Network Beamforming and Interference Cancellation

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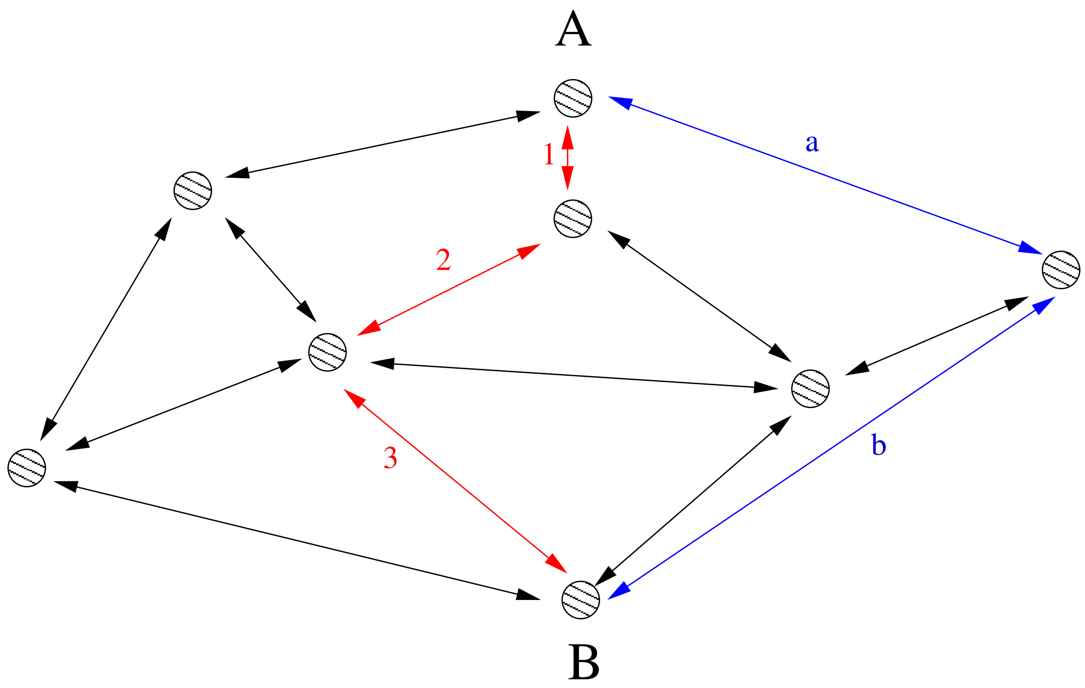
Outline

- Introduction
- Network Beamforming
- Partial Channel Information (Quantized Feedback)
- Interference Cancellation
- Conclusions

Cooperative Diversity

The improvement in reliability (error rate) achieved by having the relays cooperate.



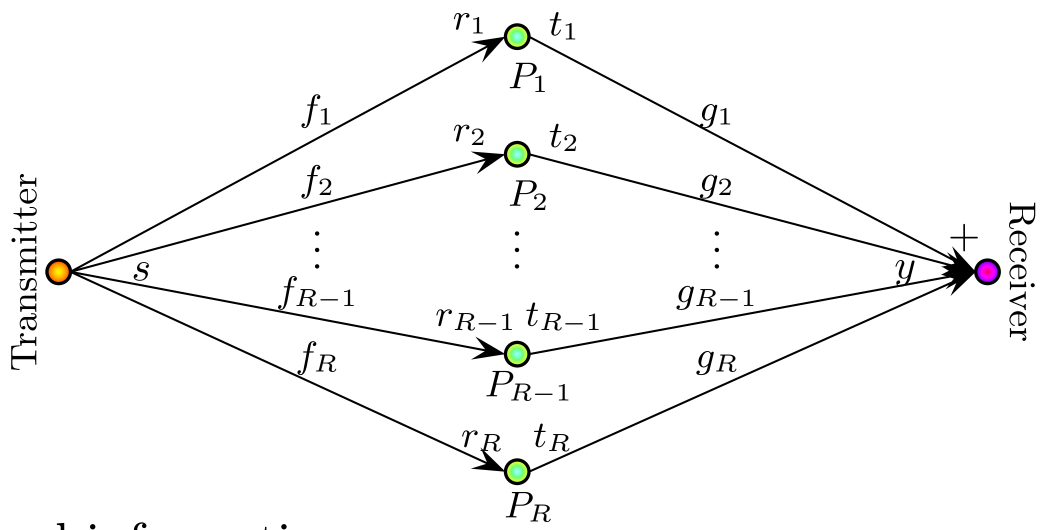
Cooperative Schemes

- Amplify-and-forward
- Decode-and-forward
- Relay selection
- Coded-cooperation
- $\cdots$

The missing piece: **Adaptive relay power control**

Network Beamforming

Wireless Relay Network



- Full channel information

transmitter	Relay i	receiver
/	f_i and g_i	All $(f_1, g_1, \ldots, f_R, g_R)$

- A **separate short-term** power constraint on each node:
  - A node CANNOT save its power to favor better/worse channel realizations.

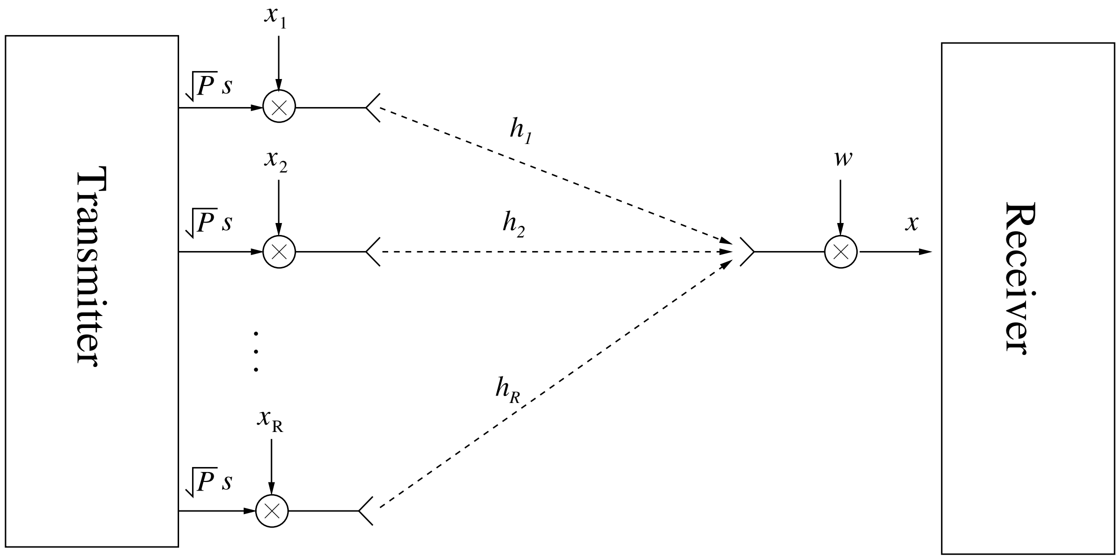
Protocol: Amplify-and-Forward

- Amplify-and-forward + adaptive transmit power and direction.
- Two-phase protocol:

Phase 1: transmitter $\longrightarrow$ relays	**Phase 2:** relays $\longrightarrow$ receiver
$\alpha_0\sqrt{P_0}s \longrightarrow r_i = \alpha_0\sqrt{P_0}f_i s + v_i$	$t_i \;=\; \frac{\alpha_i\sqrt{P_i}}{\sqrt{1+\alpha_0^2\|f_i\|^2 P_0}} r_i e^{j\theta_i}$ $\longrightarrow y = \sum_{i=1}^{R} g_i t_i + w$

- Transmitter uses power $\alpha_0^2 P_0$. Relay i uses power $\alpha_i^2 P_i$.
- Our Problem: **Optimize α_0 and $\mathbf{x} = \begin{bmatrix} \alpha_1 e^{j\theta_1} & \cdots & \alpha_R e^{j\theta_R} \end{bmatrix}$ (relay beamforming vector).**

Multiple-Antenna Systems



- **Beamforming:** $\mathbf{x}_{opt} = \mathbf{h}^*/\|\mathbf{h}\|$ $(\alpha_i = |h_i|/\|\mathbf{h}\|,\ \theta_i = -\arg h_i)$.
  1) Antennas can share power. 2)Each antenna knows s.
- Similar idea but totally different results for networks

Network Beamforming Problem Formulation

- Optimal angles: $\theta_i = -\arg f_i - \arg g_i$.
- **The transmitter should always use its full power.**
- The SNR optimization problem becomes:

$$\max_{\alpha_{\mathbf{i}} \in [0,1]} \frac{P_0 \left(\sum_{i=1}^{R} \frac{\alpha_{\mathbf{i}} |f_i g_i| \sqrt{P_i}}{\sqrt{1+|f_i|^2 P_0}} \right)^2}{1 + \sum_{i=1}^{R} \frac{\alpha_{\mathbf{i}}^{\mathbf{2}} |g_i|^2 P_i}{1+|f_i|^2 P_0}}.$$

Formulation Continued

- Define

$$\mathbf{x} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_R \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \frac{|f_1 g_1|\sqrt{P_1}}{\sqrt{1+|f_1|^2 P_0}} \\ \vdots \\ \frac{|f_R g_R|\sqrt{P_R}}{\sqrt{1+|f_R|^2 P_0}} \end{bmatrix}, \mathbf{a} = \begin{bmatrix} \frac{|g_1|\sqrt{P_1}}{\sqrt{1+|f_1|^2 P_0}} \\ \vdots \\ \frac{|g_R|\sqrt{P_R}}{\sqrt{1+|f_R|^2 P_0}} \end{bmatrix}, \mathbf{c} = \begin{bmatrix} |f_1| \\ \vdots \\ |f_R| \end{bmatrix}, A = \mathrm{diag}\{\mathbf{a}\}.$$

- Transformation $\mathbf{y} = A\mathbf{x}$,

$$\max_{\mathbf{y}} \frac{\langle \mathbf{c}, \mathbf{y} \rangle^2}{1 + \|\mathbf{y}\|^2} \quad \text{s.t.} \quad 0_R \preceq \mathbf{y} \preceq \mathbf{a}.$$

- **It is not a concave function.**

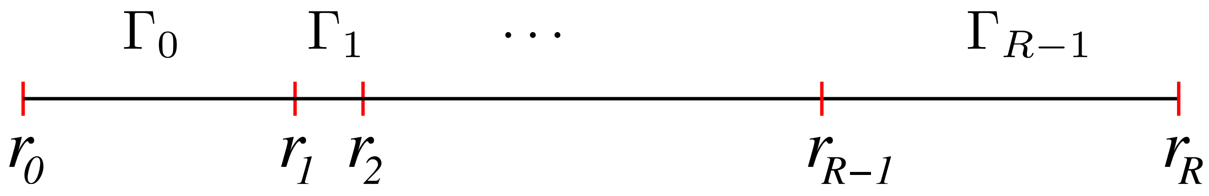
Ideas to Solve the Optimization

- The difficulty comes from the shape of the feasible region.
- If $\|\mathbf{y}\|$ is fixed, the solution is the vector on the hypersphere that has the smallest angle with $\mathbf{c}$.
- Find the optimal solution for each feasible length of $\mathbf{y}$, then find the optimal length.
- The feasible interval of the length of $\mathbf{y}$ is $[0, \|\mathbf{a}\|]$.
- To solve the problem, we need to decompose the interval $[0, \|\mathbf{a}\|]$ into R smaller intervals.

Radius Decomposition

- Define $\phi_j = \phi(f_j, g_j, P_j) = c_j/a_j = |f_j/g_j|\sqrt{1 + |f_j|^2 P_0/P_j}$.
- Order $\phi_{\tau_1} \geq \phi_{\tau_2} \geq \cdots \geq \phi_{\tau_R}$.
- $[0, \|\mathbf{a}\|] = [r_0, r_1] \cup \cdots [r_{R-1}, r_R]$, where $r_0 = 0$ and

$$r_i = \sqrt{\phi_{\tau_i}^{-2}\|\mathbf{c}_{\tau_i,\ldots,\tau_R}\|^2 + \|\mathbf{a}_{\tau_1,\ldots,\tau_{i-1}}\|^2} = \sqrt{\phi_{\tau_i}^{-2}\|\mathbf{c}_{\tau_{i+1},\ldots,\tau_R}\|^2 + \|\mathbf{a}_{\tau_1,\ldots,\tau_i}\|^2}$$



Steps to Solve the Problem

$$\underbrace{\max_{i=0,\ldots,R} \underbrace{\max_{r\in\Gamma_i} \frac{1}{1+r^2}\left(\underbrace{\max_{\|\mathbf{y}\|=r} \langle \mathbf{c},\mathbf{y}\rangle}_{\mathbf{z}^{(i,r)}}\right)^2}_{\mathbf{x}^{(i)} \text{ or } \mathbf{y}^{(i)}}}_{\mathbf{x}^*}.$$

Step 1	**Step 2**	**Step 3**
Find $\mathbf{z}^{(i,r)}$	Find $\mathbf{x}^{(i)}$ or equivalently $\mathbf{y}^{(i)}$	Find $\mathbf{x}^*$

Step 1: the Inner Product Maximization

- $\mathbf{z}^{(i,r)}$ is the vector on the hypersphere of radius r and within the feasible region that has the minimum angle with $\mathbf{c}$.

Lemma 1. $\mathbf{z}^{(i,r)}_{\tau_1,\ldots,\tau_i} = \mathbf{a}_{\tau_1,\ldots,\tau_i}$ *and* $\mathbf{z}^{(i,r)}_{\tau_{i+1},\ldots,\tau_R} = \lambda_{r,i}\mathbf{c}_{\tau_{i+1},\ldots,\tau_R}$, *where*

$$\lambda_{r,i} = \frac{\sqrt{r^2 - \|\mathbf{a}_{\tau_1,\ldots,\tau_i}\|^2}}{\|\mathbf{c}_{\tau_{i+1},\ldots,\tau_R}\|}$$

and $\mathbf{a}_{i_1,\ldots,i_k} = \begin{bmatrix} a_{i_1} & \cdots & a_{i_k} \end{bmatrix}^T$. *Thus,*

$$\max_{\|\mathbf{y}\|=r\in\Gamma_i,\mathbf{y}\in\Lambda} \langle \mathbf{c}, \mathbf{y} \rangle = \sum_{m=1}^{i} b_{\tau_m} + \lambda_{r,i}\|\mathbf{c}_{\tau_{i+1},\ldots,\tau_R}\|^2.$$

Step 2: the ith Subproblem

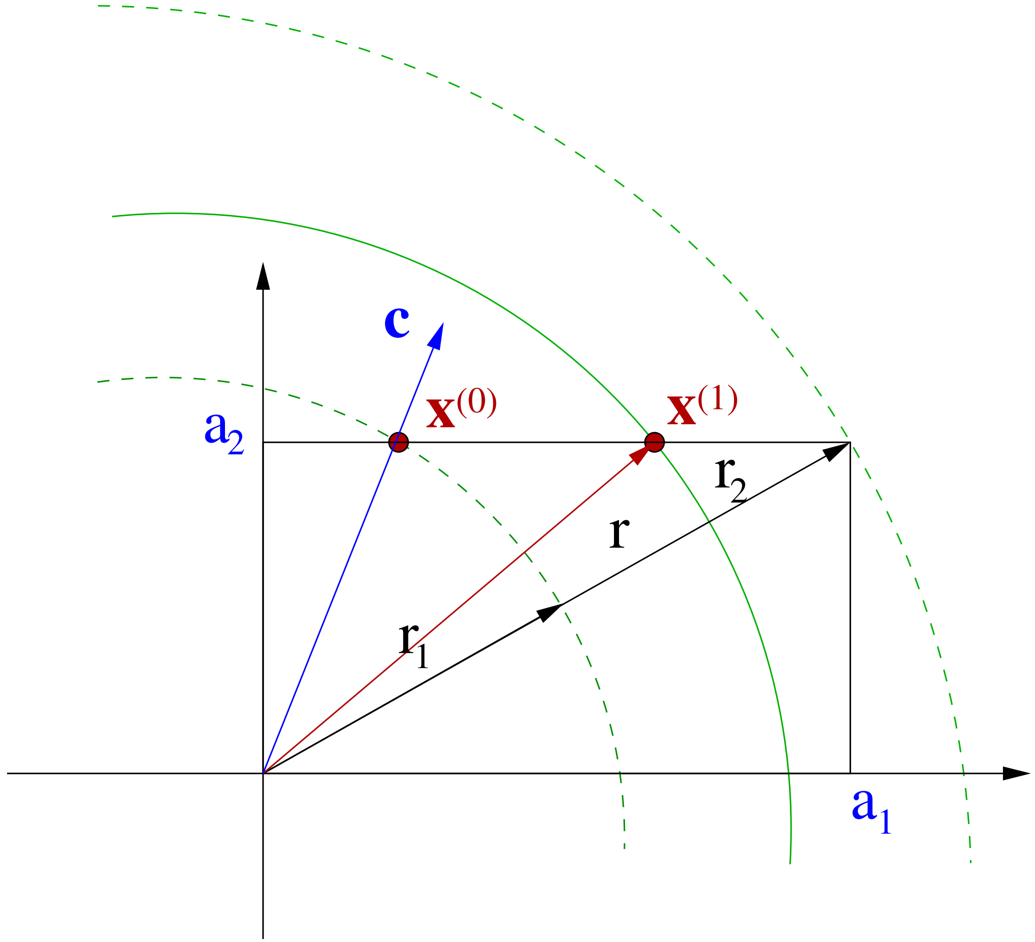
The optimization of $\mathbf{y}^{(i)}$ is thus transformed into a 1D problem:

$$\max_{\lambda\in[\phi_{\tau_i}^{-1},\phi_{\tau_{i+1}}^{-1}]} \frac{\left(\sum_{m=1}^{i} b_{\tau_m} + \|\mathbf{c}_{\tau_{i+1},\dots,\tau_R}\|^2\lambda\right)^2}{1+\|\mathbf{a}_{\tau_1,\dots,\tau_i}\|^2+\|\mathbf{c}_{\tau_{i+1},\dots,\tau_R}\|^2\lambda^2}.$$

Lemma 2.

- $\mathbf{y}^{(0)} = \phi_{\tau_1}^{-1}\mathbf{c}$
- $\begin{cases} \mathbf{y}^{(i)}_{\tau_1,\dots,\tau_i} = \mathbf{a}_{\tau_1,\dots,\tau_i} \\ \mathbf{y}^{(i)}_{\tau_{i+1},\dots,\tau_R} = \min\left\{\frac{1+\|\mathbf{a}_{\tau_1,\dots,\tau_i}\|^2}{\sum_{m=1}^{i} b_{\tau_m}}, \phi_{\tau_{i+1}}^{-1}\right\}\mathbf{c}_{\tau_{i+1},\dots,\tau_R} \end{cases}.$

Illustration of a 2D Case



The Main Theorem

- Define $\phi_j = \phi(f_j, g_j, P_j) = c_j/a_j = |f_j/g_j|\sqrt{1 + |f_j|^2 P_0/P_j}$.
- Order $\phi_{\tau_1} \geq \phi_{\tau_2} \geq \cdots \geq \phi_{\tau_R}$.

Theorem 1. *Define* $\mathbf{x}^{(i)}$ *as*

$$x_j^{(i)} = \begin{cases} 1 & j = \tau_1, \ldots, \tau_i \\ \lambda_i \phi_j & j = \tau_{i+1}, \ldots, \tau_R \end{cases}$$

and

$$\lambda_i = \frac{1 + \|\mathbf{a}_{\tau_1, \ldots, \tau_i}\|^2}{\sum_{m=1}^{i} b_{\tau_m}}.$$

- *The solution is* $\mathbf{x}^{(i_o)}$ *with* i_o *the smallest* i *such that* $\lambda_i < \phi_{\tau_{i+1}}^{-1}$.
- $i_0 \geq 1$.

Discussion

- The optimal transmit power of Relay i is not binary and not even a differentiable function of channel coefficients.
- A relay transmit power is determined not only by its own channels but all others' as well. P_0 and all the $P_1, \ldots, P_R$ play roles as well.
- There exists an i_0 such that the i_0 relays with the largest ϕ's use their full power.
- At least one relay uses its full power. This can be represented by $\mathcal{X} \triangleq \{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } \|\mathbf{x}\|_\infty = 1\}$.
- For the remaining $R - i_0$ relays, the power used at the jth relay is $\lambda_{i_0}^2 \phi_j^2 P_j$ with λ_{i_0} a constant for each channel realization.

Distributed Scheme 1

- For networks with a small number of relays
- The receiver broadcasts

$(b_1, \dots, b_R)$	λ_{i_0}

- If $b_i = 1$, Relay i uses its full power. Otherwise, it uses power $\lambda_{i_0}^2 \phi_j^2 P_j$.
- Feedback bits: $R + B_1$, with B_1 the number of bits for λ_{i_0}.

Distributed Scheme 2

- More economical in networks with a lot of relays
- The receiver broadcasts

d $(\phi_{\tau_{i_0}} > d > \phi_{\tau_{i_0+1}})$	λ_{i_0}

- If $\phi_j > d$, Relay j uses its full power. Otherwise, it uses power $\lambda_{i_0}^2 \phi_j^2 P_j$.
- Feedback bits: $2B_1$.

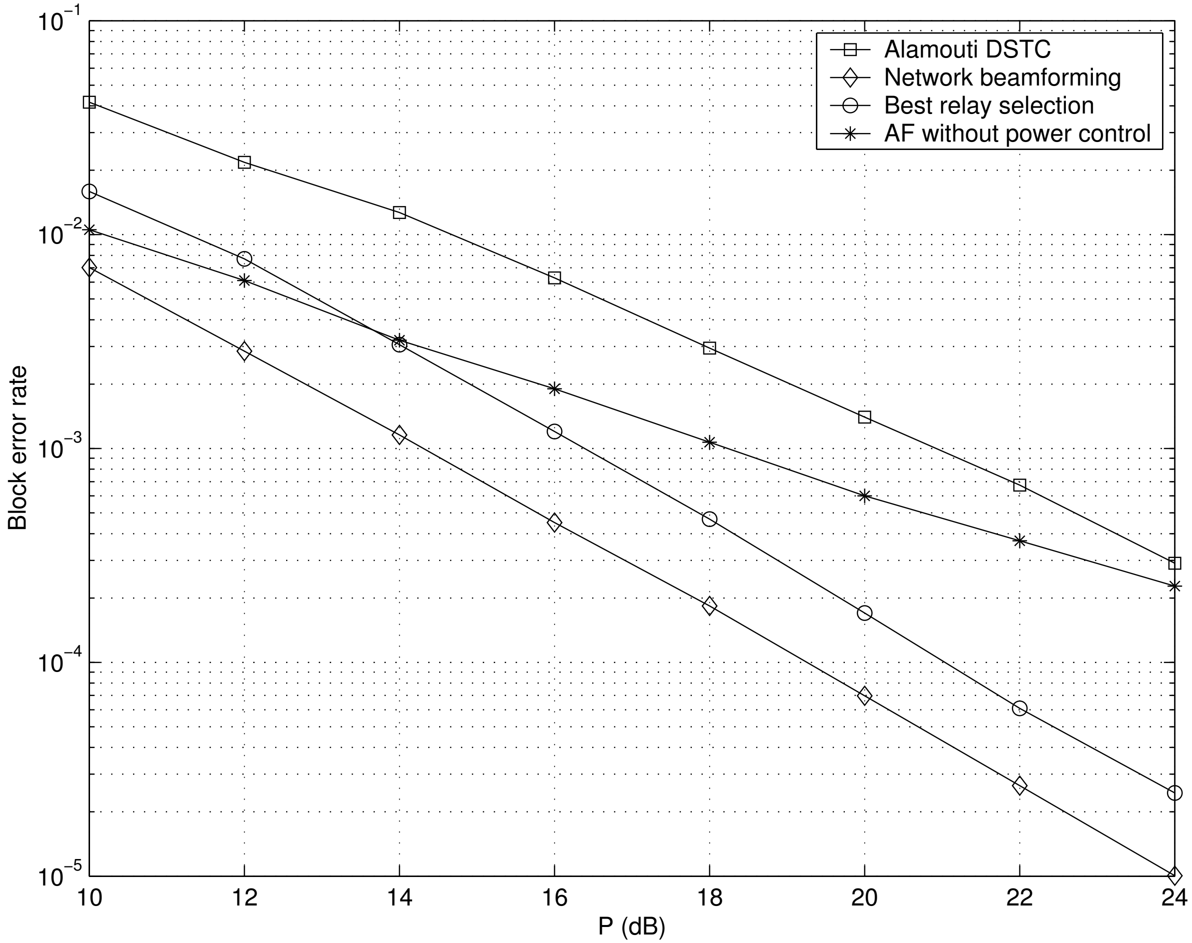
Simulation Setup

- Schemes
  - Network beamforming: match filters and power control at relays
  - Coherent amplify-and-forward: match filters only
  - Distributed space-time coding: no channel information at relays
  - Best-relay selection: choose the relay with the highest

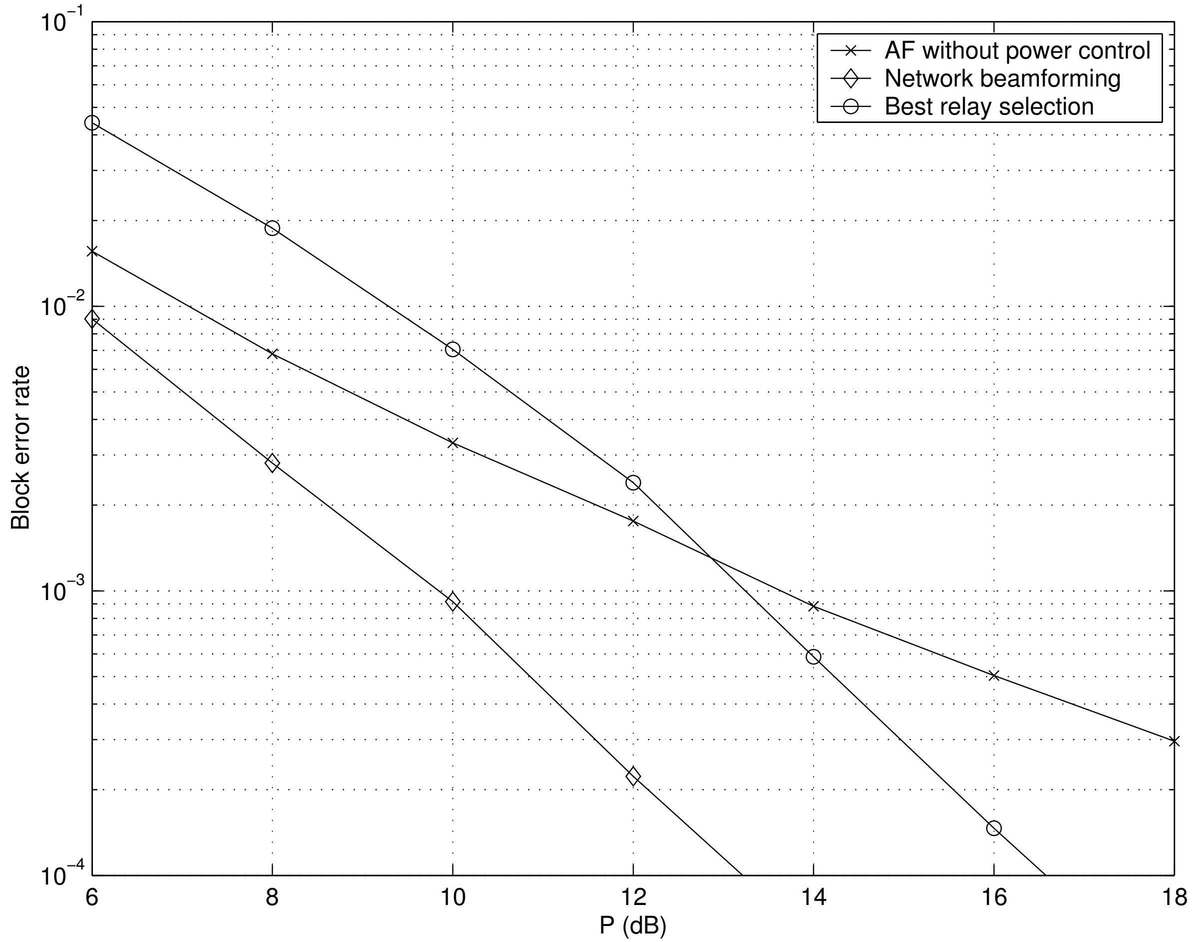
$$h_j = h(f_j, g_j, P_j) = \frac{P_j |f_j g_j|^2}{1 + |f_j|^2 P_0 + |g_j|^2 P_j}.$$

  - P_i/P is a constant.

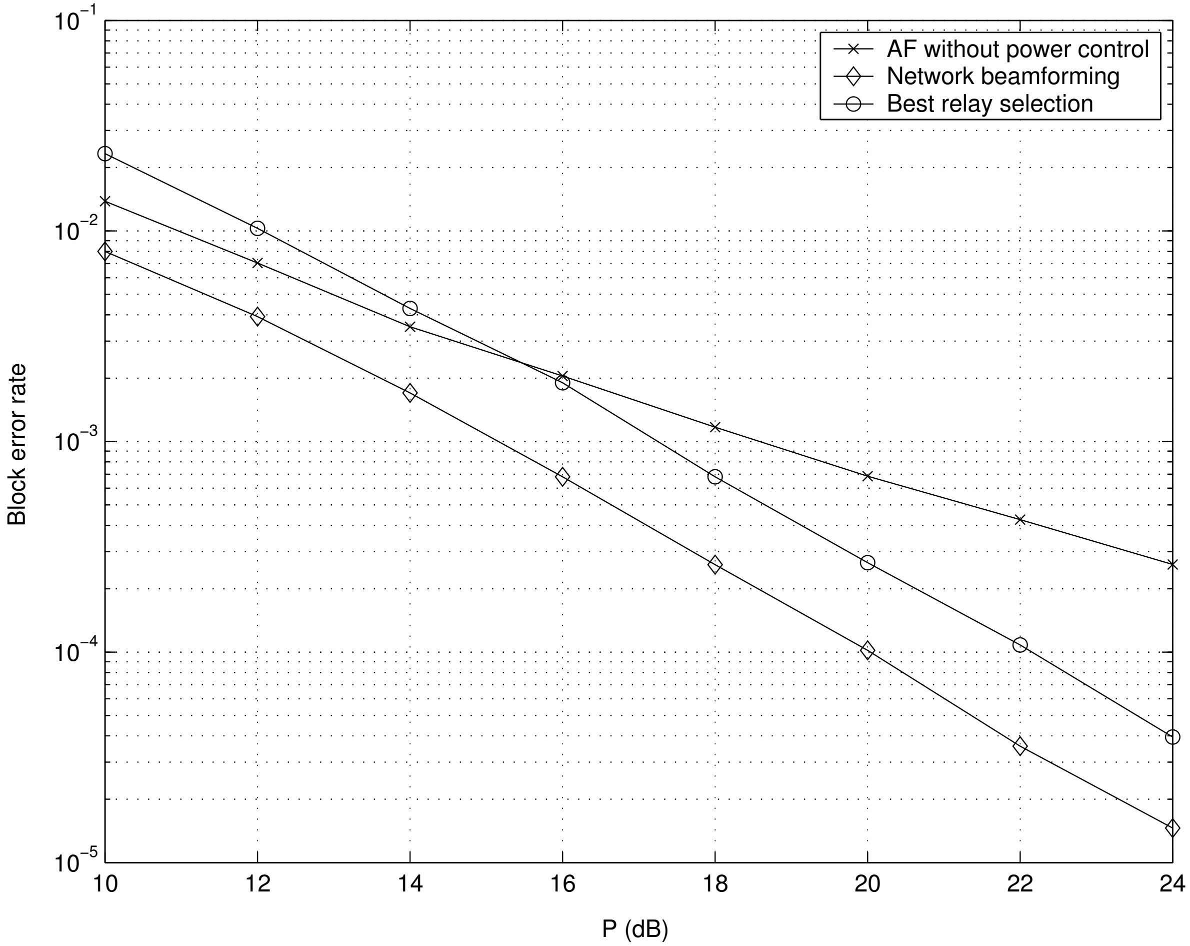
Simulation: Rayleigh Fading and $P_i = P$ (2 relays)



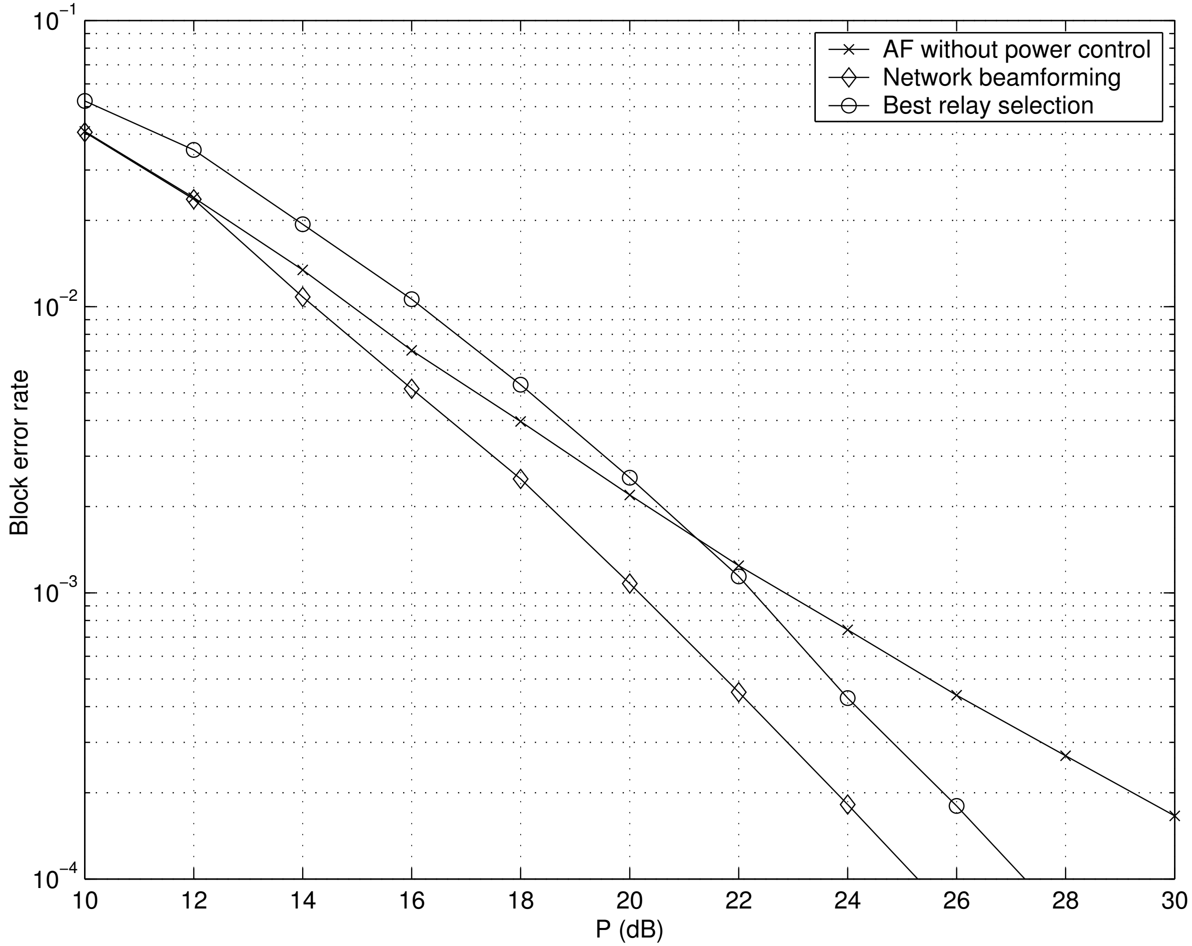
Simulation: Rayleigh Fading and $P_i = P$ (3 relays)



Network with different relay powers $P_0 = P_1 = 2P_2$



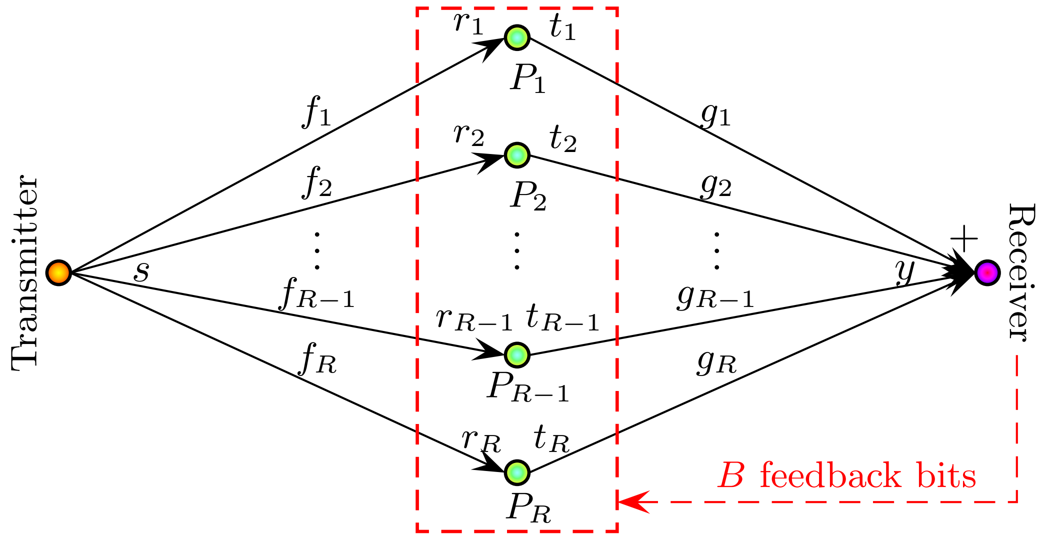
Network with path-loss plus fading channels with $\rho = 2$



What's Next?

- Beamforming in wireless relay networks with perfect channel information at relays.
  - Adaptive relay power control
  - Analytical solution with linear complexity
  - Distributive schemes
  - Full diversity and optimal array gain
- Next: **network with partial channel information**
  - Quantized feedback
  - Channel statistics (mean and covariance)

Wireless Relay Network with Quantized Feedback



- The ith relay knows f_i.
- Each relay has B bits of partial CSI provided by the receiver.
- The feedback channel is error-free and delay-free.
- The information each relay receives from the feedback is the same.
  - Common codebook: $\mathcal{C} = \{\mathbf{x}^1, \ldots, \mathbf{x}^M\}$, contains $M = 2^B$ beamforming vectors.

Feedback Transmission Scheme

- Each channel realization $\mathbf{h} = (f_1, g_1, \ldots, f_R, g_R)$, is quantized using a quantizer $\mathcal{Q} \triangleq \mathcal{D}(\mathcal{E}(\mathbf{h}))$:
  - The encoder mapping $\mathcal{E} : \mathbb{C}^{2R} \rightarrow \mathcal{I}$.
  - The decoder mapping $\mathcal{D} : \mathcal{I} \rightarrow \mathcal{C}$.
  - $\mathcal{I} = \{1, \ldots, M\}$.
- The receiver:
  - Performs the encoding operation $\mathcal{E}(\mathbf{h}) = m$,
  - Feeds back B bits, representing m.
- Each relay:
  - Decodes the feedback bits, $\mathcal{D}(m) = \mathbf{x}^m = (x_1^m, x_2^m, \ldots, x_R^m)$.
  - Uses the complex number x_i^m to adjust its transmit power and phase.

Performance Measure

- The received SNR is given by:

$$\gamma(\mathcal{Q}(\mathbf{h}),\mathbf{h}) = P_0 \frac{\left|\sum_{i=1}^{R} x_i^m f_i g_i \sqrt{\frac{P_i}{1+|f_i|^2 P_0}}\right|^2}{1+\sum_{i=1}^{R} |x_i^m|^2 |g_i|^2 \frac{P_i}{1+|f_i|^2 P_0}} \text{ for } \mathcal{Q}(\mathbf{h}) = \mathbf{x}^m \in \mathcal{C}.$$

- The performance measure is the bit error rate (BER):

$$P_e = \mathrm{E}_{\mathbf{h}}\left[\mathrm{Q}\left[\sqrt{2\gamma(\mathbf{h};\mathcal{Q},\mathcal{C})}\right]\right], \text{ for BPSK.}$$

- Goal: *Jointly optimize the quantizer $\mathcal{Q}$ and the quantization codebook $\mathcal{C}$, such that the BER is minimized.*

The Optimal Quantizer

- Given a fixed codebook $\mathcal{C}$, the optimal quantizer chooses the SNR maximizing beamforming vector in $\mathcal{C}$:

$$\mathcal{Q}^*_{\mathcal{C}}(\mathbf{h}) = \arg\max_{\mathbf{x}\in\mathcal{C}} \gamma(\mathbf{x}, \mathbf{h}).$$

- Codebook design: The Generalized Lloyd Algorithm (GLA)
  - For a given codebook $\{\mathbf{x}^m : m = 1, \ldots, M\}$, the optimal partition of the channel state space is given by:

$$\mathcal{R}_m \triangleq \{\mathbf{h} : \gamma(\mathbf{x}^m, \mathbf{h}) > \gamma(\mathbf{x}^n, \mathbf{h}), \forall n \neq m\}.$$

  - For a given partition $\{\mathcal{R}_m : m = 1, \ldots, M\}$, the optimal codebook vectors satisfy:

$$\mathbf{x}^m = \arg\min_{\mathbf{x}\in\mathcal{X}} \mathrm{E}_{\mathbf{h}} \left[\mathrm{Q}\left(\sqrt{2\gamma(\mathbf{x}, \mathbf{h})}\right) \mid \mathbf{h} \in \mathcal{R}_m \right].$$

Performance Analysis - Diversity

- Idea: Introduce structured codebook designs that provide full diversity. The GLA can be used to further improve the performance.
- The relay selection scheme:
  - For a given $\mathbf{h}$, *only one of the relays* is allowed to cooperate.
  - Codebook: $\mathcal{C}_e \triangleq \{\mathbf{e}^m : m = 1, \ldots, M\}$, $\mathbf{e}_i^m = 1$ if $i = m$, and $\mathbf{e}_i^m = 0$ for $i \neq m$.

Main Theorem

- **Theorem 2:** Let $P_i = \lambda_i P_0$, where $\lambda_i > 0$ are constants, and $\lambda \triangleq \max_i\{1 + \lambda_i^{-1/2}\}$. Then, the BER of the relay selection scheme is bounded by:

$$P_e < RP_0^{-R}\lambda^{2R}\sum_{n=0}^{R-1}\binom{R-1}{n}n!.$$

- The relay selection scheme achieves the full diversity order R for $B = \log_2 R$.

- In general, the diversity order $\min(2^B, R)$ is achievable with quantized feedback.

Performance Analysis - Average SNR Loss and Capacity Loss

- The *pointwise* SNR loss of $\mathcal{C}$: The difference between the received SNRs with perfect feedback and with $\mathcal{C}$, e.g.

$$D(\mathbf{h}) \triangleq \gamma(\mathcal{Q}_{\mathcal{X}}^{*}(\mathbf{h}), \mathbf{h}) - \gamma(\mathcal{Q}_{\mathcal{C}}^{*}(\mathbf{h}), \mathbf{h}).$$

- **Theorem 3:** The average SNR loss with quantized feedback decays at least exponentially as the number of feedback bits B:

$$\mathrm{E}_{\mathbf{h}}\left[D(\mathbf{h})\right] < \mu 2^{-\frac{B}{2(R-1)}},$$

where μ is independent of B.

- **Corollary:** The capacity loss with quantized feedback decays at least exponentially as the number of feedback bits B:

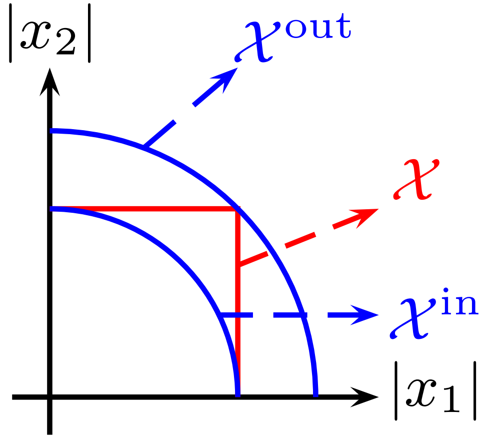
$$C_{Loss} < \mu 2^{-\frac{B}{2(R-1)}}.$$

Performance Analysis - BER

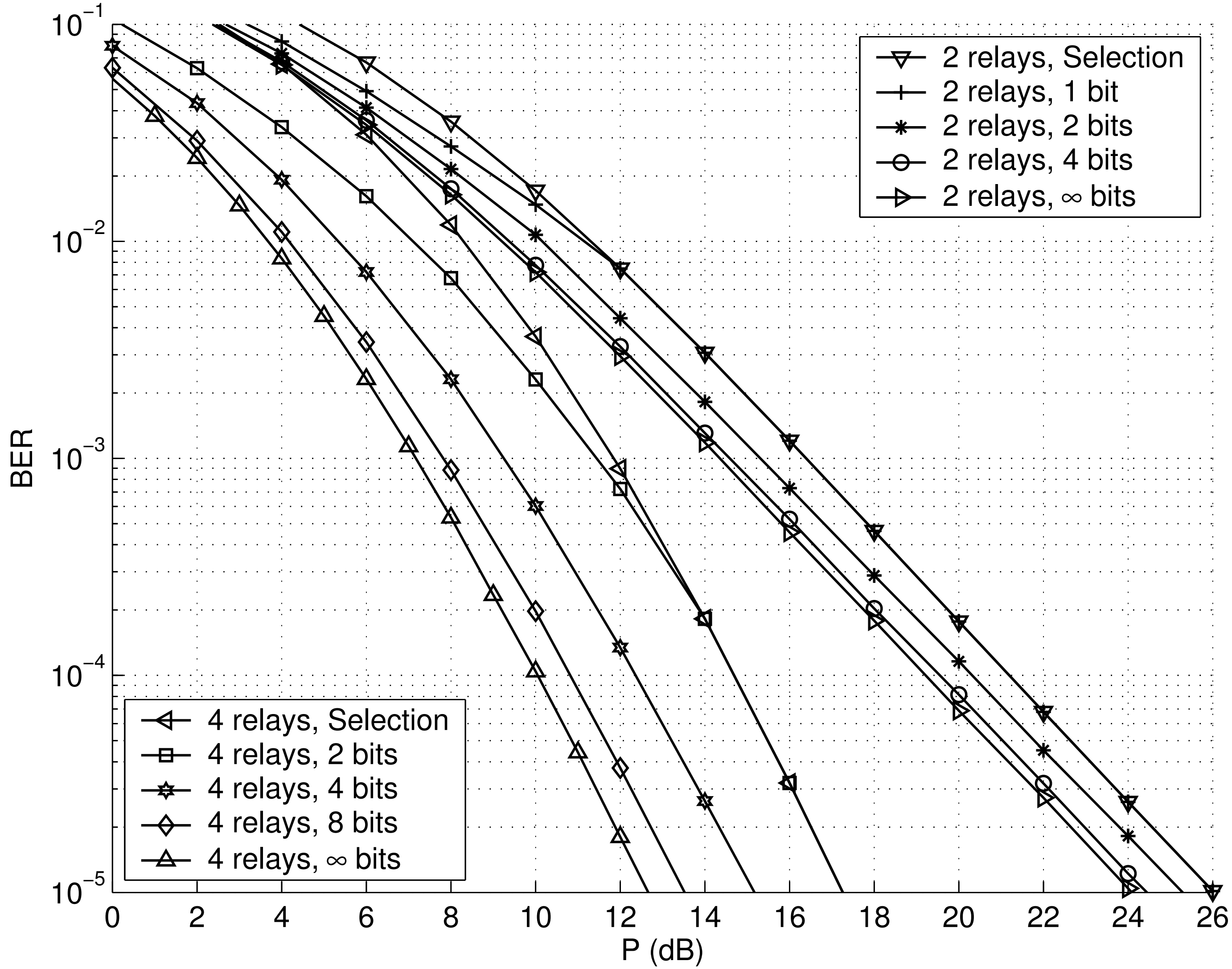
- An exact BER analysis is not tractable, mostly because of the shape of the feasible set $\mathcal{X} \triangleq \{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } \|\mathbf{x}\|_\infty = 1\}$.
- Solution: Bound the BER with $\mathcal{X}$ by BERs with feasible sets that correspond to a sum-power constraint on relays, e.g. $\mathcal{X}^{\text{in}}$ and $\mathcal{X}^{\text{out}}$. These sets are much easier to deal with.

$$\mathcal{X}^{\text{in}} \triangleq \{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } ||\mathbf{x}||^2 = 1\},$$
$$\mathcal{X}^{\text{out}} \triangleq \{\mathbf{x} : \mathbf{x} \in \mathbb{C}^R \text{ and } ||\mathbf{x}||^2 = R\}.$$

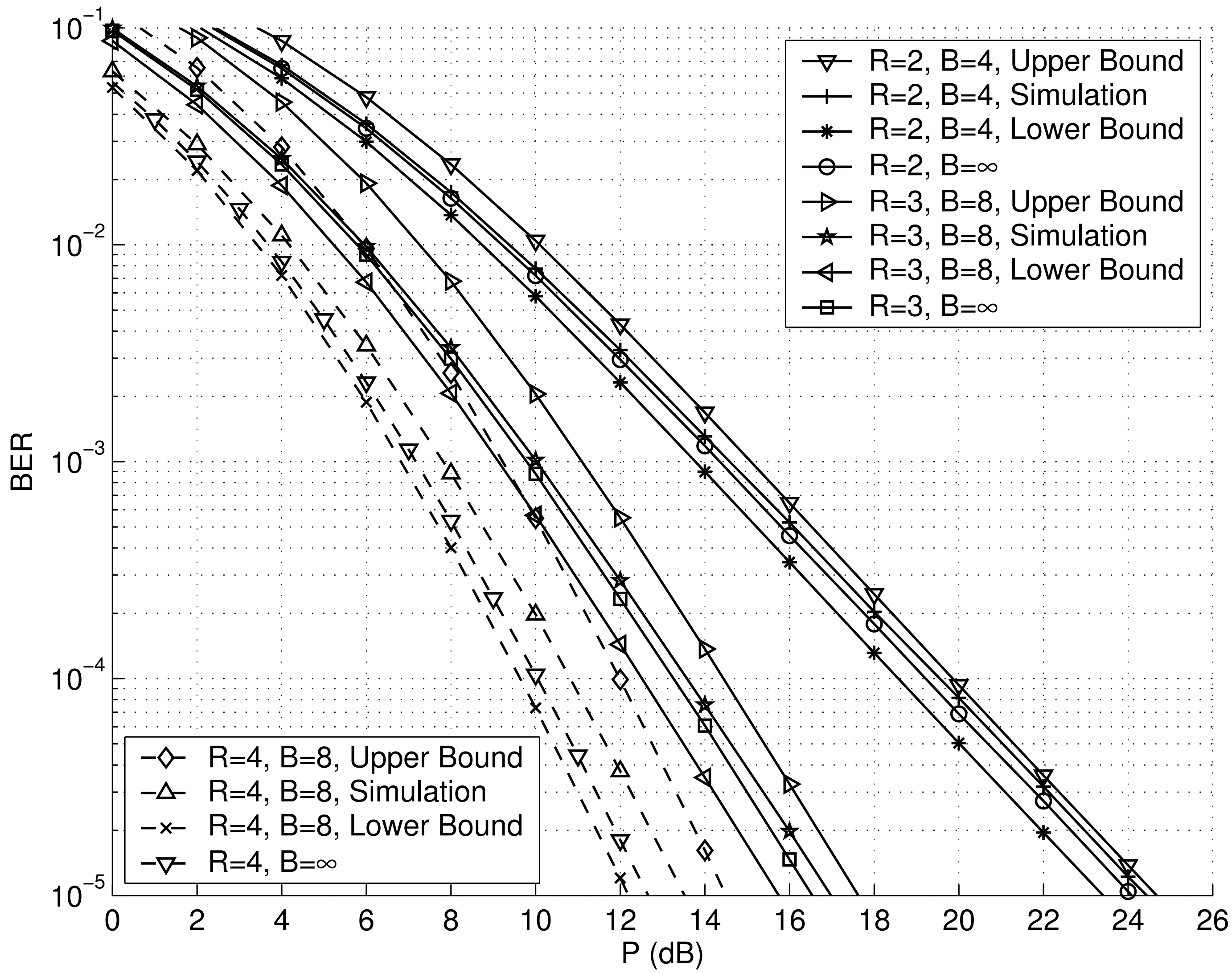
- An illustration for a 2-relay network:



Simulation Results ($P_i = P$)



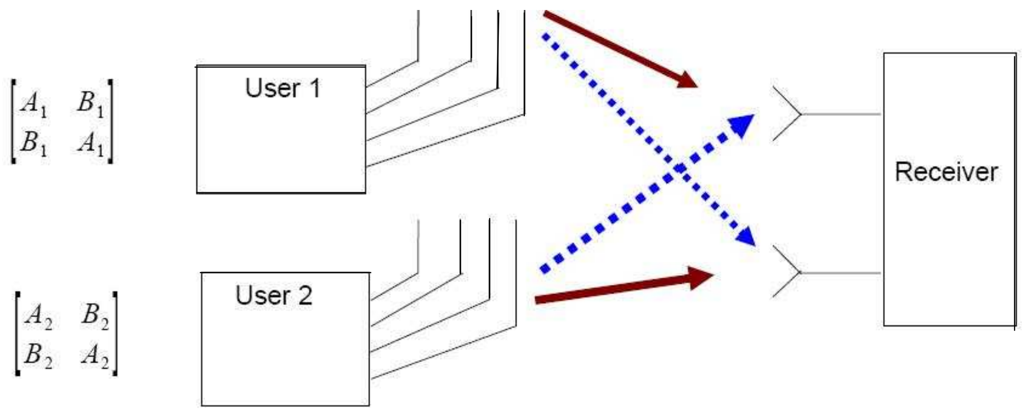
Simulation Results - BER Analysis ($P_i = P$)



Conclusions

- We introduced Network Beamforming
  - Adaptive relay power control
  - Analytical solution with linear complexity
  - Distributive schemes
  - Full diversity and optimal array gain
- We also proposed distributed codebook designs using quantized feedback information
- Network beamforming with quantized feedback achieves full diversity

Using MIMO for Interference Cancellation



- To cancel the interference of J users, we only need J receive antennas

Properties

- Advantages
  - Simple Decoding
  - Rate One
  - Full (transmit) Diversity
  - Extendable to any Number of Users
- Disadvantage
  - Needs Synchronization