\tilde{W})^* = \tilde{P} \] (B.1)

where \( R \) is the input autocorrelation matrix, \( \tilde{P} \) is the cross correlation vector, and \( \tilde{W} \) is the vector of tap coefficients.

For a process with rational spectral density the spectrum can be written as

\[
S_{xx}(f) = \prod_{m=1}^{M} (z - e^{-\beta_m} + j\omega_m)(z^{-1} - e^{-\beta_m} - j\omega_m)
\]

with the time domain autocorrelation function

\[
\phi_{xx}(k) = \begin{cases} 
\sum_{n=1}^{N} A_n e^{-\alpha_n|k| + j\omega_n k}, & k \geq 0 \\
\sum_{n=1}^{N} A_n e^{-\alpha_n|k| + j\omega_n k}, & k < 0 \end{cases}
\] (B.3)

Using standard mathematical techniques, the general solution of (B.1) with rational spectral densities (B.2) can be shown to be of the form

\[
w^*(k) = \sum_{m=1}^{M} \left\{ B_m^+ e^{-\beta_m k + j\omega_m k} + B_m^- e^{-\beta_m(L-1-k) + j\omega_m k} \right\}
\]

\[+ \sum_{r=1}^{N-M} \left\{ C_r^+ \delta(k - r + 1) + C_r^- \delta(k + r - L) \right\}, \quad k = 0, 1, \cdots, L - 1 \] (B.4)

consisting of damped exponentials and impulses.

The \( B \)'s in \( w(k) \) are found by solving the following matrix equations:

\[
\sum_{m=1}^{M} \left\{ \frac{B_m^+}{1 - e^{\alpha_n - \beta_m - j(\omega_n - \theta_m)}} + \frac{B_m^-}{1 - e^{-\alpha_n + \beta_m - j(\omega_n - \theta_m)}} \right\}
\]

\[+ \sum_{r=1}^{N-M} C_r^+ e^{\alpha_n(r-1) - j\omega_n(r-1)} = e^{-(\alpha_n - j\omega_n) \Delta}, \quad n = 1, \cdots, N, \]

\[
\sum_{m=1}^{M} \left\{ \frac{B_m^+ e^{-\beta_m L + j\theta_m L}}{1 - e^{-\alpha_n - \beta_m - j(\omega_n - \theta_m)}} + \frac{B_m^- e^{\beta_m L + j\theta_m L}}{1 - e^{-\alpha_n + \beta_m - j(\omega_n - \theta_m)}} \right\}
\]

\[+ \sum_{r=1}^{N-M} C_r^- e^{\alpha_n + j\omega_n r} = 0, \quad n = 1, \cdots, N. \] (B.5)

These are found by substituting \( (B.5) \) into \( (B.1) \), carrying out the summations, and equating the exponential terms. A detailed derivation is found in [14].

B. Wiener Filter Coefficients of Narrowband Process Embedded in White Noise

In general when white noise is present at the input, the order of the numerator and the denominator of the input power spectrum is the same \( (N = M) \), and in this case the coefficients of the impulses \( \{C^+, C^-\} \) in (B.4) and (B.5) are \{0\}.

For any narrowband processes in white noise, the input spectrum is described by

\[
S_{xx}(z) = G(z - e^{-\mu_1})(z - e^{-\mu_2})(z - e^{-\mu_3})(z - e^{-\mu_4})(z - e^{-\mu_5})(z - e^{-\mu_6})\]

\[
(z - e^{-\psi_1})(z - e^{-\psi_2})(z - e^{-\psi_3})(z - e^{-\psi_4})\] (B.6)

with the parameters \( (\alpha, \omega) \) and \( (\beta, \psi) \) found in Section III-A. The optimum filter coefficients can be solved using \( (B.5) \), which can be reordered into the matrix equations (shown in (B.7) at the top of the next page) and \( \Delta \) is the delay in the prediction or the bulk delay.

For \( \beta L \gg 1 \) sufficiently large, the \( B^- \) terms can be neglected and \( B^+ \) is found to be

\[
B_{1,2}^+ = |B^+|e^{\pm i\theta}
\] (B.8)

where

\[
\theta = \arctan \left( \frac{1}{Z} \cot \psi \right)
\]

\[
|B^+| = \frac{2\sin \omega \cos(\psi - \theta)}{y e^{(\beta - \alpha)}}
\] (B.9)

and

\[
Z = e^{-(\beta - \alpha)} \sin \omega \sin \psi \left[ \frac{x}{y} + \cot \omega \right]
\]

\[
x + iy = e^{-(\alpha - \omega L)} \left( e^{-(\alpha - \beta)(\omega + \psi)} - e^{(\alpha - \beta)(\omega - \psi)} \right).
\] (B.10)

Substituting the solutions into (B.4) the optimum filter weights simplify to

\[
w^*(k) = 2|B^+|e^{-\beta k} \cos(\psi k + \theta) \quad k = 0, 1, \cdots, L - 1.
\] (B.11)

For the special case of \( \omega = 0 \) these simplify to the AR1 case and can be solved exactly [4], i.e.,

\[
w^*(k) = B_1 b^k + B_2 b^{L-1-k} \quad k = 0, \cdots, L - 1
\] (B.12)

where

\[
b = A - \sqrt{A^2 - 1} \quad A = \frac{1}{2\alpha} \left[ 1 + a^2 \right] = \frac{\sigma^2}{(1 + \sigma^2)} (1 - a^2)
\] (B.13)
\[ M \tilde{B} = \tilde{V} \] 

(B.7)

where

\[
M = \begin{bmatrix}
\frac{1}{1 - e^{-\beta - j(\omega_0 - \psi)}} & \frac{1}{1 - e^{-\alpha - j(\omega_0 + \psi)}} & e^{-\beta L} & e^\beta & e^{-\beta L} & e^\beta \\
\frac{1}{1 - e^{-\beta - j(-\omega_0 - \psi)}} & \frac{1}{1 - e^{-\alpha - j(-\omega_0 + \psi)}} & e^{-\beta L} & e^\beta & e^{-\beta L} & e^\beta \\
e^{-\beta L} & e^{-\beta L} & e^{-\beta jL} & e^{-\beta jL} & 1 - e^{-\alpha - j(\omega_0 - \psi)} & e^{-\alpha jL} \\
e^{j\beta L} & e^{j\beta L} & e^{j\beta - jL} & e^{j\beta - jL} & 1 - e^{-\alpha - j(\omega_0 + \psi)} & e^{j\alpha jL}
\end{bmatrix}
\]

\[ \tilde{B} = [B_1^+ \quad B_2^+ \quad B_1^- \quad B_2^-] \]

\[ \tilde{V} = [e^{-(\alpha-j\omega)\Delta} \quad e^{-(\alpha+j\omega)\Delta} \quad 0 \quad 0]^T \]

and

\[ B_1 = \frac{(a-b)(1-ab)^2}{(1-ab)^2 - b^2L(a-b)^2} \]

\[ B_2 = \frac{bL(a-b)^2(1-ab)}{(1-ab)^2 - b^2L(a-b)^2} \]  

(B.14)

The \( \beta \) found here is equivalent to \( \beta = e^{-(\beta_{PC}+\psi_{PC})} \) and \( \alpha = e^{-\alpha} \) as in Section III.

APPENDIX C

OUTPUT POWER OF THE ALE SUPPRESSION FILTER

The output power of the filter when configured as an ALE is given by the minimum estimation error

\[ \xi_{\text{min}} = \sigma_{xx}^2 - \tilde{W}^T R \tilde{W}, \]  

(C.1)

but for the optimum Wiener filter (\( \tilde{W} = \tilde{W}^* \)), this simplifies to

\[ \xi_{\text{min}} = \sigma_{xx}^2 - \tilde{P}^T \tilde{W}^* \]  

(C.2)

where relation (B.1) has been used. The cross correlation vector is given by

\[ \tilde{P}_l = \sigma_2^2 e^{-\alpha(l+\Delta \Delta)} \cos \omega(l + \Delta) \quad \Delta \geq 1. \]  

(C.3)

Using the Wiener coefficients found in (B.11),

\[ \xi_{\text{min}} = (1 + \sigma_n^2) + \sigma_2^2 \]

Denoting (C.5) at the bottom of the page, the predictable narrowband interference power \( \text{PWR}_j \) may be written in closed form as

\[ \text{PWR}_j = \sigma_j^2 |B|^2 e^{-\alpha \Delta} \left[ \Sigma[\alpha + \beta, \omega + \psi, \omega \Delta + \theta, L] + \Sigma[\alpha + \beta, \omega - \psi, \omega \Delta - \theta, L] \right]. \]  

(C.6)

For the case \( \Delta = 1, \omega = \pi/2, \psi = \pi/2, \theta = \pi/2 \)

\[ \text{PWR}_j = 2\sigma_j^2 |B|^2 e^{-\alpha \Delta} \left[ \Sigma[\alpha + \beta, \pi, \pi, L] + \Sigma[\alpha + \beta, 0, 0, 0] \right] = \sigma_j^2 \sinh(\beta - \alpha) e^{-(3\alpha + \beta)\Delta} \]

\[ \frac{1 - e^{-2L/2}(\alpha + \beta)}{1 - e^{-2(\alpha + \beta)}} \]  

(C.7)

where \([x]\) denotes the largest integer smaller than \(x\).

For the \( \Delta = 1, \omega = 0 \) case, this reduced to the AR1 process. The exact expression for \( \text{PWR}_j \) is \([4]\).

\[ \text{PWR}_j = \sigma_j^2 (1 - a) \frac{[(1 - ab) + (a - b)2L + 1]}{[(1 - ab)^2 - (a - b)^22L]} \]  

(C.8)

where

\[ a = e^{-\alpha} \quad b = e^{-(\beta + j\phi)} \]  

\[ \Sigma[\mu, a, b, L] = \sum_{l=0}^{L-1} e^{-\mu l} \cos(\alpha l + b) \]

\[ = \frac{\cos(b) - e^{-\mu} \cos(a - b) - e^{-\mu L} [\cos(\alpha L + b) - e^{-\mu} \cos(\alpha(L + 1) + b)]}{1 - 2e^{-\mu} \cos(a) + e^{-2\mu}} \]  

(C.5)
To compare the fit of this second order approximation to the actual BPSK, Fig. 18 plots the ratio of the output powers of the Wiener filter. Using $\alpha = 0.01$ and $L = 50$, the output power of the Wiener filter was determined numerically for a range of SNR and $\omega$, for the narrowband and BPSK inputs. The output power of the second order approximation was determined using (C.6) above. Fig. 19 shows that the narrowband approximation to the BPSK input models the output power very well for the large SNR values of interest. The fit is improved both as SNR or the product $\alpha L$ is increased.

Using $\Sigma[i]$ defined above, the sum of the square of the filter weights can be written as

$$\sum_{i=0}^{L-1} |w^*(i)|^2 = 2|B^+|^2|\Sigma[2\beta,0,0,L] + \Sigma[2\beta,2\psi,2\theta,L]|.$$ (C.9)
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