MIMO Channel Equalization and Multiple Access Interference Suppression at the Transmitter

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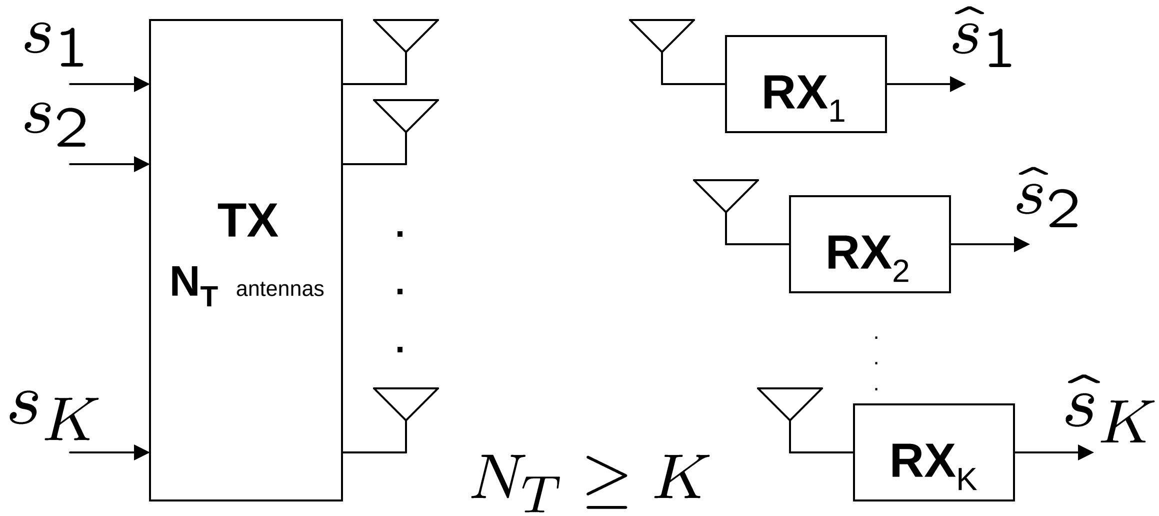
List of Publications

- **Journal Papers**
  1. P. Amihood, E. Masry, L. B. Milstein, and J. G. Proakis, “Performance Analysis of a Pre-BLAST DFE Technique for MISO Channels with Decentralized Receivers," *IEEE Transactions on Communications*, vol. 55, pp. 1385-1396, July 2007.
  2. P. Amihood, E. Masry, L. B. Milstein, J. G. Proakis, “Performance Analysis of High Data Rate MIMO Systems in Frequency Selective Fading Channels," accepted in *IEEE Transactions on Information Theory*, August 2007.
  3. P. Amihood, E. Masry, L.B. Milstein, J. G. Proakis, “The Effects of Channel Estimation Errors on a Nonlinear Precoder for MISO Channels with Decentralized Receivers,” submitted to *IEEE Transactions on Communications,* Sept. 11, 2007.

- **Conference Papers**
  1. P. Amihood, E. Masry, L. B. Milstein, J. G. Proakis, “Asymptotic Performance of Multicode MIMO Systems in Frequency Selective Fading Channels,” in *Proc. IEEE Military Commun.Conf. (MILCOM)*, Atlantic City, New Jersey, USA, vol. 5, pp. 3036-3041, October 17 - 20, 2005.
  2. P. Amihood, E. Masry, L. B. Milstein, J. G. Proakis,”Analysis of a MISO Pre-BLAST-DFE Technique for Decentralized Receivers,” in *Proc. of the 40th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, USA, October 29 - November 1, 2006.
  3. J. G. Proakis and P. Amihood, “Equalization of MIMO Systems,” in *Proc. of 15th European Signal Processing Conference (EUSIPCO)*, Poznan, Poland, September 3 - 7, 2007.

Point-to-Multipoint (Broadcast) MIMO System



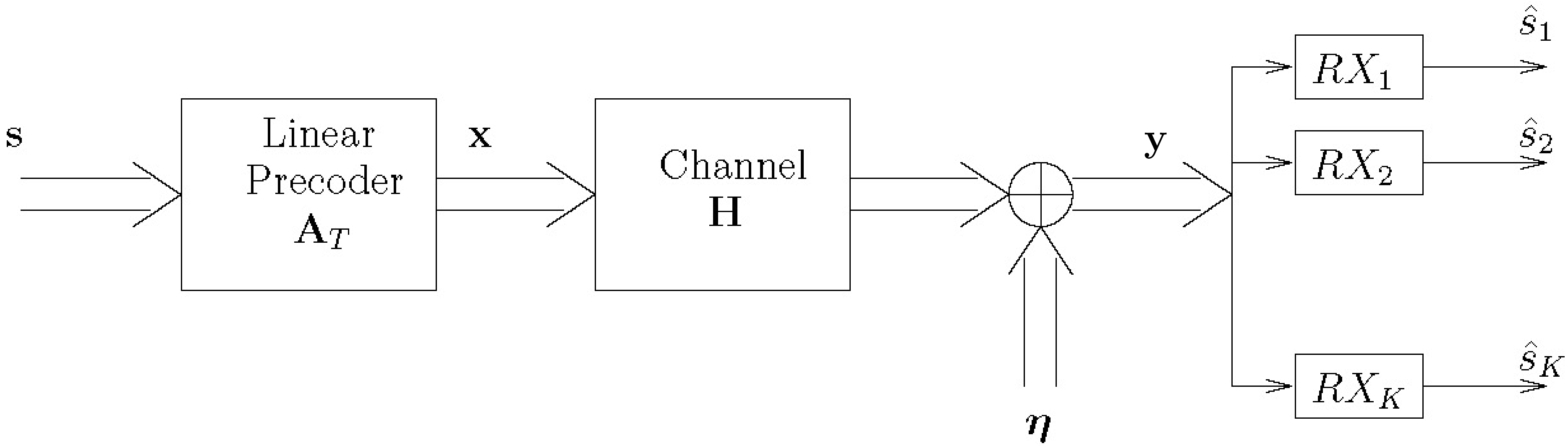
- Objectives:
  1) Simplify the receiver processing
  2) Suppress ISI and MAI
- Receivers have limited processing power
- Channel is known at the transmitter
- Each user has a single antenna (this is easily generalized to multiple receive antennas)

Transmit Processing (Precoding) Methods

- Linear Processing
  1. ZF Criterion
  2. MMSE Criterion

- Nonlinear processing
  1. QR Decomposition
  2. ZF Vector Precoding
  3. MMSE Vector Precoding
  4. Lattice Reduction Precoding

Linear ZF Precoder (Equalizer)



- Received signal (Flat Fading Channel):

$$\mathbf{y} = \mathbf{H}\mathbf{A}_T\mathbf{s} + \boldsymbol{\eta}$$

$$\mathbf{A}_T = \alpha\mathbf{H}^\dagger$$

$$\mathbf{H}^\dagger = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$$

α is a scale factor selected to satisfy the total transmitted power constraint, i.e. $\|\mathbf{A}_T\mathbf{s}\|^2 = \mathbf{P}$

Minimum MSE Precoder (Equalizer)

$$J(\mathbf{A}_T,\, \alpha) = \underset{\alpha,\ \mathbf{A}_T}{\arg\min} E\|\frac{1}{\alpha}(\mathbf{H}\mathbf{A}_T\mathbf{s} + \boldsymbol{\eta}) - \mathbf{s}\|^2$$

$$\mathbf{A}_T = \alpha\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \beta\mathbf{I})^{-1}$$

α is selected to satisfy the power constraint

β is selected to maximize the signal-to-interference-plus-noise ratio (SINR) at the receiver $(\beta = K/P)$

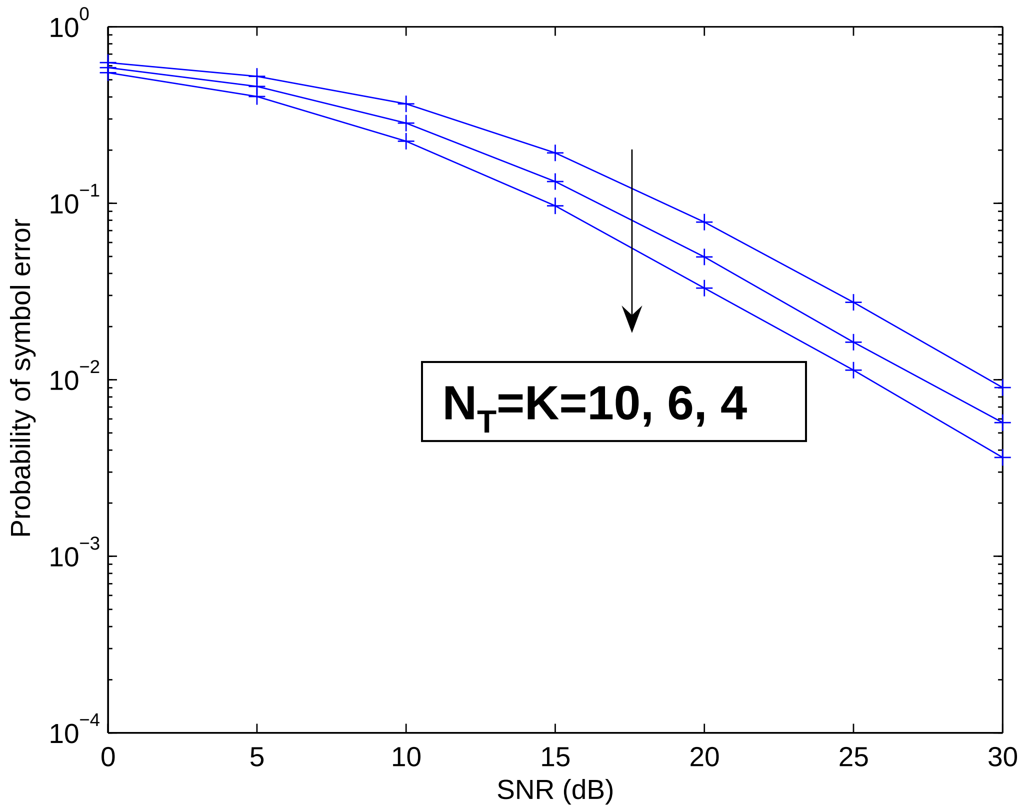
System Performance and Signal Diversity

- Signal Diversity (D) is achieved by
  1. Use of multiple antennas (N_T)
  2. Use of channel coding with interleaving (d_{min})
  3. Exploitation of resolvable multipath components (L)

- Primary Objective:

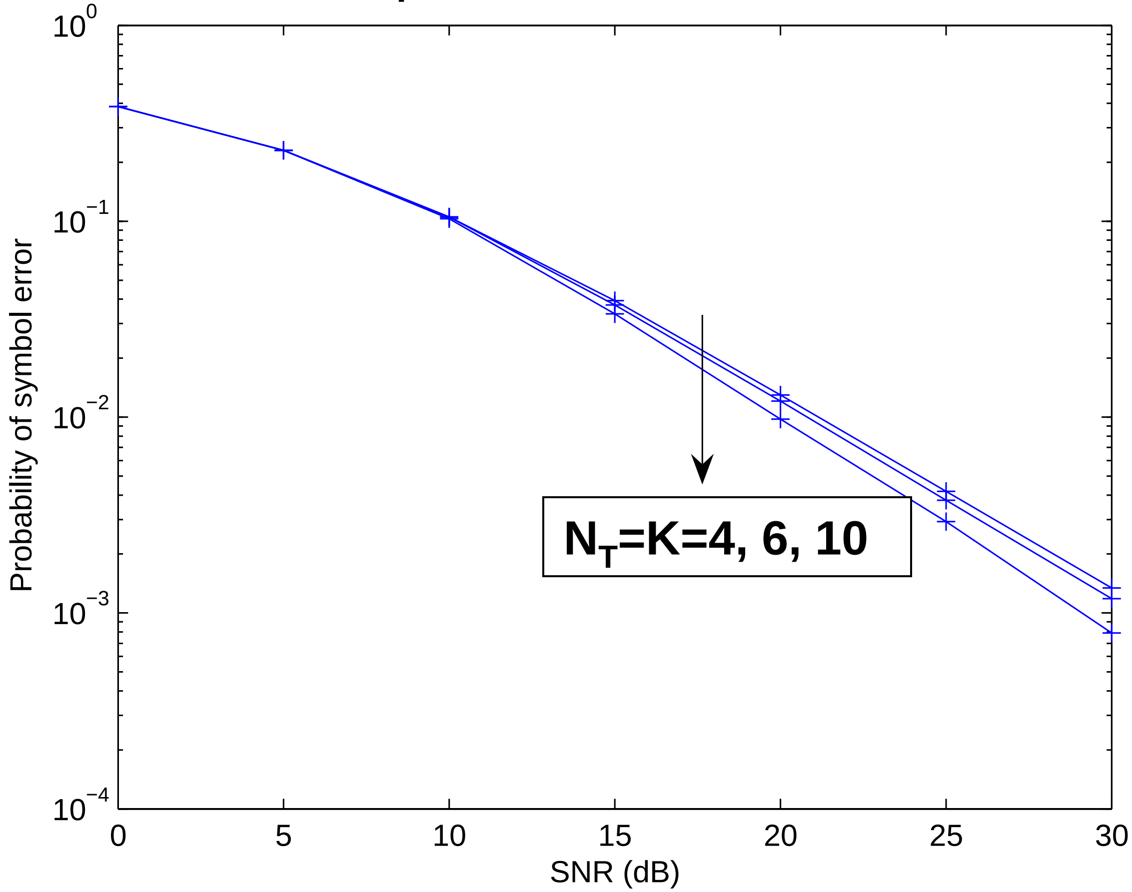
  Achieve the largest possible signal diversity inherent in the received signal

Performance of ZF linear precoding with N_T=K=4, 6, 10



- Performance deteriorates as the number of users increases
- Signal diversity is not exploited: $D=N_T-K+1$ for uncoded transmission and flat fading
- Performance of ZF precoder suffers due to ill-conditioning of the channel matrix $\mathbf{H}$

Performance of MMSE linear precoding with N_T=K=4, 6, 10



- Performance improves with an increase in the number of users
- Ill-conditioning of the channel matrix is avoided
- Signal diversity is not exploited: D=N_T-K+1 for uncoded transmission and flat fading

Nonlinear Precoding – The QR Decomposition

- Channel impulse response between the ith transmit antenna and the receive antenna of the kth user

$$h_{ki}(t) = \sum_{l=0}^{L-1} h_{kl}^{(l)} \delta(t - lT)$$

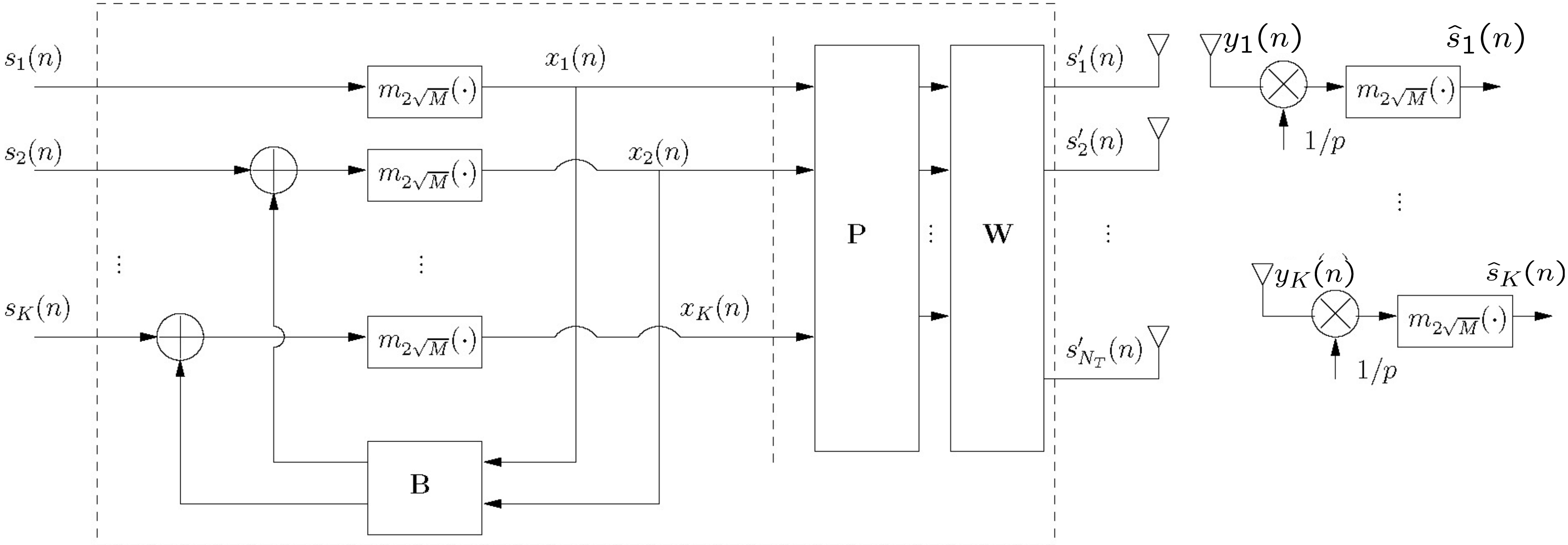
L ~ number of multipath components

T~ symbol interval

- Arrange the channel coefficients for the lth path in a K x N_T matrix $\mathbf{H}^{(l)}$, with elements

$$\left[\mathbf{H}^{(l)}\right]_{ki} = h_{ki}^{(l)},\ i = 1, 2, \ldots, N_T;\ k = 1, 2, \ldots, K,\ l = 0, 1, \ldots, L-1$$

QR Decomposition with Tomlinson-Harashima Precoding



Signal: M-ary square QAM

- $\left[\mathbf{H}^{(0)}\right]^H = \mathbf{QR}$

  $\mathbf{QQ}^H = \mathbf{I}$

  $\mathbf{R} \sim K \times K$ upper triangular matrix with diagonal elements $\{r_{ii}\}$
- $\mathbf{W} = \mathbf{QA}$ ~ Precoding Matrix

  $\mathbf{A} \sim K \times K$ diagonal matrix, with diagonal elements $\{1/r_{ii}\}$
- $\mathbf{P} = p\mathbf{I} \sim K \times K$ diagonal matrix used for power scaling
- $\mathbf{B} = \left[\mathbf{I} - \mathbf{H}^{(0)}\mathbf{W},\ -\mathbf{H}^{(1)}\mathbf{W},\ -\mathbf{H}^{(2)}\mathbf{W}, \ldots,\ -\mathbf{H}^{(L-1)}\mathbf{W}\right]$

Optimum Ordering of the Receivers

- Ordering of the K receivers affects the construction of the $K \times N_T$ channel matrix $\mathbf{H}^{(0)}$ and, hence, the decomposition $\left[\mathbf{H}^{(0)}\right]^H = \mathbf{QR}$
- There are $K!$ possible permutations of $\left[\mathbf{H}^{(0)}\right]^H$ and, hence, each permutation results in a different $\mathbf{QR}$ decomposition and requires a different transmit power
- We need to find the permutation (ordering) that minimizes the transmit power. A method for simplifying the search is described in a paper by Foschini et. al. (1999) (as in V-BLAST algorithm).

- Suppose we have found the optimal ordering for a given $\mathbf{H}^{(0)}$

- Result:

$$\begin{aligned} \mathbf{H}^{(0)}\mathbf{W}\mathbf{P} &= [\mathbf{Q}\mathbf{R}]^H \mathbf{Q}\mathbf{A}\mathbf{P} \\ &= p\mathbf{R}^H\mathbf{A} \end{aligned}$$

$\mathbf{R}^H\mathbf{A} \sim K \times K$ lower triangular matix with unit diagonal elements

$\sim$ has full rank K when $N_T \geq K$

- Thus, user k sees interference from users 1, 2, …, k-1.

- Interference subtraction at the transmitter

  Subtract the interference at the transmitter that each user would observe at their respective receiver. Thus, when the channel adds the same interference to the transmitted signal, the received signal at each receiver would be free of interference.

Transmit: $x_k = s_k - i_k$

Channel Output: $x_k + i_k = s_k$

- Successive interference cancellation by feedback filter (as in V-BLAST algorithm)

$$\mathrm{B} = \left[\mathbf{I} - \mathbf{H}^{(0)}\mathbf{W},\ -\mathbf{H}^{(1)}\mathbf{W},\ -\mathbf{H}^{(2)}\mathbf{W}, \ldots,\ -\mathbf{H}^{(L-1)}\mathbf{W}\right]$$

$\mathbf{I} - \mathbf{H}^{(0)}\mathbf{W} \sim$ cancels interference due to other users arising in the current symbol

$-\mathbf{H}^{(i)}\mathbf{W},\ i = 1,\ 2, \ldots,\ L-1 \sim$ cancels interference due to previous symbols

- Tomlinson-Harashima Precoding

  (for square constellations)

$$\begin{aligned}\mathbf{x}(n) &= \mathrm{mod}_{2\sqrt{M}}[\mathbf{s}(n) + \mathbf{B}\hat{\mathbf{x}}(n)] \\ &= \mathbf{s}(n) + \mathbf{B}\hat{\mathbf{x}}(n) - 2\sqrt{M}\mathbf{z}_x(n)\end{aligned}$$

where the modulo operation is performed on each real and imaginary component of the vector $\mathbf{s}(n) + \mathbf{B}\hat{\mathbf{x}}(n)$

$$\hat{\mathbf{x}}(n) = [\mathbf{x}(n)^T \,,\; \mathbf{x}(n-1)^T, ..., \; \mathbf{x}(n-L+1)^T]^T$$

$\mathbf{z}_x(n) \sim K \times 1$ vector with complex-valued integer real and imaginary coefficients

- Transmitted signal vector:

$$\begin{aligned} \mathrm{s}'(n) &= \mathbf{WPx}(n) \\ &= p\mathbf{Wx}(n) \end{aligned}$$

- Received signal vector:

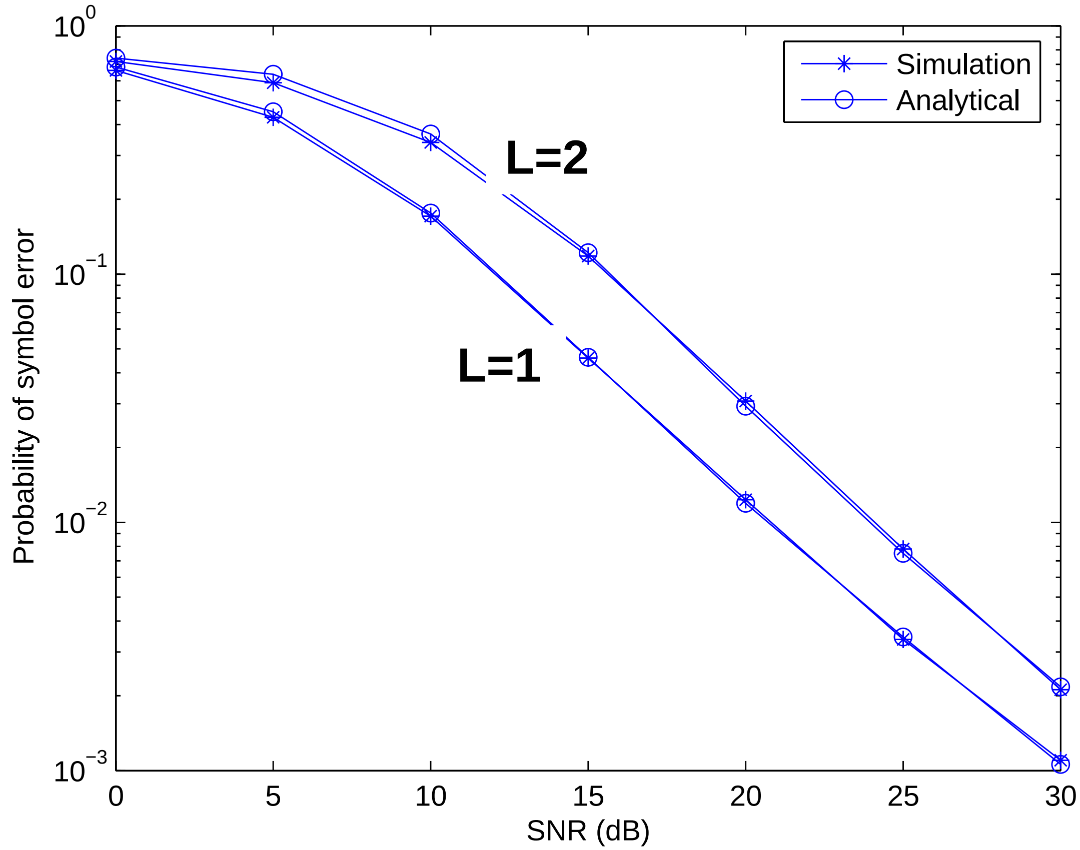
$$\mathbf{y}(n) = p \sum_{i=0}^{L-1} \mathbf{H}^{(i)} \mathbf{W} \mathbf{x}(n-i) + \boldsymbol{\eta}(n)$$

Compute:

$$\frac{1}{p}\mathbf{y}(n) = \mathrm{s}(n) + \boldsymbol{\eta}'(n) - 2\sqrt{M}\mathbf{z}_x(n)$$

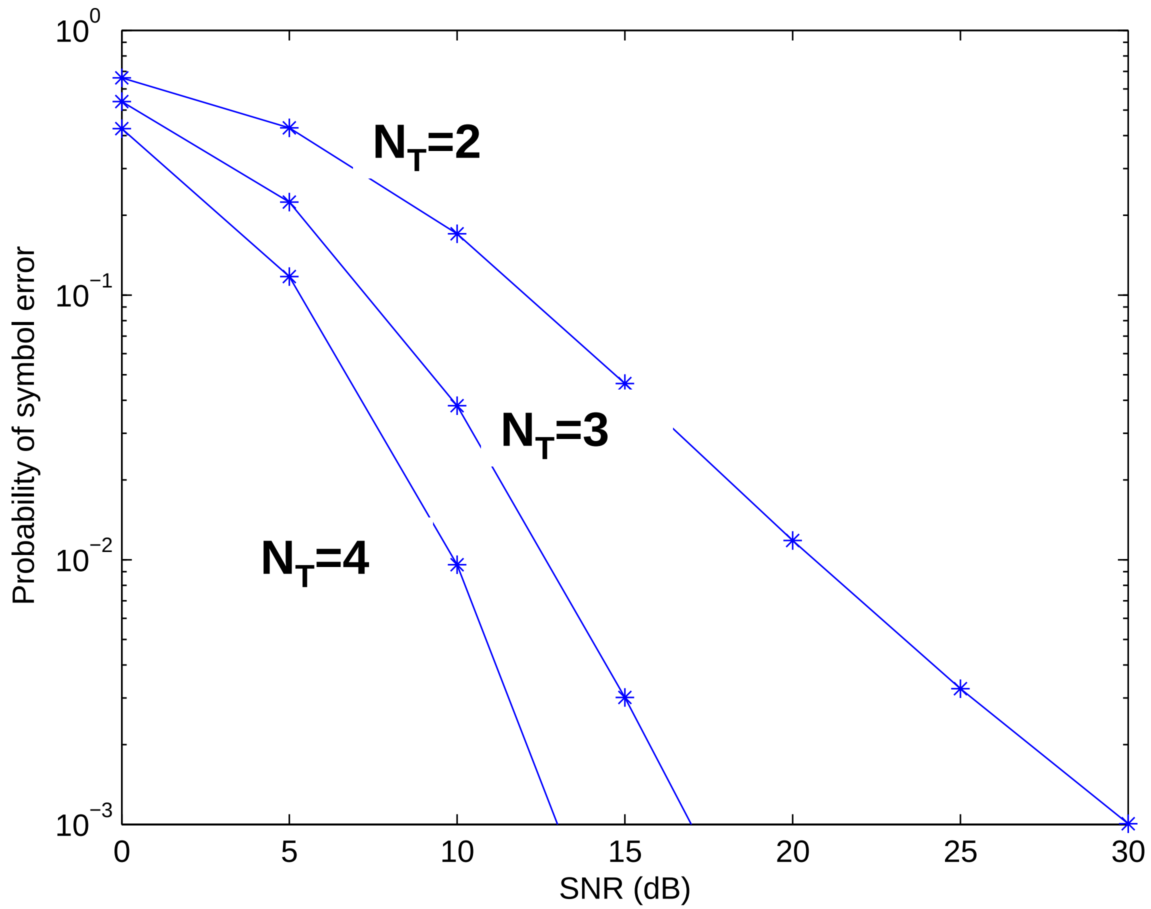
$$\hat{\mathrm{s}}(n) = \mathrm{mod}_{2\sqrt{M}}\left[\frac{1}{p}\mathbf{y}(n)\right]$$

Performance of optimal ordered QR decomposition with N_T=K=2 and L=1, 2.



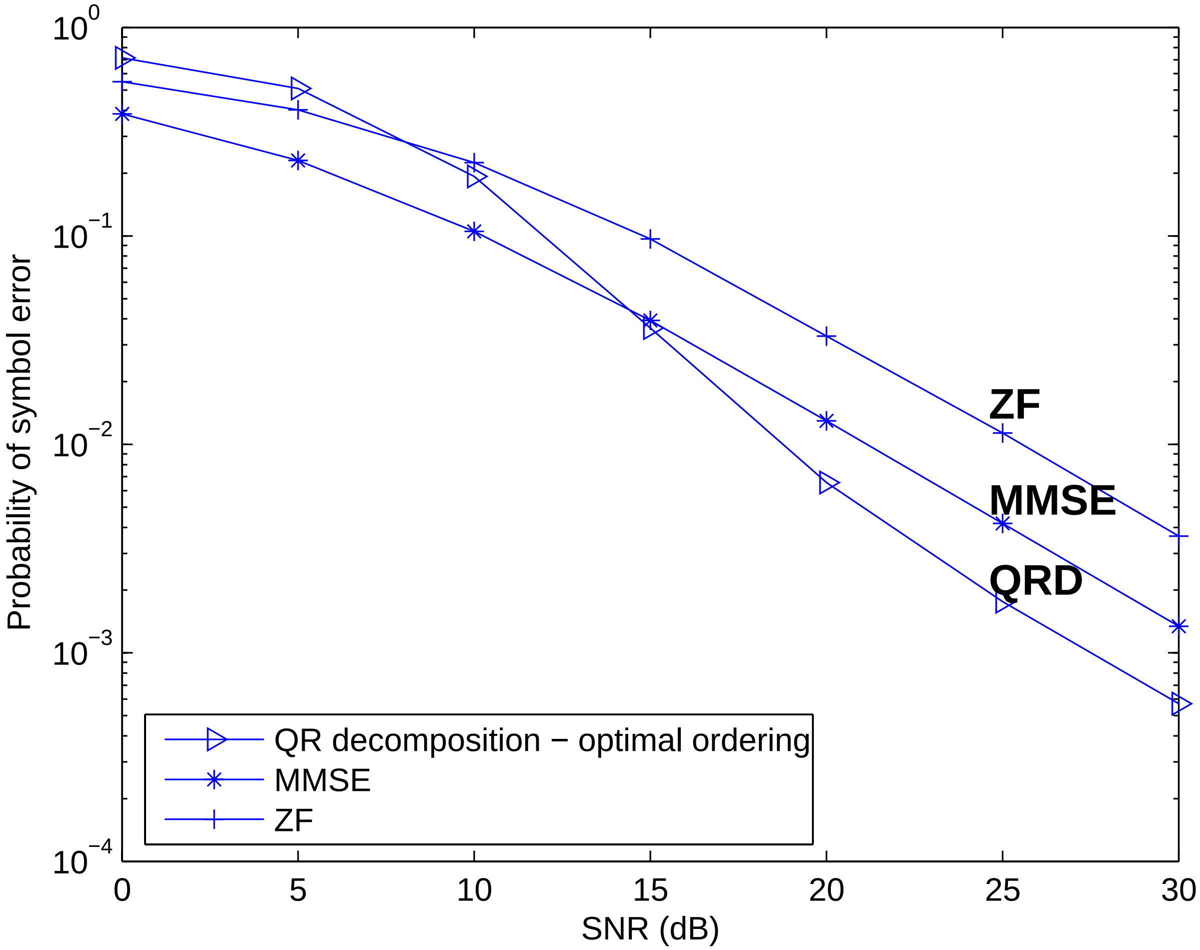
- Performance degrades with multipath
- QRD Method does not exploit the signal diversity inherent in channel multipath

Performance of optimal ordered QR decomposition with K=2, L=1, and N_T=2, 3, and 4.



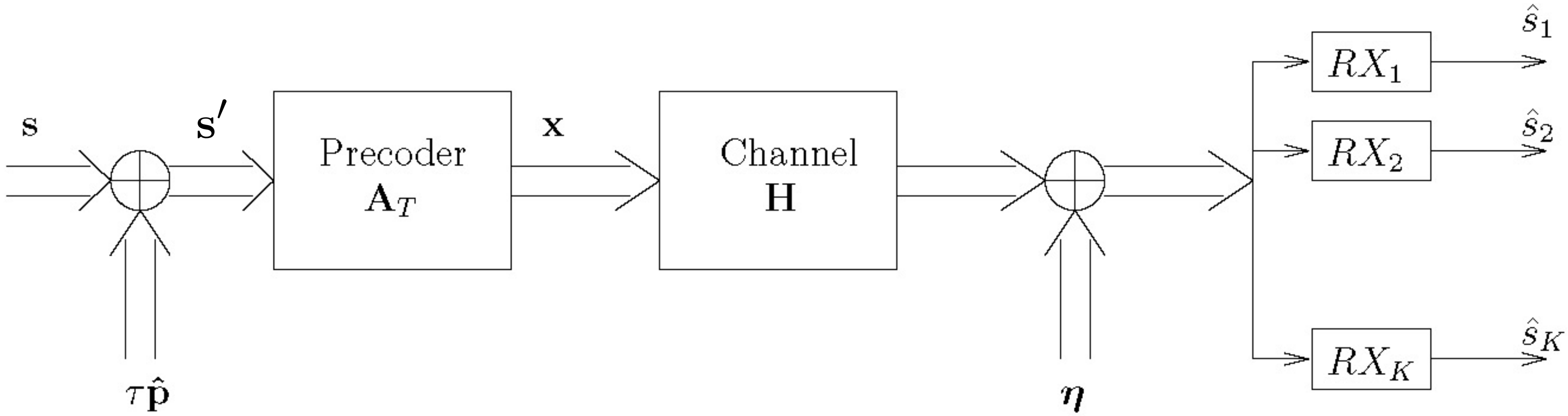
- Signal diversity can be achieved by increasing the number of transmit antennas

Comparison of the QR decomposition and the linear precoders with N_T=K=4



- QRD method outperforms the linear precoders at high SNR
- Performance of QRD method at low SNR can be improved by proper scaling

Vector Precoding
(Peel et. al. (2005), Hochwald et. al. (2005))



- Flat fading channel
- The data vector $\mathbf{s}$ is modified by the addition of a precoding vector $\hat{\mathbf{p}}$ so that

$$\mathbf{s}' = \mathbf{s} + \tau\hat{\mathbf{p}}$$

$\tau \sim$ real positive number

$\hat{\mathbf{p}} \sim$ K-dimensional vector with complex-valued elements where the real and imaginary components are integers

ZF Vector Precoding

(Peel et. al. (2005), Hochwald et. al. (2005))

- Transmitted Signal Vector

$$\mathbf{x} = \mathbf{A}_T(\mathbf{s} + \tau\hat{\mathbf{p}})$$

where

$$\mathbf{A}_T = \alpha\mathbf{H}^\dagger = \alpha\mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$$

Hence,

$$\mathbf{x} = \alpha\mathbf{H}^\dagger(\mathbf{s} + \tau\hat{\mathbf{p}})$$

- The offset vector $\hat{\mathbf{p}}$ is selected to have complex-valued integer elements and is chosen to minimize the power in the transmitted signal, i.e.,

$$\hat{\mathbf{p}} = \text{arg}\min_{\mathbf{p}}\|\alpha\mathbf{H}^\dagger(\mathbf{s} + \tau\mathbf{p})\|^2$$

- By constraining the real and imaginary components of $\hat{\mathbf{p}}$ to be integers, the receivers may use the modulo operation, as in Tomlinson-Harashima precoding, to recover the data.
- $\hat{\mathbf{p}}$ can be determined by sphere encoding search

ZF Vector Precoding

(Peel et. al. (2005), Hochwald et. al. (2005))

- The scalar τ is selected large enough, so that each receiver applies the modulo operation to the real and imaginary components of each element of the received vector

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\eta}$$

- The k[th] user assumes that its received signal has the form

$$y_k = \alpha(s_k + \tau p_k) + \eta_k$$

- The k[th] user performs the modulo operation on y_k to remove p_k and passes the result to the decoder.
- To achieve a symmetric decoding region around the real and imaginary components of every signal constellation symbol, τ is selected as

$$\tau = 2|s_k|_{max} + \Delta$$

$|s_k|_{max} \sim$ signal constellation symbol having the largest magnitude

$\Delta \sim$ distance between adjacent constellation symbols

MMSE Vector Precoding

(Peel et. al. (2005), Hochwald et. al. (2005))

- Apply the perturbation vector to the data vector

$$\mathbf{s}' = \mathbf{s} + \tau\hat{\mathbf{p}}$$

- Transmit the vector $\mathbf{x}$, where

$$\mathbf{x} = \mathbf{A}_T(\mathbf{s} + \tau\hat{\mathbf{p}})$$

- and

$$\mathbf{A}_T = \alpha\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \beta\mathbf{I})^{-1}$$

- $\hat{\mathbf{p}}$ has complex-valued elements where the real and imaginary components are integers and is selected to minimize the power in the transmitted signal, i.e.,

$$\hat{\mathbf{p}} = \arg\min_{\mathbf{p}} \|\alpha\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \beta\mathbf{I})^{-1}(\mathbf{s} + \tau\mathbf{p})\|^2$$

$\alpha \sim$ selected as in the ZF vector precoder

$\beta \sim$ selected to satisfy the transmitted power allocation constraint

$\tau \sim$ selected to maximize SINR

MMSE Vector Precoding

(Peel et. al. (2005), Hochwald et. al. (2005))

- Received Signal

$$\mathbf{y} = \alpha \mathbf{H}\mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \beta \mathbf{I})^{-1}(\mathbf{s} + \tau \hat{\mathbf{p}}) + \boldsymbol{\eta}$$

- The kth user assumes that its received signal has the form

$$y_k = \alpha(s_k + \tau p_k) + \eta'_k$$

$\eta'_k \sim$ AWGN + residual interference

- kth user performs the modulo operation on y_k to remove p_k and passes the result to the decoder

Vector Precoding Methods

- Achieve the full signal diversity N_T in a flat fading channel
- The method of sphere encoding (decoding) is computationally complex

Lattice Reduction Technique for Precoding

- Lattice signal constellations are commonly used in designing signal sets for communication systems, e.g. square QAM constellations
- Lattice in n (complex) dimensions

$$\Lambda = \left\{ \mathbf{x} \mid \mathbf{x} = \sum_{k=1}^{n} c_k \mathbf{g}_k \right\}$$

$\mathbf{G} = [\mathbf{g}_1\ \mathbf{g}_2\ \ldots \mathbf{g}_n] \sim$ generator matrix, where $\{\mathbf{b}_i\}$ are basis vectors

$\mathbf{C} = [c_1\ c_2 \ldots c_n] \sim$ n-dimensional vector of complex integer weights

- $\mathbf{G}$ is not unique. If $\mathbf{G}$ is a generator matrix, then $\mathbf{G}' = \mathbf{GF}$ is also a generator matrix of the lattice Λ if $\mathbf{F}$ is a square matrix with integer entries such that $\det(\mathbf{F}) = \pm 1,\ \pm j$. Then $\mathbf{F}^{-1}$ also exists. The new generator matrix $\mathbf{G}'$ defines a new basis for Λ

$$\mathbf{G} = \mathbf{G}'\mathbf{F}^{-1} \quad \text{or} \quad \mathbf{G}' = \mathbf{GF}$$

Lattice Reduction Technique for Precoding

- Received signal vector: $\mathbf{y} = \mathbf{H}\mathbf{s} + \boldsymbol{\eta}$
- The noiseless received vector $\mathbf{H}\mathbf{s}$ is a subset of a lattice that is transformed by the channel matrix $\mathbf{H}$, whose columns are the basis of this transformed lattice. In general, the columns of $\mathbf{H}$ are not orthogonal.
- The basis vectors of the transformed lattice may be orthogonalized and reduced in magnitude:

  New Generator Matrix: $\mathbf{G}' = \mathbf{H}\mathbf{F}$
- Columns of $\mathbf{G}'$ are orthogonal (or nearly orthogonal)
- $\mathbf{F}$ is a square matrix having integer real and imaginary components such that $\mathbf{F}$ satisfies the condition: $|\det(\mathbf{F})| = 1$
- $\mathbf{F}^{-1}$ also exists.

Lenstra-Lenstra-Lovász Lattice Basis Reduction Algorithm

- The LLL lattice basis reduction algorithm which takes a given lattice basis as input and outputs a basis with short, nearly orthogonal vectors. The algorithm was developed originally for factoring polynomials with rational coefficients into irreducible polynomials.
- LLL algorithm is designed for matrices having real elements.
- Extension to complex matrices has been published.

Lattice Reduction Technique for Precoding

- Apply the LLL algorithm on the columns of:

$$\mathbf{H}^{\dagger} = \mathbf{H}^{H}(\mathbf{H}\mathbf{H}^{H})^{-1} \quad \text{for ZF}$$

$$\mathbf{H}^{\dagger}_{MSE} = \mathbf{H}^{H}(\mathbf{H}\mathbf{H}^{H} + \beta\mathbf{I})^{-1} \quad \text{for MMSE}$$

- Thus, we obtain

$$\mathbf{H}^{\dagger} = \mathbf{W}_{ZF}\mathbf{F}_{ZF}$$

$$\mathbf{H}^{\dagger}_{MSE} = \mathbf{W}_{MSE}\mathbf{F}_{MSE}$$

where the K columns of the N_T x K matrix $\mathbf{W}$ are approximately orthogonal and constitute a reduced basis.

- The approximate offset vector is

$$\hat{\mathbf{p}}_{ZF} = -\mathbf{F}^{-1}Q(\mathbf{F}\mathbf{s})$$

where $Q(\cdot)$ denotes the component-wise rounding of K-dimensional vector to the integer lattice.

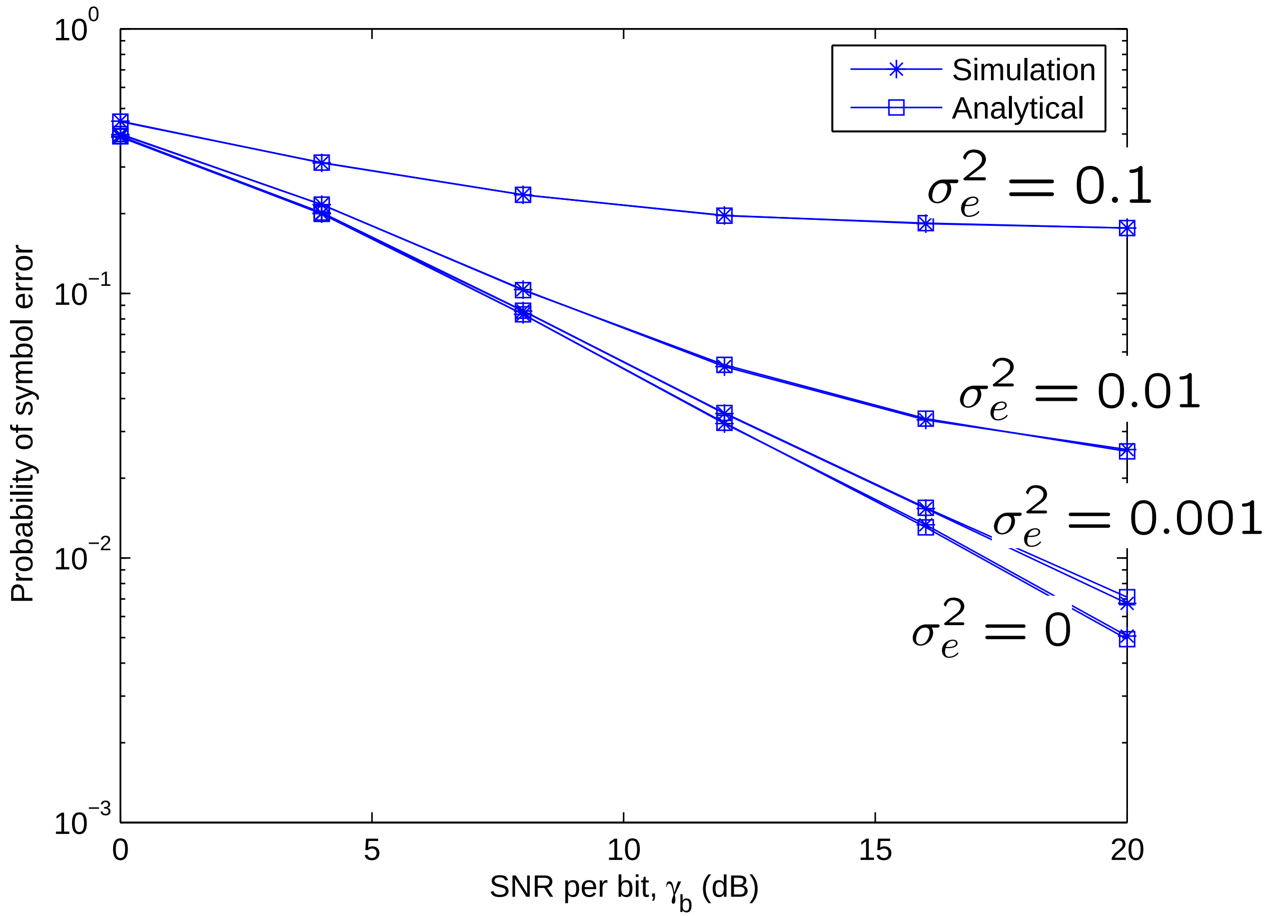
Effect of Channel Estimation on Performance of QRD Method

- The estimate of the channel matrix is modeled as

$$\hat{\mathbf{H}} = \mathbf{H} + \Delta\mathbf{H}$$

  where $\Delta\mathrm{H}$ can be modeled as a matrix of i.i.d. complex circularly symmetric Gaussian random variables with zero-mean and variance σ_e^2 , which becomes smaller as the training length increases.
- In the case of imperfect channel estimation, it is not possible to remove the effect of the multiuser interference due to a mismatch between the precoder and the channel.
- Consequently, it is necessary to include the estimation error in deriving the probability of symbol error.

Effect of Channel Estimation on Performance of QRD Method with N_T=K=2, and $\sigma_e^2 = 0,\ 0.001,\ 0.01,\ 1$



- As σ_e^2 increases, performances degrades significantly compared to the case when the channel is known.

More Work Needed

1. Effect of Channel Estimation Errors on Performance of Nonlinear Precoding Methods
2. Effect of ISI on Performance of Nonlinear Precoding Methods
3. Reduction in Computational Complexity of Nonlinear Precoding Methods in Frequency Selective Channels

Selected References on MIMO Precoding

1. A.K. Lenstra, H. W. Lenstra and L. Lovász, “Factoring Polynomials with Rational Coefficients,” Math. Ann., pp. 515-534, 1982.
2. L. Babai, “On Lovász’ Lattice Reduction and the Nearest Lattice Point Problem,” Combinatorica, vol. 6, pp. 1-13, 1986.
3. Y. H. Gan and W. H. Mow, “Accelerated Complex Lattice Reduction Algorithm Applied to MIMO Detection,” Proc. 2005 Globecom, pp. 2953-2957, St. Louis, MO, November 28 – December 2, 2005.
4. Y. H. Gan, C. Ling, and W. H. Mow, “Complex Lattice Reduction Algorithm for Low-Complexity MIMO Detection,” submitted to IEEE Trans. On Signal Processing, January 2007.
5. E. Agrell, T. Eriksson, A. Vardy and K. Zeger, “Closest Point Search in Lattices,” IEEE Trans. Inf. Theory, vol. 48, pp. 2201-2214, August 2002.
6. H. Yao and G. W. Wornell, “Lattice-Reduction-Aided Detectors for MIMO Communication Systems," Proc. IEEE Globecom, 2002, Taipai, Taiwan, November 2002.
7. C. Windpassinger, R. F. H. Fischer, and J. B. Huber, “Lattice-Reduction-Aided Broadcast Precoding,‘‘ IEEE Trans. Commun., vol. 52, pp. 2057-2060, December 2004.
8. R. F. H. Fischer, C. Windpassinger, A. Lampe, and J. B. Huber, “MIMO Precoding for Decentralized Receivers,” Proc. IEEE ISIT 2002, p. 496, Lausanne, Switzerland, June 2002.
9. C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, “A Vector-Perturbation Technique for Near-Capacity Multiantenna Multiuser Communication - Part I: Channel Inversion and Regularization,” IEEE Trans. Commun., vol. 53, pp. 195-202, January 2005.
10. B. M. Hochwald, C. B. Peel, and A. L. Swindlehurst, “A Vector-Perturbation Technique for Near-Capacity Multiantenna Multiuser Communication - Part II: Perturbation,” IEEE Trans. Commun., vol. 53, pp. 537-544, March 2005.
11. G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky, “Simplified processing for high spectral efficiency wireless communication employing multi-element arrays,‘‘ IEEE J. Sel. Areas in Communications, vol. 17, pp. 1841-1852, November 1999.
12. G. D. Golden, G. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, “Detection Algorithm and Initial Laboratory Results Using V-BLAST Space-Time Communication Architecture,” Electronics Letters, vol. 35, pp. 14-15, January 1999.

Selected References on MIMO Precoding

13. P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, “V-BLAST: An Architecture for Realizing Very High Data Rates Over Rich-Scattering Wireless Channels,” Proc. Inter. Symp. On Signals, Systems and Electronics (ISSSE), Pisa, Italy, September 1998.
14. P. Amihood, E. Masry, L. B. Milstein, and J. G. Proakis, “Performance Analysis of a Pre-BLAST DFE Technique for MISO Channels with Decentralized Receivers,'' IEEE Trans. Communications, vol. 55, pp. 1385-1396, July 2007.
15. P. Amihood, E. Masry, L. B. Milstein, J. G. Proakis, “Performance Analysis of High Data Rate MIMO Systems in Frequency Selective Fading Channels,'' accepted in *IEEE Transactions on Information Theory*, August 2007.
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17. P. Amihood, E. Masry, L. B. Milstein, J. G. Proakis, “Asymptotic Performance of Multicode MIMO Systems in Frequency Selective Fading Channels,” in *Proc. IEEE Military Commun.Conf. (MILCOM)*, Atlantic City, New Jersey, USA, vol. 5, pp. 3036- 3041, October 17 - 20, 2005.
18. P. Amihood, E. Masry, L. B. Milstein, J. G. Proakis,”Analysis of a MISO Pre-BLAST-DFE Technique for Decentralized Receivers,” in *Proc. of the 40th Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, USA, October 29 - November 1, 2006.
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