Finite Rate Feedback for Spatially and Temporally Correlated MISO Channels in the Presence of Estimation Errors and Feedback Delay

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***Abstract*—In this paper, the problem of finite-rate feedback for spatially and temporally correlated Rayleigh fading Multiple Input Single Output (MISO) channels with estimation errors at the receiver and feedback delay is addressed. A model that captures estimation errors, feedback delay, and finite-rate quantization of the channel is developed. A novel codebook design algorithm that minimizes the loss in ergodic capacity is proposed. Simulation results show that the new codebook designed under the consideration of estimation errors and feedback delay outperforms the codebook designed assuming ideal conditions. Analysis for the loss in ergodic capacity for spatially i.i.d channels with channel estimation errors and delay (EED) is presented and validated through simulations.**

Index Terms: MISO systems, transmit beamforming, channel quantization, spatial correlation, channel estimation errors

I. Introduction

In a MISO system, if the channel state information (CSI) is available at the transmitter, both the diversity and array gains can be achieved with transmit beamforming [1]. In this paper the focus is on MISO systems where CSI is conveyed from the receiver to the transmitter through a finite-rate feedback link. An information theoretic approach to transmit beamforming with imperfect feedback is presented in [7]. In [8], for spatially i.i.d channels with estimation errors, the authors study the bounds on ergodic capacity of Multiple Input and Multiple Output (MIMO) systems. The lower bounds on ergodic capacity for spatially correlated MIMO channels with estimation errors are studied in [9]. When required, in both [8] and [9], an un-quantized CSI is assumed at the transmitter. The problem of feedback delay is addressed in [10] and [11].

Most of the existing work on finite rate quantization assumes perfect channel estimation (PCE) and no feedback delay [2]-[6]. Under PCE and no feedback delay, for spatially correlated channels, optimum codebook design for ergodic capacity loss is proposed in [3]. A grassmannian codebook design for correlated channels with PCE and no delay is proposed in [6]. In the context of determining a transmit weighting matrix that improves the performance of orthogonal space-time block codes, a mean-squared error criteria is used to design codebook for channels with feedback delay and feedback channel bit errors [12]. With randomized vector codebook, the ergodic capacity for spatially i.i.d channels with estimation errors and finite-rate quantization is studied in [13].

This paper focuses on optimum vector quantization (VQ) algorithm that is directly related to the loss in ergodic capacity of a spatially and temporally correlated channel with channel estimation errors and delay (EED). Following the approach taken in [2] and [3] the new design criteria can be shown to be amenable to codebook design using a Lloyd-type VQ algorithm. The codebook design for spatially i.i.d channel with EED then becomes a special case of the correlated case. With this codebook, analysis for loss (for the correlated case only codebook design is proposed) in ergodic capacity is presented for the spatially i.i.d scenario with EED.

The rest of this paper is organized as follows. In Section II, the system model is introduced. In Section III, codebook design is considered. The ergodic capacity loss analysis for the i.i.d case is presented in Section IV. Numerical and simulation results form Section V. The paper is concluded in Section VI.

Notation: Small and upper case bold letters indicate vector and matrix respectively. $E(.)$, $(.)^T$, $(.)^H$, $|.|$, and $\|.\|$ denote expectation, transpose, Hermitian, absolute value, and norm respectively. $\mathbf{x} \sim \mathcal{NC}(\mu, \mathbf{\Sigma})$ indicates a circularly symmetric complex Gaussian random variable $\mathbf{x}$ with mean μ and covariance $\mathbf{\Sigma}$.

II. System Model

A MISO system with t antennas at the base station (BS) and one antenna at the mobile station (MS) is considered. The channel between the BS and the MS is modeled as a frequency-flat, slowly varying Rayleigh fading channel. The vector valued channel at time k, $\mathbf{h}[k] = [h_1[k], h_2[k], \ldots, h_t[k]]^T$, is the MISO channel response with spatial distribution given by $\mathbf{h} \sim \mathcal{NC}(\mathbf{0}, \mathbf{\Sigma}_{\text{hh}})$. Let $\mathbf{w} \in \mathbb{C}^{t \times 1}$ be the unit norm beamforming vector (BV) at the BS. Then, the received signal at the MS is given by

$$y[k] = \mathbf{w}^H[k]\mathbf{h}[k]s[k] + \eta[k], \quad (1)$$

where $\eta \sim \mathcal{NC}(0, 1)$. The transmitted symbol is denoted by s. Average signal to noise ratio (SNR) is $E[|s|^2] = \gamma_s$.

In this paper, the CSI is assumed to be imperfectly estimated at the receiver and is partially available at the transmitter

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through a finite-rate feedback link of B bits per channel update. Also a delay of D is assumed between channel estimation and its actual use. More specifically, a quantization codebook $\mathcal{W} = \{\widehat{\mathbf{v}}_1, \cdots, \widehat{\mathbf{v}}_N\}$, of size $N = 2^B$ composed of unit-norm transmit BVs is assumed to be known to both the receiver and the transmitter. Based on the channel estimate, the receiver selects the best code point $\widehat{\mathbf{v}}$ from the codebook and sends the corresponding index back to the transmitter through an error free link.

A. Modeling of Estimation Errors and Delay

In MISO systems with feedback, much of the past work makes the simplifying assumption of perfect channel knowledge at receiver and instantaneous feedback. In practice, this is not true and both channel estimation error and feedback delay can result in significant degradation in system performance. Our work deals with these sources of error by making the following assumption about the channel. As indicated earlier, first the actual channel is modeled as a zero mean Gaussian random process with certain correlation structure. By modifying the correlation structure, which typically is a function of mobility, one can understand the impact of feedback delay on performance. Second, the channel estimate and the actual channel are modeled as jointly Gaussian random processes. The correlation between the two processes provides a mechanism to control the quality of the channel estimate and study the impact of channel estimation errors on system performance. The two together allow for a mechanism to deal with EED in MISO feedback systems. The modeling can be justified for pilot based channel estimation schemes and as shown in later sections, the modeling is mathematically tractable.

Since the channel is modeled as a Gaussian random process, for a feedback delay of D, the channel $\mathbf{h}[k]$ and its delayed version $\mathbf{h}[k-D]$, are jointly Gaussian with zero mean and are related in the following manner [15]

$$\mathbf{h}[k] = \boldsymbol{\Sigma}_{\text{hd}}\boldsymbol{\Sigma}_{\text{dd}}^{-1}\mathbf{h}[k-D] + \mathbf{e}[k-D] = \mathbf{h}_c[k-D] + \mathbf{e}[k-D], \quad (2)$$

where $\mathbf{h}_c \sim \mathcal{NC}(\mathbf{0}, \boldsymbol{\Sigma}_{\text{cc}})$, $\boldsymbol{\Sigma}_{\text{cc}} = \boldsymbol{\Sigma}_{\text{hd}}\boldsymbol{\Sigma}_{\text{dd}}^{-1}\boldsymbol{\Sigma}_{\text{dh}}$, and the uncorrelated error component $\mathbf{e} \sim \mathcal{NC}\left(\mathbf{0}, \left(\boldsymbol{\Sigma}_{\text{hh}} - \boldsymbol{\Sigma}_{\text{hd}}\boldsymbol{\Sigma}_{\text{dd}}^{-1}\boldsymbol{\Sigma}_{\text{dh}}\right)\right)$. $\boldsymbol{\Sigma}_{\text{hh}}$, $\boldsymbol{\Sigma}_{\text{dd}}$ are the autocorrelation matrices of $\mathbf{h}[k]$ and $\mathbf{h}[k-D]$ respectively. $\boldsymbol{\Sigma}_{\text{hd}}$ and $\boldsymbol{\Sigma}_{\text{dh}}$ are the cross-correlation matrices.

In the above equation, in the absence of estimation errors, $\mathbf{h}_c[k-D]$ is the quantity available at the transmitter. In the presence of estimation errors, by virtue of the joint Gaussianity assumption the ideal channel component in (2), $\mathbf{h}_c[k-D]$, and its estimate, $\mathbf{h}_e[k-D]$, are jointly Gaussian with zero mean and are related in the following manner [15]

$$\mathbf{h}_c[k-D] = \boldsymbol{\Sigma}_{\text{ce}}\boldsymbol{\Sigma}_{\text{ee}}^{-1}\mathbf{h}_e[k-D] + \varepsilon[k-D], \quad (3)$$

where $\varepsilon \sim \mathcal{NC}\left(\mathbf{0}, \left(\boldsymbol{\Sigma}_{\text{cc}} - \boldsymbol{\Sigma}_{\text{ce}}\boldsymbol{\Sigma}_{\text{ee}}^{-1}\boldsymbol{\Sigma}_{\text{ec}}\right)\right)$. $\boldsymbol{\Sigma}_{\text{cc}}$, $\boldsymbol{\Sigma}_{\text{ee}}$ are the autocorrelation matrices of $\mathbf{h}_c[k-D]$ and $\mathbf{h}_e[k-D]$ respectively. $\boldsymbol{\Sigma}_{\text{ce}}$ and $\boldsymbol{\Sigma}_{\text{ec}}$ are the cross-correlation matrices.

The relation between actual channel and the delayed version of the channel estimate can be obtained by substituting (3) in (2)

$$\mathbf{h}[k] = \widetilde{\mathbf{h}}[k-D] + \mathbf{n}[k], \quad (4)$$

where $\widetilde{\mathbf{h}} \sim \mathcal{NC}(\mathbf{0}, \boldsymbol{\Sigma}_{\text{im}})$, $\boldsymbol{\Sigma}_{\text{im}} = \boldsymbol{\Sigma}_{\text{ce}}\boldsymbol{\Sigma}_{\text{ee}}^{-1}\boldsymbol{\Sigma}_{\text{ec}}$ and the uncorrelated noise $\mathbf{n} \sim \mathcal{NC}(\mathbf{0}, \boldsymbol{\Sigma}_{\text{n}})$, $\boldsymbol{\Sigma}_{\text{n}} = \boldsymbol{\Sigma}_{\text{hh}} - \boldsymbol{\Sigma}_{\text{ce}}\boldsymbol{\Sigma}_{\text{ee}}^{-1}\boldsymbol{\Sigma}_{\text{ec}}$. In summary, (2) represents the actual channel in terms of its delayed version only and (4) represents the actual channel in terms of its estimated and delayed version. The receiver has the instantaneous knowledge of $\widetilde{\mathbf{h}}[k-D]$ and the statistical knowledge of $\boldsymbol{\Sigma}_{\text{n}}$. In the next section, the optimum beamforming vector that maximizes the received SNR is derived based on $\widetilde{\mathbf{h}}[k-D]$, $\boldsymbol{\Sigma}_{\text{n}}$ and operating SNR γ_s.

Note that it is possible to construct slightly different versions of the above modeling. However, as long as the actual channel is written as a sum of a channel related component with $\mathcal{NC}(\mathbf{0}, \boldsymbol{\Sigma}_{\text{sh}})$ and an orthogonal noise component with $\mathcal{NC}(\mathbf{0}, \boldsymbol{\Sigma}_{\text{sn}})$, such that $\boldsymbol{\Sigma}_{\text{sh}} + \boldsymbol{\Sigma}_{\text{sn}} = \boldsymbol{\Sigma}_{\text{hh}}$, then the rest of the sections in the paper are directly applicable without any modification.

III. Transmit Beamforming with Estimation Errors and Delay (EED)

In this section, we first discuss optimum transmit beamforming in the EED context and then develop an optimum feedback strategy based on vector quantization.

A. Optimum Beamforming

The received signal with an arbitrary unit norm beamforming vector $\mathbf{w}$ is given by

$$\begin{aligned} y[k] &= \mathbf{w}^H[k]\mathbf{h}[k]s[k] + \eta[k] \\ &= \mathbf{w}^H[k]\left(\widetilde{\mathbf{h}}[k-D] + \mathbf{n}[k]\right)s[k] + \eta[k] \\ &= \mathbf{w}^H[k]\widetilde{\mathbf{h}}[k-D]s[k] + \hat{\zeta}[k], \end{aligned} \quad (5)$$

where conditioned on $\mathbf{w}$, $\hat{\zeta}[k] \sim \mathcal{NC}\left(0, 1 + \gamma_s\, \mathbf{w}^H\boldsymbol{\Sigma}_{\text{n}}\mathbf{w}\right)$. The appearance of signal term in the noise is due to the fact that only $\widetilde{\mathbf{h}}[k-D]$ is available at the transmitter instead of actual channel $\mathbf{h}[k]$. The lower bound on ergodic capacity without channel quantization is given by [16] (for simplicity all time indexes are ignored)

$$C = E\left[\log_2\left(1 + \frac{\gamma_s \mathbf{w}^H\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}^H\mathbf{w}}{1 + \gamma_s\, \mathbf{w}^H\boldsymbol{\Sigma}_{\text{n}}\mathbf{w}}\right)\right]. \quad (6)$$

Selection of $\mathbf{w}_{opt}$, the optimum BV is based on

$$\begin{aligned} \mathbf{w}_{opt} &= \arg\max_{\|\mathbf{w}\|=1}\left(\frac{\mathbf{w}^H\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}^H\mathbf{w}}{1 + \gamma_s\, \mathbf{w}^H\boldsymbol{\Sigma}_{\text{n}}\mathbf{w}}\right) \\ &= \arg\max_{\|\mathbf{w}\|=1}\left(\frac{\mathbf{w}^H\,\widetilde{\mathbf{h}}\widetilde{\mathbf{h}}^H\,\mathbf{w}}{\mathbf{w}^H\,\boldsymbol{\Sigma}_{\text{d}}\,\mathbf{w}}\right), \end{aligned}$$

where $\boldsymbol{\Sigma}_{\text{d}} = \gamma_s\boldsymbol{\Sigma}_{\text{n}} + \mathbf{I}$. The solution to the above maximization problem is given by

$$\mathbf{w}_{opt} = \boldsymbol{\Sigma}_{\text{d}}^{-1}\widetilde{\mathbf{h}}/\|\boldsymbol{\Sigma}_{\text{d}}^{-1}\widetilde{\mathbf{h}}\|. \quad (7)$$

With this selection of the beamforming vector, the lower bound on the ergodic capacity is given by

$$C = E\left[\log_2(\omega)\right], \quad (8)$$

where $\omega = 1 + \gamma_s\widetilde{\mathbf{h}}^H\boldsymbol{\Sigma}_{\text{d}}^{-1}\widetilde{\mathbf{h}}$. If there are no estimation errors and delay, then $\boldsymbol{\Sigma}_{\text{n}} = \mathbf{I}$ and $\boldsymbol{\Sigma}_{\text{d}} = \mathbf{I}(1+\gamma_s)$. The maximization

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then (without EED) is achieved by the well known solution of normalized channel direction. Because of EED there is a different solution given by (7).

B. Beamforming with Quantized Feedback

If the beamforming vector is fedback through a low rate channel, this results in additional errors. To minimize the effect of quantization errors, one is naturally led to a VQ framework and finding an optimum codebook of $N = 2^B$ beamforming vectors. We now discuss the design of such a codebook. The beamforming vector after quantization is given by

$$\mathbf{w}[k] = \widehat{\mathbf{v}}[k-D] = \mathcal{Q}\left(\mathbf{w}_{opt}\right), \tag{9}$$

where $\mathcal{Q}$ is the quantization function. The lower bound on the ergodic capacity with channel quantization is given by (with $\widehat{\mathbf{v}}[k-D]$ from (9) plugged into (6)),

$$C_Q = E\left[\log_2\left(1 + \frac{\gamma_s\,\|\widetilde{\mathbf{h}}\|^2|\vartheta|^2}{1+\gamma_s\,\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{n}}\widehat{\mathbf{v}}}\right)\right], \tag{10}$$

where $\vartheta = \widehat{\mathbf{v}}^H\mathbf{v}$ and $\mathbf{v} = \widetilde{\mathbf{h}}/\|\widetilde{\mathbf{h}}\|$. The additional loss in ergodic capacity (there is already some loss because of EED) due to channel quantization, is given by

$$C_L = C - C_Q = E\left[\log_2\left(\frac{\omega + \gamma_s\,\omega\,\widehat{\mathcal{V}}}{1+\gamma_s\cdot\left(\widehat{\mathcal{V}} + \alpha\cdot|\vartheta|^2\right)}\right)\right], \tag{11}$$

where $\alpha = \|\widetilde{\mathbf{h}}\|^2$ and $\widehat{\mathcal{V}} = \widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{n}}\widehat{\mathbf{v}}$. Note that C_L quantifies the loss due to quantization alone precisely, and unlike C and C_Q, it is not a bound.

1) Codebook Design: The criteria for designing the codebook is to minimize the loss in ergodic capacity C_L (11). However, C_L in the above form is not very convenient because it complicates the centroid finding step in the VQ design and some modification is necessary. After some manipulation, the loss term C_L can be written as

$$C_L = -E\,\log_2\left[1 - \left(1 - \left(\frac{1+\gamma_s\cdot\left(\widehat{\mathcal{V}} + \alpha\cdot|\vartheta|^2\right)}{\omega + \gamma_s\,\omega\,\widehat{\mathcal{V}}}\right)\right)\right].$$

After taking the first order approximation using $-\log(1-x) \simeq x$, the approximated loss, C_{LA}, can be written as

$$\begin{aligned} C_{LA} &= \frac{1}{\ln 2}E\left(\frac{\omega - 1 + \gamma_s\left(\widehat{\mathcal{V}}(\omega-1) - \alpha|\vartheta|^2\right)}{\omega + \gamma_s\,\omega\,\widehat{\mathcal{V}}}\right) \\ &= \frac{1}{\ln 2}E\left(\frac{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{v}}\widehat{\mathbf{v}}}{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{d}}\widehat{\mathbf{v}}}\right), \end{aligned} \tag{12}$$

where

$$\boldsymbol{\Sigma}_{\mathrm{v}} = \left\{\frac{(\omega-1)\mathbf{I} + \gamma_s(\omega-1)\boldsymbol{\Sigma}_{\mathrm{n}} - \gamma_s\alpha\mathbf{v}\mathbf{v}^H}{\omega}\right\}. \tag{13}$$

It will be shown that in this form C_{LA} results in a convenient VQ design. The above approximation is important as the codebook design will be developed using C_{LA}. The approximation is well justified in the high SNR and high resolution (higher N) regime.

Codebook Design Criterion: Design a quantizer $\mathcal{Q}$ ($\mathcal{Q}$: $\mathbb{C}^t \to \mathcal{W}$) to minimize C_{LA}, which can be written as

$$\min_{\mathcal{Q}(.)} E\left(\frac{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{v}}\widehat{\mathbf{v}}}{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{d}}\widehat{\mathbf{v}}}\right) \tag{14}$$

where $\widehat{\mathbf{v}} = \mathcal{Q}(\mathbf{v})$, $\|\widehat{\mathbf{v}}\| = 1$ and $\widehat{\mathbf{v}} \in \mathcal{W}$. The above objective function is amenable to codebook design using Lloyd type VQ-algorithm with a monotonic convergence property.

Codebook Design Algorithm: Lloyd algorithm has two conditions, the Nearest Neighborhood Condition and Centroid Condition. The details of these two conditions are discussed below. Generate a large sample set of vectors **v**, which are the normalized vectors of the delayed channel estimates.

Nearest Neighborhood Condition: Beginning with an arbitrary set of unit vectors $\widehat{\mathbf{v}}_i,\ i = 1,\cdots,N$ forming the codebook $\mathcal{W}$, the optimum Voronoi Regions $\mathcal{R}_i,\ i = 1,\cdots,N$ are found from the following condition

$$\mathcal{R}_i = \left\{\mathbf{v}\in\mathbb{C}^t : \frac{\widehat{\mathbf{v}}_i^H\boldsymbol{\Sigma}_{\mathrm{v}}\widehat{\mathbf{v}}_i}{\widehat{\mathbf{v}}_i^H\boldsymbol{\Sigma}_{\mathrm{d}}\widehat{\mathbf{v}}_i} \le \frac{\widehat{\mathbf{v}}_j^H\boldsymbol{\Sigma}_{\mathrm{v}}\widehat{\mathbf{v}}_j}{\widehat{\mathbf{v}}_j^H\boldsymbol{\Sigma}_{\mathrm{d}}\widehat{\mathbf{v}}_j}, \forall j \neq i\right\}$$

$\mathcal{R}_i$ contains all the training unit norm vectors **v** satisfying the above condition. In the above condition, $\boldsymbol{\Sigma}_{\mathrm{v}}$ as defined in (13) contains **v** (as indicated earlier, $\mathbf{v} = \widetilde{\mathbf{h}}/\|\widetilde{\mathbf{h}}\|$).

Centroid Condition: The codebook $\mathcal{W}$ is updated in this step. For a given partition $\mathcal{R}_i$ obtained from the previous step, the new set of beamforming vectors satisfy

$$\begin{aligned} \widehat{\mathbf{v}}_i &= \arg\min_{\|\widehat{\mathbf{v}}\|=1,\widehat{\mathbf{v}}\in\mathcal{R}_i} E\left\{\frac{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{v}}\widehat{\mathbf{v}}}{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{d}}\widehat{\mathbf{v}}}\right\},\ i = 1,\cdots,N \\ &= \arg\min_{\|\widehat{\mathbf{v}}\|=1,\widehat{\mathbf{v}}\in\mathcal{R}_i}\left\{\frac{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{m}}\widehat{\mathbf{v}}}{\widehat{\mathbf{v}}^H\boldsymbol{\Sigma}_{\mathrm{d}}\widehat{\mathbf{v}}}\middle|\,\mathbf{v}\in\mathcal{R}_i\right\},\ i = 1,\cdots,N, \end{aligned}$$

where $\boldsymbol{\Sigma}_{\mathrm{m}} = E(\boldsymbol{\Sigma}_{\mathrm{v}})$. In the implementation of the algorithm $\boldsymbol{\Sigma}_{\mathrm{m}}$ has to be estimated from the training unit norm vectors belonging to $\mathcal{R}_i$. To solve the above minimization problem consider the generalized eigenvalue equation for $\boldsymbol{\Sigma}_{\mathrm{d}}$ and $\boldsymbol{\Sigma}_{\mathrm{m}}$

$$\boldsymbol{\Sigma}_{\mathrm{m}}\mathbf{F} = \Lambda\boldsymbol{\Sigma}_{\mathrm{d}}\mathbf{F}, \tag{15}$$

where

$$\Lambda = \mathrm{diag}\left(\lambda_1,\cdots,\lambda_t\right), \quad \text{and}\ \ \mathbf{F} = \left(\mathbf{f}_1,\cdots,\mathbf{f}_t\right).$$

Assuming that $\lambda_1 > \lambda_2\cdots > \lambda_t$, the solution to minimization function is given by normalized version of $\mathbf{f}_t$, i.e.the i^{th} new codeword is given by

$$\widehat{\mathbf{v}}_i = \mathbf{f}_t/\|\mathbf{f}_t\|. \tag{16}$$

The above two conditions are iterated until the convergence.

Compared to the optimum codebook design presented in [3], apart from some changes in additive and multiplicative factors, the primary change due to EED is that, in the centroid condition, the new codebook design depends on the joint eigen decomposition of both signal and noise correlation matrices. In contrast, the centroid condition in [3] requires eigen decomposition of only the signal correlation matrix. Similar to [3] a new codebook has to be designed for each SNR point. However, the codebook design is an off-line process so it is not a computational burden on MS.

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2) Encoding: Beamforming Vector Selection: The optimum encoding process (selection of the code point index to be sent to the transmitter) is defined as follows

$$\widehat{\mathbf{v}} = \arg\max_{\widehat{\mathbf{v}}_i \in \mathcal{W}} \frac{|\langle \widehat{\mathbf{v}}_i, \mathbf{v}\rangle|^2}{\widehat{\mathbf{v}}_i^H \boldsymbol{\Sigma}_{\mathrm{d}} \widehat{\mathbf{v}}_i}$$

By this encoding process, the unit norm sphere $\mathcal{S}_t = \{\mathbf{v} \in \mathbb{C}^t\}$, is partitioned into N regions $\mathcal{R}_i, i = 1, ..., N$.

The above encoding process is optimum in the sense that it maximizes the received SNR. Compared to [3] the encoding process is also different. Since EED was not considered, in [3] only the numerator part is used in the encoding.

IV. Spatially IID Channel - Loss Analysis

Analyzing the loss for correlated channels with EED and quantization is a complicated problem and analytic tractability remains elusive at this time. However, we have had success with the loss in ergodic capacity for the less general but still important spatially i.i.d channel with EED and channel quantization. A closed form analytical expression for the combined loss due to the three forms of feedback imperfection (estimation errors, delay and channel quantization) is derived in this section for the spatially i.i.d channel.

Without EED and channel quantization (i.e., transmitter has $\mathbf{h}/\|\mathbf{h}\|$), the ergodic capacity is given by

$$C_{ideal} = E\left[\log_2\left(1 + \|\mathbf{h}\|^2\gamma_s\right)\right]. \quad (17)$$

It can be shown that for the spatially i.i.d channel, $\boldsymbol{\Sigma}_{\mathrm{hh}} = \mathbf{I}$ and $\boldsymbol{\Sigma}_{\mathrm{im}} = |\rho|^2\mathbf{I}$, $0 < |\rho| < 1$. More specifically ρ can be shown as the product of estimation related correlation coefficient ρ_e, and delay related correlation co-efficient ρ_d [14]. With these values the lower bound on ergodic capacity as given in (10) becomes

$$C_{quant} = E\left[\log_2\left(1 + \|\mathbf{h}\|^2\gamma_s^f\,\theta\right)\right], \quad (18)$$

where

$$\gamma_s^f = \frac{|\rho|^2\gamma_s}{1 + (1 - |\rho|^2)\gamma_s}, \text{and } \theta = |\vartheta|^2.$$

The loss in ergodic capacity between the ideal case, unquantized channel without EED, and the quantized channel with EED is given by (19). Note that this loss formulation is different (but more general) from the one in spatially correlated with EED scenario. The loss function in correlated channel case is defined as the further loss due to quantization. Where as, for i.i.d, C_{L-iid} implies the overall loss due to EED and quantization. By using the Taylor series expansion (19) can be written as

$$\begin{aligned} C_{L-iid} &= \frac{1}{\ln 2}\sum_{k=1}^{\infty}\frac{1}{k}E\left[\frac{\|\mathbf{h}\|^2(\gamma_s - \gamma_s^f\vartheta)}{1 + \|\mathbf{h}\|^2\gamma_s}\right]^k \\ &= \frac{1}{\ln 2}\sum_{k=1}^{\infty}\frac{1}{k}E\left(\left[\frac{\|\mathbf{h}\|^2\gamma_s}{1 + \|\mathbf{h}\|^2\gamma_s}\right]^k\right)E(\beta^k), \quad (21)\end{aligned}$$

where $\beta = 1 - \xi\theta$, $\xi = \frac{\gamma_s^f}{\gamma_s}$. It is easy to see that $\xi < 1$ and $\xi = 1$ *if and only if* there is no delay and no estimation error. In (21), the independence of the quantization related term and channel norm related term is due to the fact that the channel is spatially i.i.d. The codebook design for this scenario is a special case of the previously considered spatially correlated case. The statistical characterization of θ, required for the loss analysis, is studied in both [3] and [4] and an approximate pdf is given as

$$f_\theta(x) = 2^B(t-1)(1-x)^{t-2}, \quad 1 - \psi < x < 1, \quad (22)$$

where $\psi = 2^{-B/(t-1)}$. The first expectation in (21) can be evaluated as [18]

$$E\left(\left[\frac{\|\mathbf{h}\|^2\gamma_s}{1 + \|\mathbf{h}\|^2\gamma_s}\right]^k\right) = \frac{\Gamma(k+t)}{\Gamma(t)}\gamma_s^k\,{}_2F_0\left(t+k, k;;-\gamma_s\right), \quad (23)$$

where ${}_2F_0(,,;;)$ is the generalized hypergeometric function. Using change of variables, the pdf of β can be shown to be

$$f_\beta(x) = \frac{2^B(t-1)}{\xi^{t-1}}\left(\xi - 1 + x\right)^{t-2}, \quad 1-\xi < x < 1-\xi+\xi\psi.$$

$E\left(\beta^k\right)$ is evaluated in (20) using results from [19], where $\tau = 1 - \xi + \xi\psi$. Substituting (23) and (20) in (21) gives the final closed form expression for C_{L-iid}.

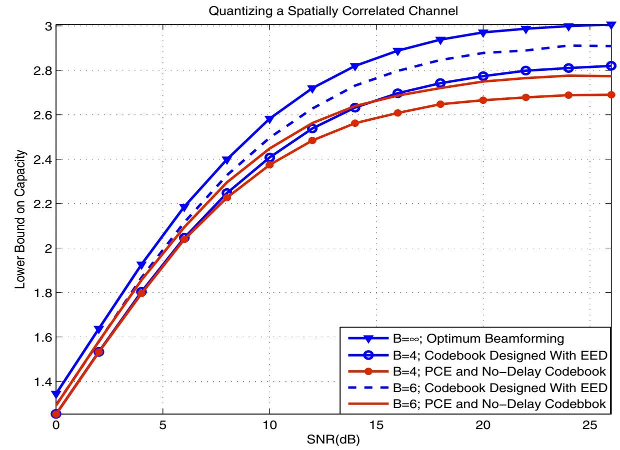

Fig. 1. Performance of new codebook deigned by considering both noise and signal correlation matrices

V. Numerical and Simulation Results

The effectiveness of the new codebook design algorithm can be seen in Fig. 1. It plots the (simulated) lower bound on ergodic capacity due to the finite rate quantization of the CSI with estimation errors and feedback delay. Simulation parameters: $t = 3$, $B \in \{4, 6\}$, the spatially correlated channel, $\boldsymbol{\Sigma}_{\mathrm{hh}}$, is simulated by the correlation model in [20]: A linear antenna array with antenna spacing of half wavelength, angle of arrival $\phi = 0°$ and an uniform angular spread of $[-\pi/5, \pi/5]$. $\boldsymbol{\Sigma}_{\mathrm{im}}$ is simulated in a similar fashion with an uniform angular spread of $[-\pi/5.5, \pi/5.5]$ and the resulting correlation matrix is scaled by 0.7582. Note that the various auto and cross correlation matrices are included in $\boldsymbol{\Sigma}_{\mathrm{im}}$, so they are not specified separately. The noise correlation is given by $\boldsymbol{\Sigma}_{\mathrm{n}} = \boldsymbol{\Sigma}_{\mathrm{hh}} - \boldsymbol{\Sigma}_{\mathrm{im}}$.

The new codebook clearly outperforms the codebook designed without taking the EED into account [3]. The difference between the two codebooks is not much in the low SNR regime. However, there is a considerable gap in the high SNR

$$
\begin{aligned}
C_{ideal} - C_{quant} &= E\left[\log_2\left(1+\|\mathbf{h}\|^2\gamma_s\right)\right] - E\left[\log_2\left(1+\|\mathbf{h}\|^2\gamma_s^f\vartheta\right)\right] \\
C_{L-iid} &= -E\left[\log_2\left(\frac{1+\|\mathbf{h}\|^2\gamma_s^f\vartheta}{1+\|\mathbf{h}\|^2\gamma_s}\right)\right] = -E\left[\log_2\left(1-\frac{\|\mathbf{h}\|^2(\gamma_s-\gamma_s^f\vartheta)}{1+\|\mathbf{h}\|^2\gamma_s}\right)\right] \qquad (19)\\
E\left(\beta^k\right) &= \frac{2^B(t-1)}{\xi^{t-1}}\int_{1-\xi}^{1-\xi+\xi\psi} x^k\left(\xi-1+x\right)^{t-2}dx
\end{aligned}
$$

$$
= \frac{2^B(t-1)(\xi-1)^{t-2}}{\xi^{t-1}(k+1)}\left[(\tau)^{k+1}{}_2F_0\left(-t+2,k+1;2+k;1+\frac{\xi\psi}{1-\xi}\right)-(1-\xi)^{k+1}{}_2F_0\left(-t+2,k+1;2+k;1\right)\right], \qquad (20)
$$

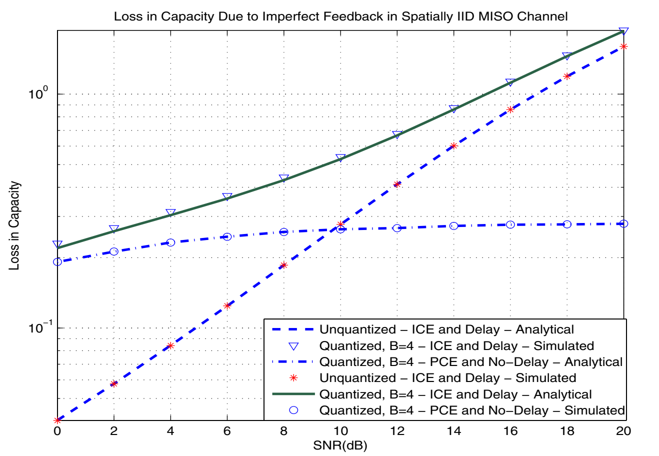

Fig. 2. Effect of feedback imperfections on ergodic capacity of spatially i.i.d channel, compared to Fig. 1, this Fig plots the loss in ergodic capacity due to EED and channel quantization.

regime. Since the signal part leaks into the noise, all the curves flatten out at high SNRs. Increasing the number of feedback bits improves the performance. Though the loss analysis for correlated channels with EED is not presented, based on the source coding perspective provided in [17], it can be shown that the loss is proportional to $2^{-B/(t-1)}$.

In Fig. 2, both the analytical and simulated curves are plotted for the loss in ergodic capacity due to estimation errors, delay and finite rate quantization. The analytical curves are in agreement with the simulations. Simulation parameters: $t = 3$, $|\rho|^2 = 0.989$, and $B = 4$. In the evaluation of the analytical expression for loss, (21), only the first 40 terms in the series were considered. The penalty of having the three forms of imperfection (solid green line) is quite severe on the system performance. The figure also shows loss due to quantization alone and EED alone. The loss due to quantization alone is seen to be much less compared to the loss due to EED alone.

VI. Conclusion

A model that can capture the estimation errors, feedback delay and channel quantization is developed for the spatially and temporally correlated Rayleigh flat-fading MISO channels. In the presence of EED, for the optimum transmit beamforming, a new codebook design algorithm that takes the noise correlation matrix into account is proposed. Simulations clearly show that the new codebook outperforms the optimum codebook designed for perfect channel estimation and no-delay case. For spatially i.i.d scenario, a closed form analytical expression for the loss (loss due to EED and channel quantization) in ergodic capacity is derived and validated through simulations.

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