

## The Effects of PN Sequences on the Misconvergence of the Constant Modulus Algorithm (CMA)

R. A. Axford, Jr., L. B. Milstein, and J. R. Zeidler

**Abstract**—The constant modulus algorithm (CMA) can misconverge when the received two-dimensional (2-D) signal is derived from encoding a binary PN sequence. The distortion of the probabilistic symmetry of the transmitted constellation can be controlled by careful selection of the underlying binary PN sequence, thereby influencing the convergence behavior of CMA.

### I. INTRODUCTION

The constant modulus algorithm (CMA) [1], [2] for blind equalization of two-dimensional (2-D) signals is based on minimization of the phase-blind cost function

$$D^{(2)} \triangleq E[ (|\hat{a}(n)|^2 - R_2)^2 ], \quad R_2 \triangleq E[|a_m|^4] / E[|a_m|^2] \quad (1)$$

where  $\hat{a}(n)$  is the equalizer output. The expected values in the definition of the modulus constant  $R_2$  are taken over the constellation  $\{a_m\}_{m=0}^{M-1}$  under the assumption that all  $M$  symbols are equally probable. The associated tap-weight update is

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu \hat{a}^*(n) (|\hat{a}(n)|^2 - R_2) \mathbf{u}(n) \quad (2)$$

where  $\mathbf{u}(n)$  is the vector of received signal samples.

In [3], Treichler *et al.* reported instances of misconvergence of CMA that were observed when the signal source was a commercial 16-QAM modem that employed a “PN-reset” data scrambler [4]. Chen *et al.* address this misconvergence phenomenon in [5] with a “blind clustering” equalization technique that offers “robustness in the presence of a nonwhite input symbol sequence.” However, unlike CMA, the algorithm presented in [5] is limited to square QAM constellations, requires a priori knowledge of the transmitted constellation, and is not phase blind.

This correspondence examines the causes of CMA misconvergence reported in [3]. The focus is on the nature of the violations of the necessary signal conditions defined in [1] and [2] that arise when a PN-reset scrambler is used in the transmitter. Guidance for the formulation of design rules for PN-reset scramblers that are used in communications systems with CMA blind equalizers is provided.

### II. SETUP AND DEFINITIONS

The setup for consideration of the blind equalization problem is shown in Fig. 1(a). The complex-valued symbol sequence  $a(n)$  is emitted by the transmitter and passes through the baud-spaced moving average (MA) channel  $\mathbf{h}$ . It is assumed that the received signal  $u(n)$  is sampled at the symbol rate and that proper baud synchronization has been established. The real and imaginary parts of the complex-valued

Manuscript received December 7, 1994; revised August 15, 1996. The associate editor coordinating the review of this paper and approving it for publication was Dr. H. Fan.

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Publisher Item Identifier S 1053-587X(98)01342-7.

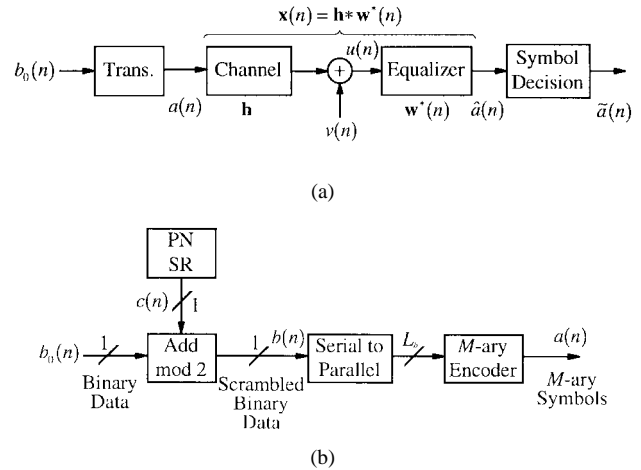


Fig. 1. (a) Overall setup. (b) PN-reset data scrambler.

additive white Gaussian noise process  $v(n)$  are assumed to have equal variance  $\frac{1}{2}\sigma_v^2$ . The combined channel/equalizer baud-spaced impulse response is

$$\mathbf{x}(n) \triangleq \mathbf{h} * \mathbf{w}^*(n).$$

The PN-reset data scrambler considered in this correspondence is shown in Fig. 1(b). The  $N$ -stage PN sequence-generating shift register (PN SR) produces the binary PN sequence  $c(n)$ . The PN sequence  $c(n)$  is added, modulo 2, to the input binary data sequence  $b_0(n)$  in a bit-by-bit fashion to produce the scrambled sequence  $b(n) = b_0(n) \oplus c(n)$ . The scrambled data are grouped  $L_b \triangleq \log_2(M)$  bits at a time and encoded into the  $M$ -ary symbol sequence  $a(n)$ . It is assumed throughout this correspondence that  $b_0(n)$  is idle and consists of all zeros. In this limiting case,  $b(n) = c(n)$ . Let

- $P_N \triangleq 2^N - 1$  maximal odd period of the binary PN sequence  $c(n)$ ;
- $P'_N \triangleq 2^N$  maximal even period of the corresponding *de-Brujn sequence*, obtained from the odd-period PN sequence by inserting an  $N$ th zero into the run of  $N - 1$  zeros; see p. 287 of [6];
- $P_M \triangleq$  period of the resulting  $M$ -ary sequence  $a(n)$ ;
- $d \triangleq$  greatest common divisor (g.c.d.) of  $L_b$  and  $P_N$  (or  $P'_N$ ).

### III. CONDITIONS FOR CMA CONVERGENCE

Godard's [1] assumptions regarding the transmitted signal are as follows. Successive data symbols are stationary and uncorrelated so that

$$E[a^*(i)a(j)] = E[|a_m|^2] \delta_{ij}. \quad (3)$$

The 2-D data symbol constellation  $\{a_m\}_{m=0}^{M-1}$  is assumed to have zero mean, i.e.,

$$E[a_m] = 0. \quad (4)$$

It is also assumed to have symmetries such that

$$E[a_m^2] = 0 \quad (5)$$

which implies that the real and imaginary parts of  $a_m$  have equal variance and are statistically uncorrelated. Finally, the constellation

is assumed to satisfy the compactness constraint

$$E[|a_m|^4] < 2(E[|a_m|^2])^2 \quad (6)$$

which is satisfied by all uniformly distributed QAM constellations that have peak-to-average energy ratios less than roughly 2, as stated in [1]. Using (3)–(6) as necessary conditions, Godard showed that the global minimum of  $D^{(2)}$  in the space of  $\mathbf{x}(n)$  is reached for the case of zero ISI. In [7], Foschini showed that Godard's global minimum is the only stable one while all others are saddles or maxima.

In [2], Treichler and Agee's consideration of the convergence of CMA was limited to constant envelope signals (e.g., frequency- and phase-modulated carriers) with baud-spaced channels and equalizers. Treichler and Agee showed that if  $a(n)$  is a constant amplitude signal of the form  $a(n) = Ae^{j\psi(n)}$ , then for  $\hat{a}(n)$  to have a constant amplitude,  $\mathbf{x}(n)$  must have only one nonzero tap (i.e., zero ISI) unless the modulation  $\psi(n)$  is periodic. However, as Treichler *et al.* point out in [3], as the period of the PN sequence becomes "long enough," the lines in the spectrum of  $a(n)$  become sufficiently densely packed so that a CMA-based equalizer will converge.

#### IV. PERIODICITY AND PROBABILITY MASS FUNCTIONS OF PSEUDO-RANDOM PSK AND QAM SIGNALS

This section examines the effects of the relative values of  $P_N$  and  $L_b$  on the periodicity of  $a(n)$  and the probability mass function (p.m.f.) of  $\{a_m\}_{m=0}^{M-1}$  when  $b_0(n)$  is idle. Only the encoding of concatenations of *complete* maximal periods of binary odd-period PN sequences and deBruijn sequences into  $M$ -ary symbol sequences is considered. In these cases, the parameter that characterizes the properties of the  $M$ -ary pseudo-random sequence is the g.c.d.  $d$  of  $L_b$  and  $P_N$  (or  $P'_N$ ). It is assumed that  $L_b \leq P_N$  (or  $P'_N$ ), and thus,  $1 \leq d \leq L_b$ . The results of this section are given in two theorems, which are stated here without proof. Proofs are given in [8]. Except where a distinction between odd period and deBruijn binary PN sequences is explicitly made,  $P_N$  can be replaced with  $P'_N$  to make the results apply to deBruijn sequences.

##### A. Periodicity

The pseudo-random  $M$ -ary sequence will start to repeat itself when an integer number of periods of the underlying binary PN sequence have been encoded.

*Theorem 1:* If  $L_b \leq P_N$ , then the period  $P_M$  of the  $M$ -ary symbol sequence  $a(n)$  is given by

$$P_M = P_N/d \quad (7)$$

where  $d$  is the g.c.d. of  $P_N$  and  $L_b$ .

If  $P_N$  and  $L_b$  are relatively prime (i.e.,  $d = 1$ ), then  $P_M$  achieves its maximum value:  $P_M = P_N$ .

##### B. Probability Mass Functions

Theorem 2 gives the p.m.f. of  $\{a_m\}_{m=0}^{M-1}$  under the condition that  $d = 1$ . For convenience, let  $a_m$  denote the symbol that corresponds to the bit pattern whose decimal equivalent is  $m$  (e.g.,  $a_0$  denotes the symbol that corresponds to  $L_b$  zeros).

*Theorem 2:* If  $L_b < N$  and  $d = 1$ , then the discrete probability mass function of  $\{a_m\}_{m=0}^{M-1}$  is given by

$$\Pr[a_m = a_m] = \begin{cases} \frac{2^{N-L_b}}{P_N} = \frac{2^N}{M(2^N - 1)}, & m \neq 0 \\ \frac{2^{N-L_b} - 1}{P_N} = \frac{2^N}{M(2^N - 1)} - \frac{1}{2^N - 1}, & m = 0 \end{cases} \quad (8)$$

when the underlying binary PN sequence has an odd period, and by

$$\Pr[a_m = a_m] = \frac{2^{N-L_b}}{P'_N} = \frac{1}{2^{L_b}} = \frac{1}{M}, \quad \forall m \quad (9)$$

when the underlying binary PN sequence is a deBruijn sequence. When (8) applies,  $\{a_m\}_{m=0}^{M-1}$  is uniformly distributed to within  $P_N^{-1}$ , and  $a_0$  occurs once less often than do all of the other symbols. When (9) applies,  $\{a_m\}_{m=0}^{M-1}$  is uniformly distributed.

When  $d \neq 1$ , the p.m.f. of  $\{a_m\}_{m=0}^{M-1}$  must be obtained by simply counting the number of times each symbol occurs during one period of the pseudo-random  $M$ -ary sequence. In addition, it is shown in [8] that the p.m.f. depends on the starting phase (relative to the  $L_b$ -bit grouping) of the underlying binary PN sequence when  $d \neq 1$ .

##### C. Discussion

Regarding the application of CMA blind equalizers, (8) and (9) of Theorem 2 represent desirable situations because the transmitted constellation during idle data periods is as uniformly distributed as possible. Indeed, if a deBruijn sequence is used in the data scrambler and if  $d = 1$ , then the distribution of the constellation will be precisely uniform during idle data periods, even though the symbol sequence is not random. However, since  $P'_N$  is always even, this can only hold for odd values of  $L_b$ . If an odd-period PN sequence is used and if  $d = 1$ , then  $P_N$  must be sufficiently large such that the deviation  $P_N^{-1}$  from a uniformly distributed constellation is negligible.

It was stated in [3] that some commercial 16-QAM ( $L_b = 4$ ) modems use PN-reset data scramblers with periods as short as 128 and 256 ( $N = 7$  and  $N = 8$ , respectively). Both are worst cases in that  $d = L_b = 4$ . The resulting periods of  $a(n)$  are 32 and 64, respectively, and the resulting constellations are as nonuniform as possible. In addition, for any given period-128 or 256 binary deBruijn sequence, there are four possible pseudo-random 16-QAM sequences [8]. Because the number of SR generated  $m$  sequences is finite, it is straightforward to catalog all possible constellation pmf's for a given number of SR stages, feedback tap connections, and starting phases (if  $d \neq 1$ ) for a given  $L_b$ . Motivated by the comments in [3], we did this for all of the deBruijn sequences generated by the  $N = 7$  and  $N = 8$  stage SR's contained in [9, Tab. III-5, p. 62] and  $L_b = 4$ . This cataloging effort, which is summarized in [8], has shown that *none* of these pmf's are uniformly distributed and that some are indeed highly distorted. For both values of  $N$ , example pmf's can be found with symbols that are completely missing. For example, consider the 16-QAM constellation pmf's in Fig. 2. The bit pattern assignments are made using a Gray code as in [10, Fig. 4–10, p. 209]. Fig. 2(a) shows the pmf of the 16-QAM constellation obtained by encoding the deBruijn sequence generated by the  $N = 8$  stage SR with feedback polynomial 455 (octal) in [9, Tab. III-5] with a starting phase of 2 bits. Note that one symbol is completely missing in the right hand corner, and this happens to correspond to bit pattern 1000. If, instead, the corresponding period-255 binary PN sequence generated by the same shift register with the same feedback connections is encoded, the 16-QAM pmf obtained is as calculated by (8), and the symbol corresponding to bit pattern 0000 is 1/255th less probable than all of the others.

##### V. VIOLATIONS OF THE SIGNAL SYMMETRY CONDITIONS

For odd-period PN sequences when  $d = 1$ ,  $E[a_m^2]$  can be calculated as follows using (8) for the probabilities.

$$E[a_m^2] = \frac{2^{N-L_b} - 1}{P_N} a_0^2 + \frac{2^{N-L_b}}{P_N} \sum_{m=1}^{M-1} a_m^2. \quad (10)$$

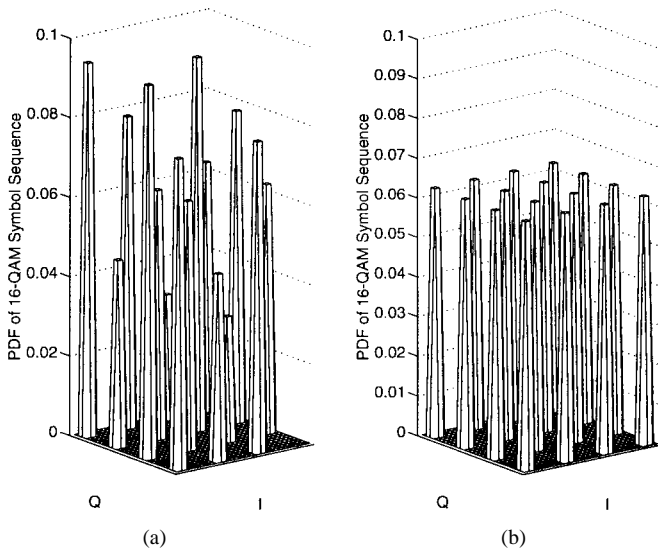


Fig. 2. (a) Probability mass function of the 16-QAM constellation obtained by encoding the deBruijn sequence generated by the  $N = 8$  stage SR with feedback polynomial 455 (octal) in [9, Tab. III-5] using a starting phase of 2 bits. Note that one symbol is completely missing (right-hand corner). (b) Probability mass function of a uniformly distributed 16-QAM constellation for comparison.

TABLE I

SUMMARY OF 8-PSK SIMULATION RESULTS DISPLAYED IN FIG. 3. THE DECIDING FACTOR IN THE PERFORMANCE OF CMA FOR THE PERIOD 32 AND PERIOD 31 SEQUENCES IS THE VIOLATION OF SYMMETRY CONDITION (5). (THE CURVES LABELLED '127' AND '128' RESULTED FROM THE SYMBOL SEQUENCES OBTAINED BY ENCODING THE ODD-PERIOD AND DEBRUIJN, RESPECTIVELY, BINARY  $m$ -SEQUENCES OBTAINED WITH THE SEVEN-STAGE SR WITH FEEDBACK CONNECTION 203 (OCTAL) [9]. SINCE  $L_b = 3$  IS RELATIVELY PRIME WITH BOTH 127 AND 128, THE PERIODS OF THE RESULTING 8-PSK SEQUENCES ARE ALSO 127 AND 128, RESPECTIVELY (THEOREM 1). THE P.M.F.'S OF THESE SEQUENCES ARE GIVEN BY (8) AND (9), RESPECTIVELY (THEOREM 2). THE CURVES LABELLED '31' AND '32' RESULTED FROM THE SYMBOL SEQUENCES OBTAINED BY ENCODING THE ODD-PERIOD AND DEBRUIJN, RESPECTIVELY, BINARY  $m$ -SEQUENCES OBTAINED WITH THE FIVE-STAGE SR WITH FEEDBACK CONNECTION 45 (OCTAL) [9]. SINCE  $L_b = 3$  IS RELATIVELY PRIME WITH BOTH 31 AND 32, THE PERIODS OF THE RESULTING 8-PSK SEQUENCES ARE ALSO 31 AND 32, RESPECTIVELY. THE P.M.F.'S OF THESE SEQUENCES ARE ALSO GIVEN BY (8) AND (9), RESPECTIVELY.)

Curve in Figure 3	Type of 8-PSK Sequence	$E[a_m^2]$	Result
R	Random sequence.	0	CMA converges.
128	Period 128 deBruijn sequence, satisfies (5).	0	CMA converges slightly more slowly than for random sequence.
127	Period 127 m-sequence, violates (5).	$1/127 = 0.007874$	CMA converges slightly more slowly than for random sequence.
32	Period 32 deBruijn sequence, satisfies (5).	0	CMA converges somewhat more slowly than for random sequence.
31	Period 31 m-sequence, violates (5).	$1/31 = 0.03226$	CMA fails to converge.

If the constellation satisfies (5) when all symbols are equally probable, then the geometry of the constellation is such that  $\sum_{m=0}^{M-1} a_m^2 = 0$ , and thus

$$E[a_m^2] = \frac{2^{N-L_b} - 1}{P_N} a_0^2 + \frac{2^{N-L_b}}{P_N} \left( \sum_{m=0}^{M-1} a_m^2 - a_0^2 \right) = -\frac{a_0^2}{P_N}. \quad (11)$$

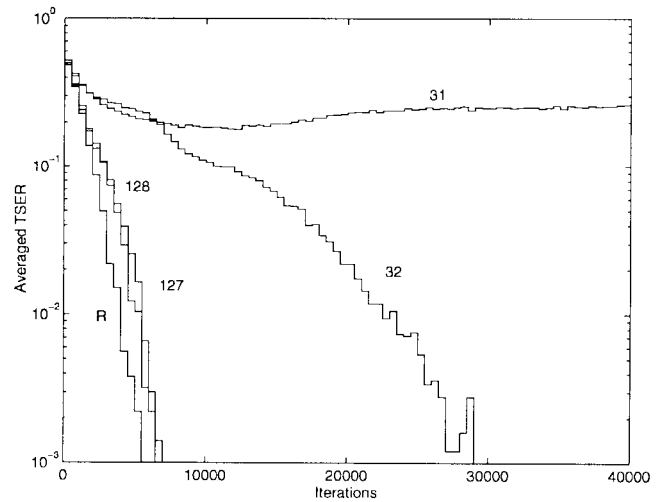


Fig. 3. Averaged transitional symbol error rate (TSER) curves for the eight-PSK sequences of Section VI-A and Table I. Curve 'R' results from a random eight-PSK sequence. The other curve labels denote the periods of pseudo-random eight-PSK sequences.

TABLE II

SUMMARY OF 16-QAM SIMULATION RESULTS DISPLAYED IN FIGS. 4-7. THE DECIDING FACTOR IN THE PERFORMANCE OF CMA IN THE FACE OF THE PSEUDO-RANDOM SYMBOLS 10001 TO 20000, AFTER SUCCESSFUL CONVERGENCE IN RESPONSE TO RANDOM SYMBOLS 1 TO 10000, IS THE DEGREE OF THE VIOLATION OF SYMMETRY CONDITION (5). (THE SECOND 10000 SYMBOLS IN FIGS. 4 AND 5 WERE DERIVED FROM ENCODING THE PERIOD-511 BINARY  $m$ -SEQUENCE OBTAINED FROM THE NINE-STAGE SR WITH FEEDBACK CONNECTION 1021 (OCTAL) [9]. THE SECOND 10000 SYMBOLS IN FIGS. 6 AND 7 WERE DERIVED FROM ENCODING THE PERIOD-511 BINARY DEBRUIJN SEQUENCE OBTAINED FROM THE SAME SR WITH THE SAME FEEDBACK CONNECTIONS.)

Figures	Parameters of pseudo-random 16-QAM sequence, symbols 10,001 to 20,000	Result
4 and 5	Period 511, $E[a_m^2] = 0.035225$	CMA remains converged.
6 and 7	Period 128, $E[a_m^2] = 0.8149$	CMA diverges from desired stationary point.

Therefore, for odd-period PN sequences when  $P_N$  and  $L_b$  are relatively prime, the violation of the symmetry condition (5) decreases monotonically with  $P_N$ . For QAM, this violation can be minimized by placing the symbol  $a_0$  as close to the origin as possible. Similarly,

$$E[a_m] = -\frac{a_0}{P_N} \quad (12)$$

for odd-period PN sequences when Theorem 2 applies.

By (9), for deBruijn sequences when  $d = 1$ , the constellation is uniformly distributed in spite of the fact that the data are nonrandom, and the moments on the left-hand sides of (11) and (12) and will be zero. For example, an eight-PSK ( $L_b = 3$ ) constellation will be uniformly distributed if the PN-reset data scrambler employs any maximal period deBruijn sequence with  $P'_N \geq 8$ . Therefore, it is possible to construct a signal that simultaneously satisfies (4) and (5), violates (3), and violates Treichler's and Agee's [2] (non-) periodic condition.

## VI. SIMULATION RESULTS

This section presents simulations that illustrate the usefulness of the results of Sections IV and V in predicting the convergence behavior of CMA in the face of pseudo-random sequences. In all cases, the channel model used was the static two-tap FIR filter with impulse response  $h(n) = h_0\delta(n) + h_1\delta(n-1)$ , where  $h_0 = 0.8575$  and

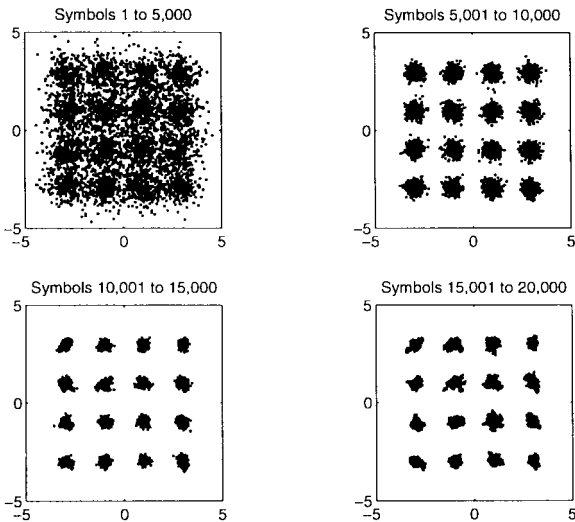


Fig. 4. Typical eye diagram. Symbols 1 to 10000 from a random 16-QAM sequence. Symbols 10001 to 20000 from a period-511 16-QAM sequence. Underlying binary PN sequence: Period-511 from the nine-stage SR with feedback connection 1021. (Corresponds to Fig. 5.)

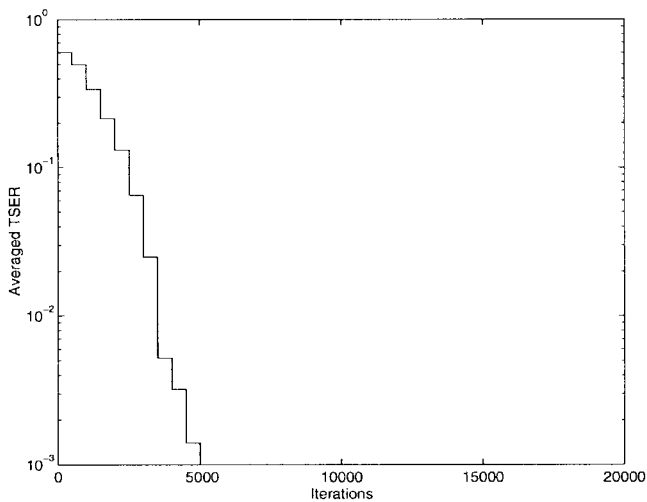


Fig. 5. Averaged TSER (10 runs). Symbols 1 to 10000 from a random 16-QAM sequence. Symbols 10001 to 20000 from a period-511 16-QAM sequence. Underlying binary PN sequence: Period-511 from the nine-stage SR with feedback connection 1021. (Corresponds to Fig. 4.)

$h_1 = -0.4456 + j0.2572$ . This minimum phase channel has a  $\sim 12$ -dB notch in its frequency response, and  $|h_0|^2 + |h_1|^2 = 1$ . In all cases,  $\sigma_v^2 = 0.002$ . In the eight-PSK simulations of Section VI-A, the signal amplitude  $A$  is 1, and thus,  $S/N = 24$  dB. The step size is  $\mu = 10^{-3}$ . In the 16-QAM simulations of Section VI-B, the signal amplitudes are drawn from  $(\pm 1, \pm 3)$ , and thus,  $S/N = 34$  dB. The step size is  $\mu = 2.5 \times 10^{-5}$ . In all cases, the number of equalizer taps is 11, and the equalizer is initialized with all taps set to zero except the center tap, which is set to one. The equalizer performance measures used here are eye diagrams and the *transitional symbol error rate* (TSER), which is the percentage of symbol errors in consecutive intervals of  $N$  received symbols. In this correspondence,  $N = 500$ , and thus, the lowest TSER that can be resolved in a single simulation run is 0.002. All TSER curves presented in this correspondence were obtained by averaging the results from 10 independent simulation runs, and thus, some values below 0.002 are shown.

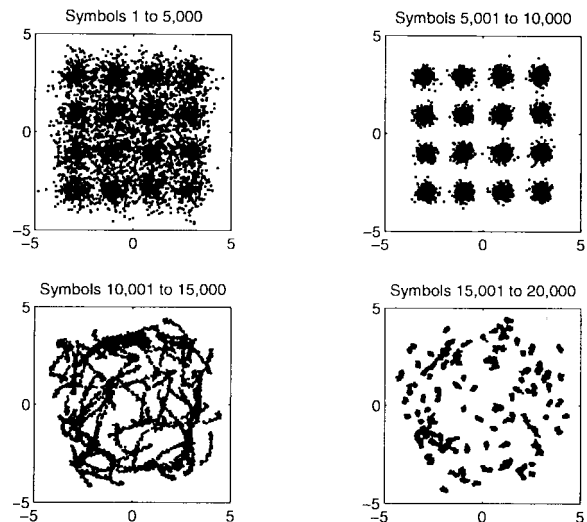


Fig. 6. Typical eye diagram. Symbols 1 to 10000 from a random 16-QAM sequence. Symbols 10001 to 20000 from a period-128 16-QAM sequence. Underlying binary PN sequence: Period-512 from the nine-stage SR with feedback connection 1021, phase shift between binary and  $M$ -ary sequences = 0. (Corresponds to Fig. 7.)

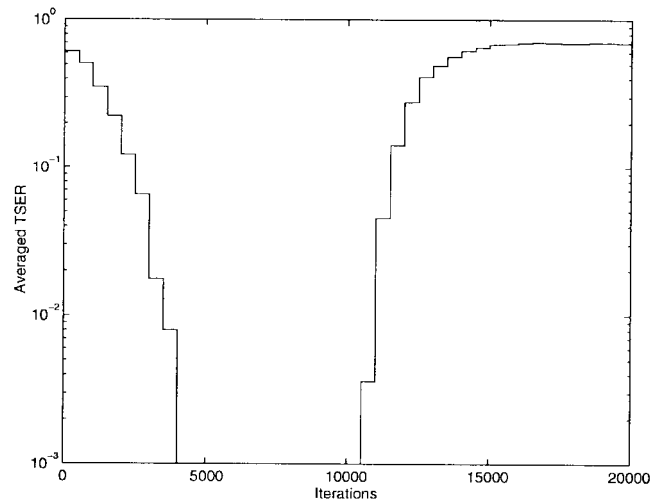


Fig. 7. Averaged TSER (10 runs). Symbols 1 to 10000 from a random 16-QAM sequence. Symbols 10001 to 20000 from a period-128 16-QAM sequence. Underlying binary PN sequence: Period-512 from the nine-stage SR with feedback connection 1021, phase shift between binary and  $M$ -ary sequences = 0. (Corresponds to Fig. 6.)

#### A. Initial (Mis)Convergence of Uniformly and "Almost" Uniformly Distributed Pseudo-Random Eight-PSK

The TSER curves in Fig. 3 correspond to the five eight-PSK symbol sequences described in Table I. The individual members of the two pairs [(‘31’, ‘32’) and (‘127’, ‘128’)] of pseudo-random eight-PSK sequences have nearly equal periods and, consequently, nearly equally spaced lines in their spectra. What distinguishes one member of each pair from the other member of the pair is the violation of Godard’s symmetry conditions in (4) and (5)—the even-period sequences satisfy these conditions, and odd-period ones do not. The convergence performances for the period-127 and -128 sequences are only slightly inferior to that obtained with the random sequence. In this case, the symmetry condition violations of the period-127 sequence are not sufficient to cause misconvergence. On the other hand, for the shorter period sequences, the uniformly distributed constellation ‘32’ eventually converges, but ‘31’ does not. Therefore, this is an example in which a violation of (5) appears to be the

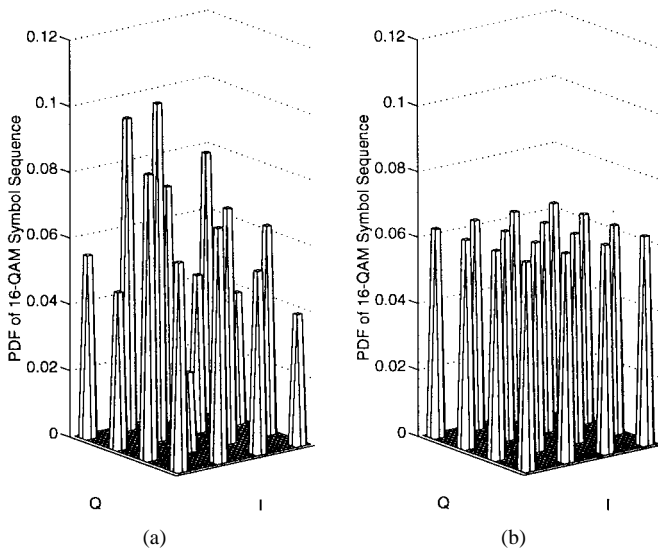


Fig. 8. Sixteen-QAM pmf's. (a) Pseudo-random pmf corresponding to symbols 10001 to 20000 in Figs. 6 and 7. (b) Uniformly random data for comparison.

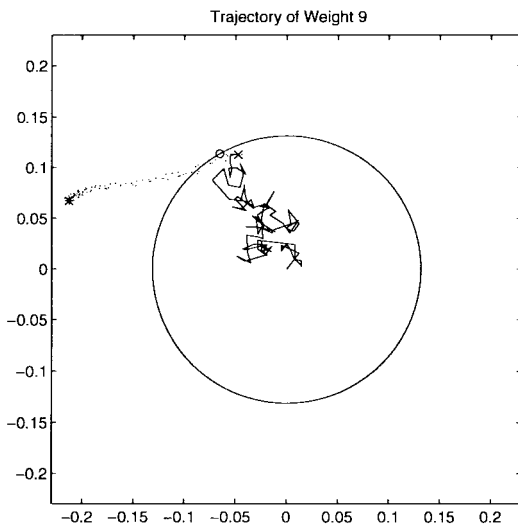


Fig. 9. Typical tap-weight trajectory number 9, in the complex plane, corresponding to the results shown in Figs. 6 and 7. (The 11 adaptive tap-weights are numbered 0 through 10.) Solid trace is in response to random symbols 1 to 10000. Dotted trace is in response to pseudo-random symbols 10001 to 20000. The large circle is drawn at the radius of the Wiener weight, the location of which is indicated by the small circle. Adaptive tap-weight number 9 is initialized at zero (the origin). The location of the weight at iteration 10000 is indicated by the 'x,' and the location at iteration 20000 is indicated by the '\*.'

dominant factor in determining whether or not the CMA equalizer will converge successfully.

### B. Misconvergence After Successful Convergence with 16-QAM

Figs. 4–7 and Table II present two simulations in each of which the first 10000 16-QAM symbols were derived from encoding a random binary sequence. The second 10000 symbols in Figs. 4 and 5 were derived from encoding the period-511 binary  $m$ -sequence obtained from the nine-stage SR with feedback connection 1021 (octal) [9]. Successful convergence of the equalizer is maintained after the introduction of this pseudo-random sequence beginning with symbol 10001. The second 10000 symbols in Figs. 6 and 7 were derived from encoding the period-512 binary deBruijn sequence obtained from the same SR with the same feedback connections. The

distorted pmf of this pseudo-random sequence is shown in Fig. 8(a). By direct calculation with this pmf, the magnitude of the violation of (5) is  $|E[a_m^2]| = 0.8149$ . Figs. 6 and 7 clearly show that the introduction of this pseudo-random sequence forces the previously converged equalizer into a state that fails to equalize the channel. This behavior, relative to that in Figs. 4 and 5, is foreshadowed by the ratio of the magnitudes of the violations of the symmetry condition (5), namely,  $0.8149/0.035225 \approx 23.1$ .

An alternative indication that the stationary points of the tap-weight update can indeed be altered by the introduction of the pseudo-random sequence is given by the plot of a typical tap-weight trajectory shown in Fig. 9. The solid trace is in response to the first 10000 random symbols, and the dotted trace is in response to the second 10000 pseudo-random symbols. The tap-weight at iteration 10000, which is indicated by the 'x,' is close to the Wiener value, which is indicated by the small circle. The introduction of the pseudo-random sequence, starting at symbol 10001, clearly causes the tap weight to move to a new location, which is indicated by the '\*' at iteration 20000, which is much farther from the Wiener weight.

These simulation results have also served to illustrate the significantly different CMA convergence performance results that can be obtained by selecting the odd-period  $m$ -sequence rather than the even-period deBruijn sequence, corresponding to the same SR and feedback connections in a PN-reset data scrambler when  $L_b$  is even.

## VII. CONCLUSIONS

This work examines how pseudo-random PSK and QAM signals can violate necessary conditions for CMA convergence. For analytical tractability, limiting cases in which continuous repetitions of complete maximal periods of a binary PN sequence are encoded into an  $M$ -ary symbol sequence were considered. In these cases, the properties of the resulting pseudo-random  $M$ -ary symbol sequence are largely determined by the greatest common divisor of the period of the underlying binary PN sequence and the number  $L_b = \log_2(M)$  of bits per symbol. To satisfy CMA convergence conditions, it is desirable that  $L_b$  and the period of the underlying binary PN sequence be relatively prime. If this condition is not met, then the probabilistic symmetry of the transmitted constellation can be highly distorted leading to serious violations of necessary conditions for CMA convergence.

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