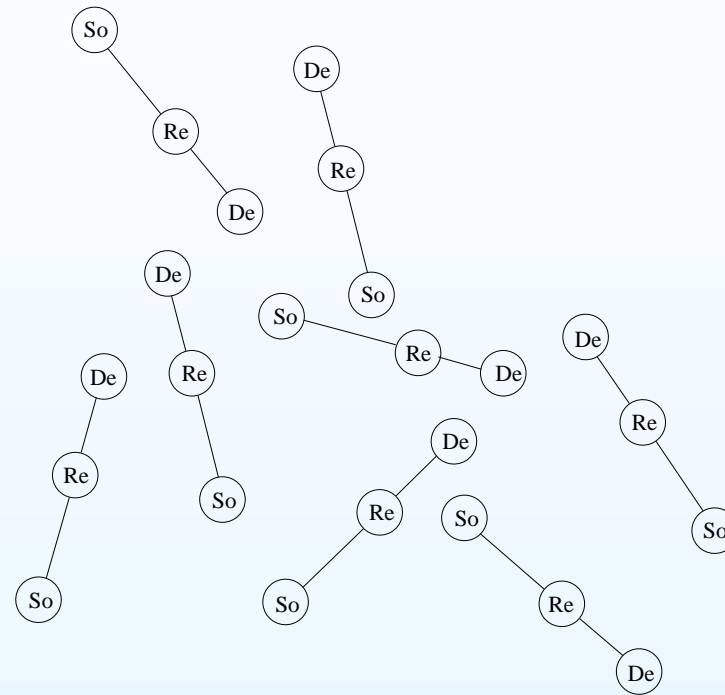


Random multi-hop networks: characterizing delay and throughput

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Annual MURI Review, 16th October 2008

Random multi-hop networks

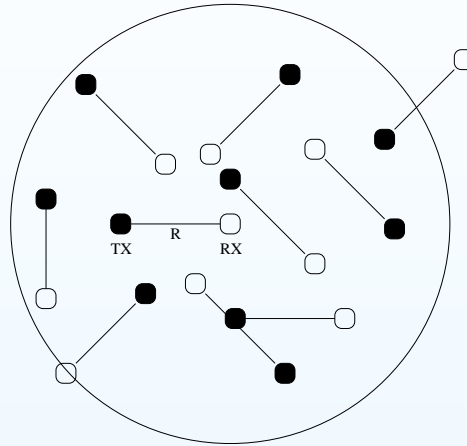


- Random source placement: Poisson Point Process (density λ)
- Packets are relayed from Source (SO) to Destination (DE)
- Random access: no coordination between hops of different routes

Random multi-hop networks

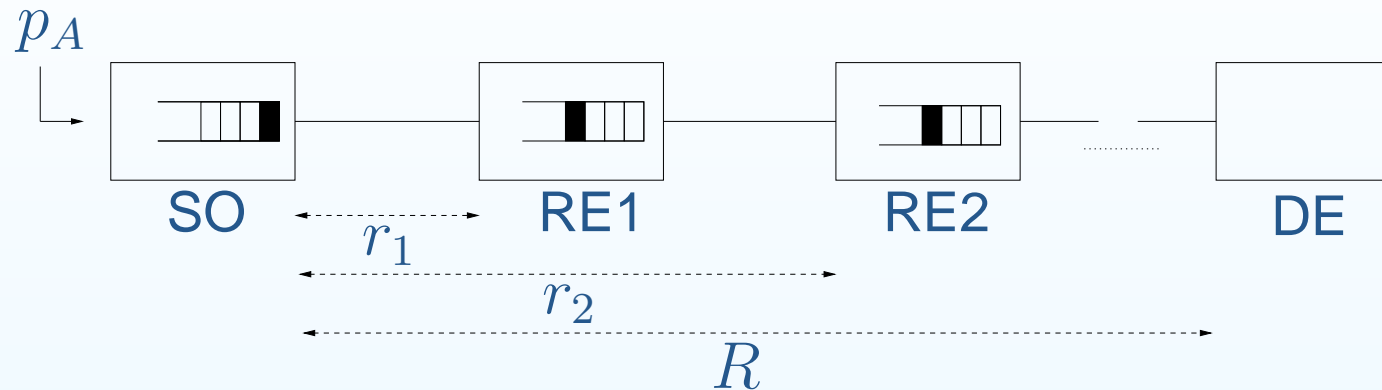
- Interference-limited scenario (nodes have same transmit power)
- Metrics
 - **E2E Delay**: travel time from SO to DE
 - **E2E Throughput**: rate of successful packet reception by DE
- TRADE-OFF: smaller hopping distance results in higher link SIR but also additional delay
- Objectives:
 - **Evaluate** metrics using “realistic” physical layer (fading, interference, MIMO, ...)
 - **Optimize** metrics over different parameters and provide design guidelines

Previous work



- Consider a “snapshot” of the network
- Implicit assumption: DE lies at infinite distance from SO
- **Devise** appropriate metrics, e.g.
 - Expected Progress = (Link Throughput) x (Hop Length)
- Physical layer: FH and coding, MIMO diversity and spatial multiplexing techniques
- Shortcoming of single-hop mentality: no E2E delay guarantees!

Network model



- N hops: $N - 1$ RE between SO and DE
- Packet arrival probability at SO every N slots: p_A
- Protocol: **synchronous TDMA/ALOHA**
(p -persistent SO, 1-persistent RE)
- Node queues: infinite size, FIFO
- Packet retransmitted until successfully received
- Network topology “changes” every slot

Physical layer

- Channel between two nodes: Rayleigh fading and path-loss
- The SIR at the n^{th} hop is

$$\text{SIR}_n = \frac{A_n (r_n - r_{n-1})^{-b}}{\sum_{t \in \mathcal{I}_{n-1}} A_t d_t^{-b}}, \quad n = 1, 2, \dots, N$$

- The packet success probability at the n^{th} hop is

$$p_n = \mathbb{P}(\text{SIR}_n > \theta) = e^{-\lambda_{n-1} c (r_n - r_{n-1})^2}$$

where

$$c = \Gamma\left(1 + \frac{2}{b}\right) \Gamma\left(1 - \frac{2}{b}\right) \pi \theta^{\frac{2}{b}}$$

Metrics: $p_A = 1$

- H_n/Q_n : mean service/waiting time at node n
- Mean E2E delay:

$$D = H_0 + \sum_{n=1}^{N-1} (Q_n + H_n)$$

- Using basic queueing theory

$$D_{\{p_A=1\}} = \frac{N}{pp_1} - N + 1 + \sum_{n=2}^N \left(N \frac{1 - p_n}{p_n - pp_1} + 1 \right)$$

- Stable operation: $pp_1 < p_n, n = 2, \dots, N$
- E2E route throughput: $\text{RT}_{\{p_A=1\}} = \frac{pp_1}{N}$

Metrics: $p_A < 1$

- Mean E2E delay:

$$D = \sum_{n=0}^{N-1} (Q_n + H_n)$$

- Similarly

$$D_{\{p_A < 1\}} = N \frac{1 - pp_1}{pp_1 - p_A} + 1 + \sum_{n=2}^N \left(N \frac{1 - p_n}{p_n - p_A} + 1 \right)$$

- Stable operation: $p_A < \min\{pp_1, p_n\}, n = 2, \dots, N$
- E2E route throughput:

$$\text{RT}_{\{p_A < 1\}} = \frac{p_A}{N}$$

Packet success probabilities: $p_A = 1$

- Symmetry and randomness: we study a stationary regime
- To ensure stable operation take **worst-case interference scenario**
- Derive **sufficient** conditions for stability
- Example: $N = 2, p_A = 1$

$$\begin{aligned} p_1 &= e^{-\lambda p c r_1^2} \\ p'_2 &= e^{-\lambda c (R - r_1)^2} < p_2 \end{aligned}$$

- Demand that

$$p p_1 < p'_2 \Leftrightarrow p e^{-\lambda p c r_1^2} < e^{-\lambda c (R - r_1)^2}$$

- If $p = 1$, $r_1 > R/2$ **ensures stability**
- For given r_1 : limit on p for stable operation derived

Packet success probabilities: $p_A = 1$

- The density of active relays in the 2nd hop is

$$\lambda_1 = \lambda \frac{pp_1}{p_2}$$

- Fixed point equation for p_2 :

$$p_2 = e^{-\lambda \frac{pp_1}{p_2} c(R-r_1)^2}$$

- Extension to N hops straightforward
- Lemma: mean E2E delay **minimized** when $p_2 = p_3 = \dots p_N$
- Degrees of freedom in backlogged scenario: p, r_1, N

Packet success probabilities: $p_A < 1$

- Example: $N = 2$, $p = 1$
- Worst-case interference scenario:

$$p'_1 = e^{-\lambda c r_1^2}$$

$$p'_2 = e^{-\lambda c (R - r_1)^2}$$

- Demand that

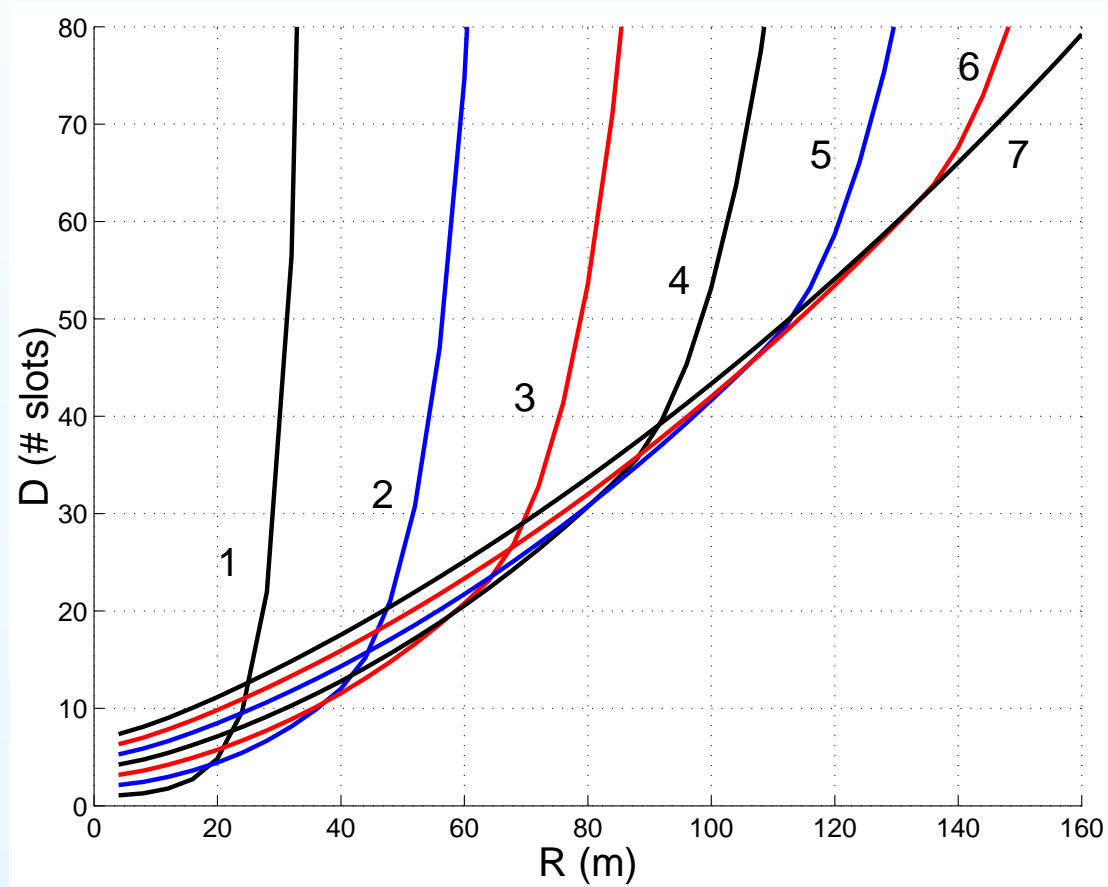
$$p_A < \min\{p'_1, p'_2\} = e^{-\lambda c \max\{r_1, R - r_1\}^2}$$

- Fixed point equations:

$$p_1 = e^{-\lambda \frac{p_A}{p_1} c r_1^2}, \quad p_2 = e^{-\lambda \frac{p_A}{p_2} c (R - r_1)^2}$$

- N hops: delay minimized when ALL hops are equidistant

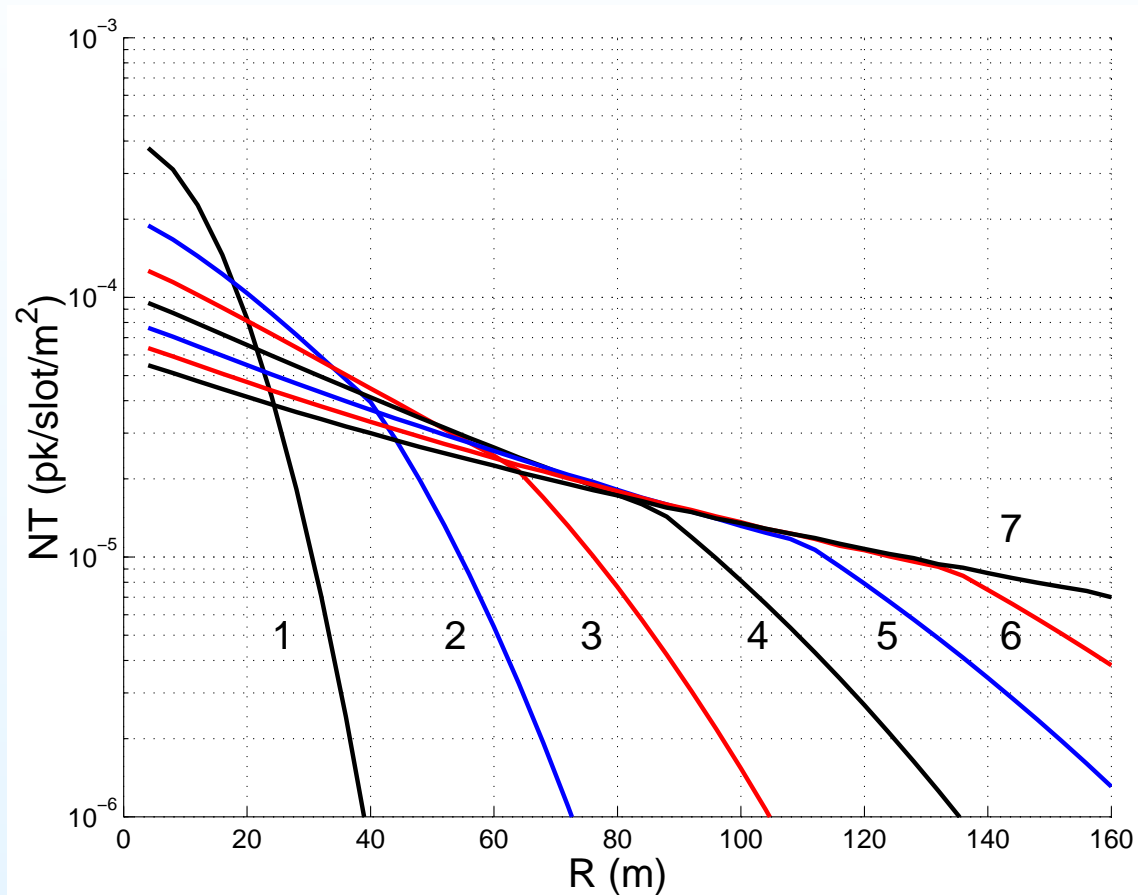
Results ($p_A = 1$)



$$p = 1, \lambda = 4 \times 10^{-4}, \theta = 6\text{dB}, b = 4$$

- Optimal number of hops determined

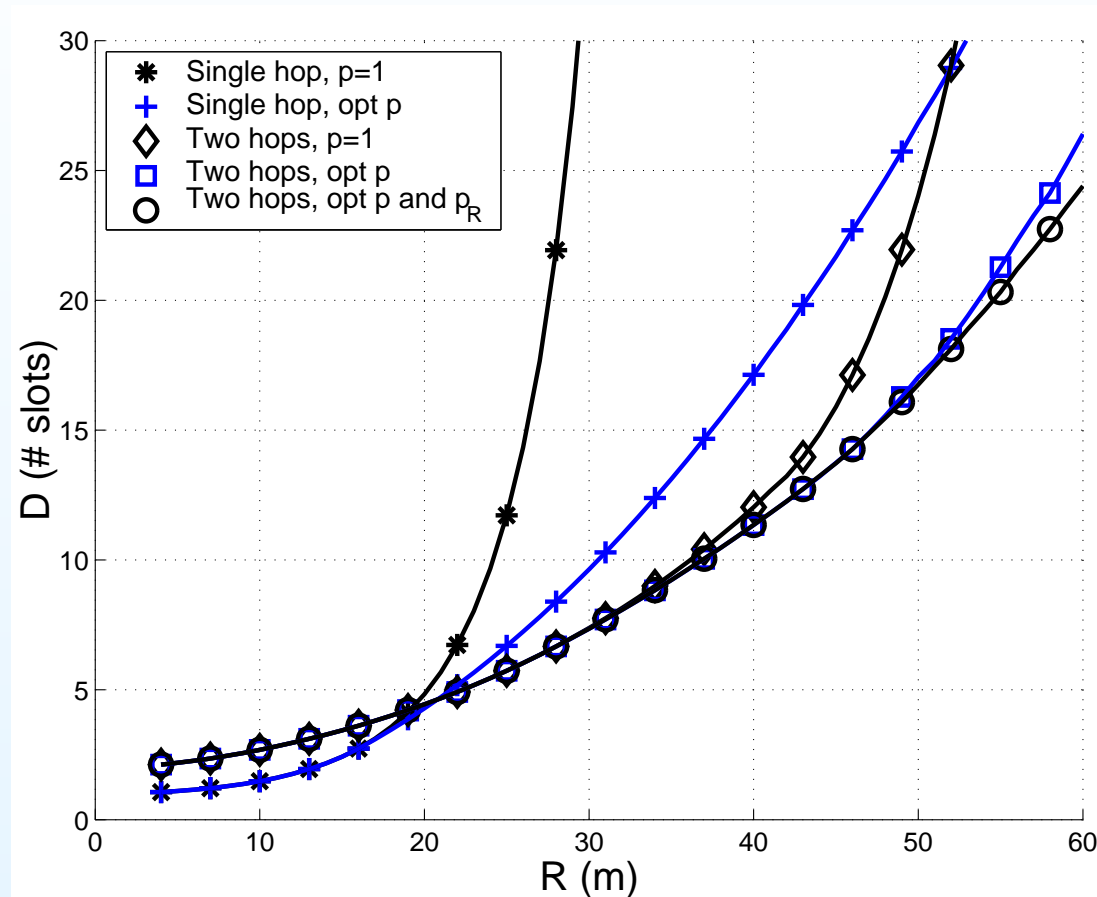
Results ($p_A = 1$)



$$p = 1, \lambda = 4 \times 10^{-4}, \theta = 6\text{dB}, b = 4$$

- Switching points approximately the same as for delay

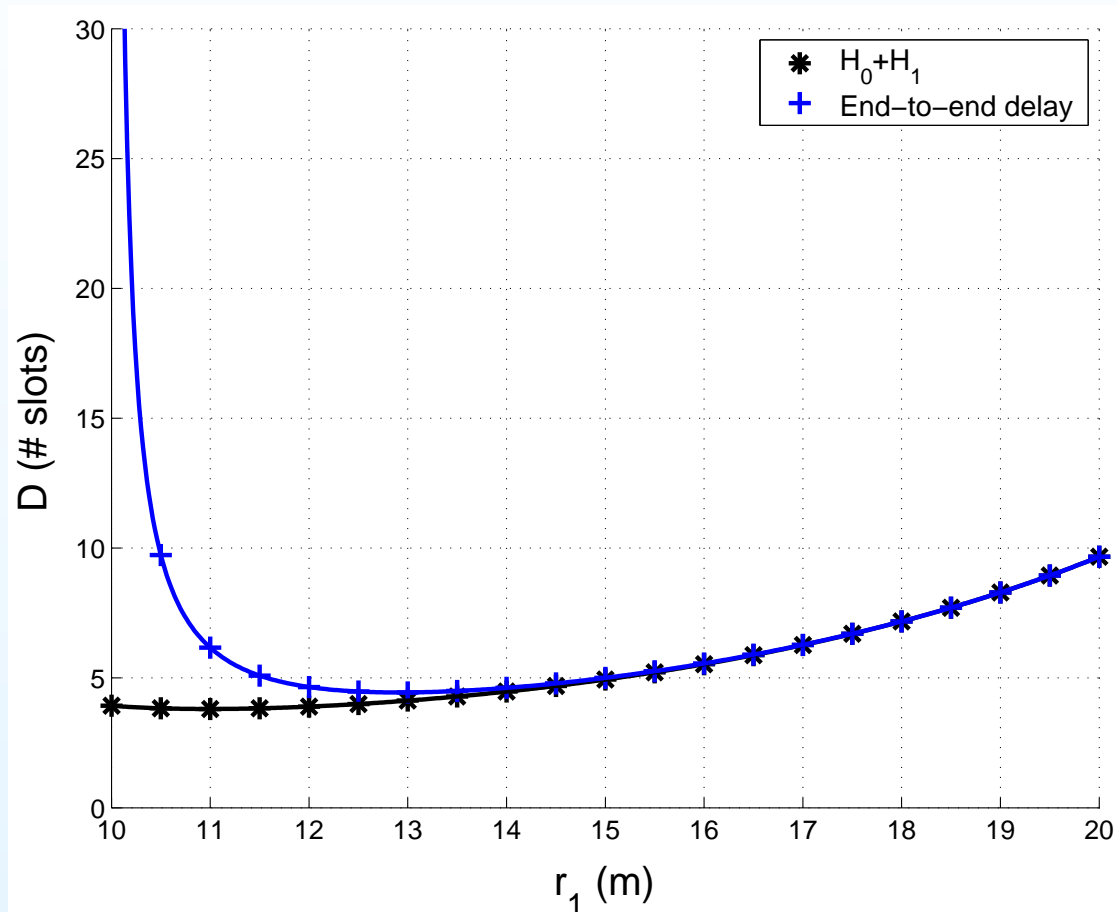
Results ($p_A = 1$)



$$N = 2, \lambda = 4 \times 10^{-4}, \theta = 6\text{dB}, b = 4$$

- Optimizing over p significant for large R

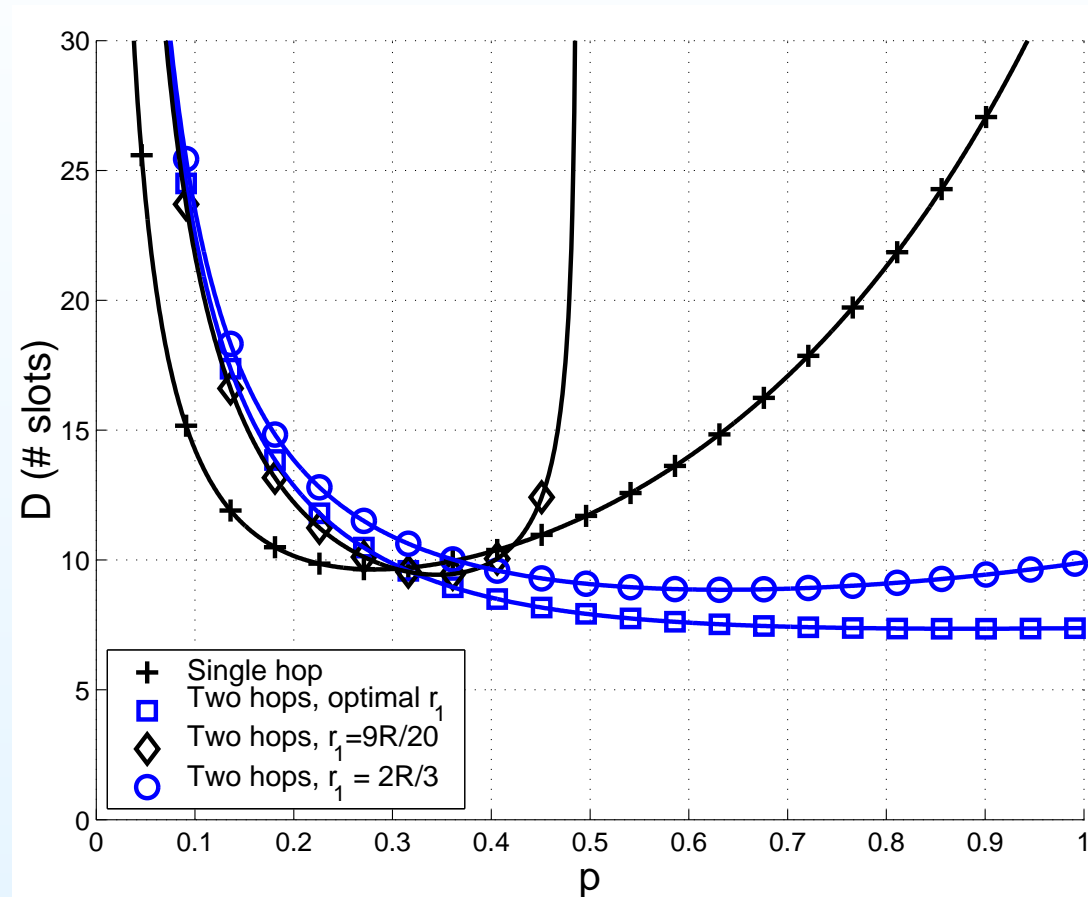
Results ($p_A = 1$)



$$N = 2, p = 1, R = 20\text{m}, \lambda = 4 \times 10^{-4}, \theta = 6\text{dB}, b = 4$$

- Preferable to place relay closer to the DE

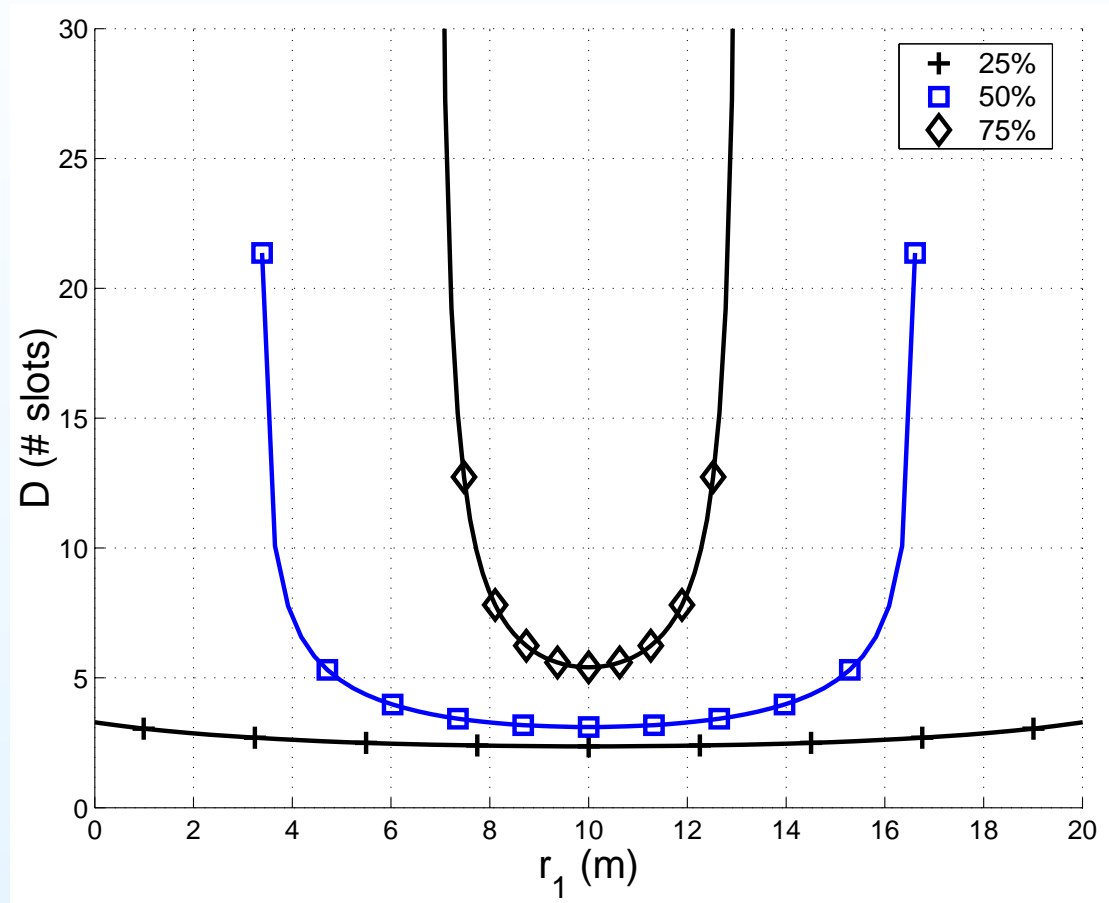
Results ($p_A = 1$)



$$N = 2, R = 30\text{m}, \lambda = 4 \times 10^{-4}, \theta = 6\text{dB}, b = 4$$

- Sensitivity to MAC characterized

Results ($p_A < 1$)



$$N = 2, p = 1, R = 20\text{m}, \lambda = 4 \times 10^{-4}, \theta = 6\text{dB}, b = 4$$

- Sensitivity to SO load characterized

In summary

- Analytical framework for random interference-limited multi-hop networks
- Performance characterized at the E2E level
 - Number of hops and relay placements that minimize delay
 - Impact of imperfect relay placement, MAC and source load
- This is a space-time project ...
 - Modify queueing analysis and success probability expressions to include MIMO
 - What MIMO strategies minimize E2E delay?