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Performance Analysis of Transmit Beamforming for MISO Systems with Imperfect Feedback

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Abstract

In this paper, we analyze the performance of transmit beamforming on multiple-antenna Rayleigh fading channels with imperfect channel feedback. We characterize the feedback imperfections in terms of noisy channel estimation, feedback delay, and finite rate channel quantization. We develop a general framework, valid for an arbitrary two-dimensional linear modulation, that captures the aforementioned imperfections, and derive the symbol and bit error probability expressions for both M-PSK and M-ary rectangular QAM constellations with Gray code mapping. We show that the proposed analytical formulation is valid for frequency-domain duplexing system with/without finite rate channel quantization and time-domain duplexing system. We validate the accuracy of the analysis through simulations, and assess the relative effects of channel estimation inaccuracy, feedback delay, and finite-rate quantization on the symbol and bit error performances for various constellations.

Keywords: Multi-antenna systems, transmit beamforming, imperfect channel knowledge, feedback delay, channel quantization.

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1 Introduction

The rapid growth in wireless industry, and the demand for high data rates with high reliability has intensified the research efforts in multiple-input and multiple-output (MIMO) wireless systems. It is well-known that the capacity of MIMO systems in a rich scattering environment increases linearly with the minimum number of transmit and receive antennas [1, 2]. In a multiple-input and single-output (MISO) system, if the channel state information (CSI) is available at the transmitter (i.e., CSIT), one can achieve both the diversity and array gains with transmit beamforming via maximal ratio transmission (MRT) [3], whereas only diversity gain can be realized with space-time coding [4, 5].

Transmit beamforming for MISO systems is an active area of research. In [6], the authors consider efficient use of CSIT for transmit beamforming. An information theoretic approach to transmit beamforming with imperfect feedback is presented in [7]. With MRT and BPSK modulation, the effect of feedback delay with perfect channel estimation (PCE) at the receiver is investigated in [8], whereas [9] studies the effect of imperfect channel estimation (ICE) without feedback delay. In [10], the authors extend the analysis of [8] accounting for the effects of channel estimation errors. The effect of feedback delay and feedback errors on the receiver signal-to-noise ratio (SNR) performance is investigated in [11] for phase-only feedback, whereas [12] analyzes the bit error probability (BEP) degradation with BPSK due to feedback errors with selection and co-phasing feedback schemes. The effects of finite-rate channel quantization and feedback delay are considered for BPSK in [13]. Finally, approaches for the design and analysis of transmit beamforming schemes under finite-rate constraints are presented in [14]-[18].

While the aforementioned works considered either BPSK (or quadrature PSK, QPSK) with estimation errors and perfect quantization [8]-[13], or ideal channel estimation with finite-rate quantization for general modulations [14]-[18], combined effects of various channel imperfections for general modulations is not yet investigated. This contribution is targeted to fill in this important void. In this work, we present a general framework for the performance analysis of transmit beamforming for MISO systems on Rayleigh fading channels with imperfect channel feedback. The feedback imperfections are characterized in terms of noisy channel estimation at the receiver side, quantization of CSI, and feedback delay. This formulation is shown to be applicable for any linear two-dimensional modulation scheme on spatially independent and identically distributed (i.i.d) Rayleigh fading channels. Our analytical framework encompasses three popular MISO system models, namely, frequency-domain duplexing (FDD), FDD with finite rate quantization of CSI (FDDQ) and time-domain duplexing (TDD). We analyze average symbol error probability (SEP) and bit error probability (BEP) performances of M-PSK and M-ary rectangular QAM constellations with Gray code mapping. Our numerical and simulation results show that channel estimation inaccuracy and feedback delay are more harmful to the system performance compared to the effects of finite-rate channel quantization.

The rest of this paper is organized as follows. In Section 2, we introduce our system model, present a model for imperfect channel feedback, and show that it captures the essential features of FDD system with/without feedback and TDD system . The decision variable (DV) at the receiver with imperfect feedback is also derived in Section 2. The average SEP and BEP expressions for M-PSK and M-QAM modulations are presented in Section 3. Numerical and simulation results are presented in Section 4. We conclude this paper in Section 5.

2 System Model

We consider a MISO system with N antennas at the base station (BS) and one antenna at the mobile station (MS) as shown in Fig. 1. The channel between the BS and the MS is modeled as a frequency-flat, slowly varying Rayleigh fading channel that is assumed to be constant over a block of L symbols. Specifically, let us denote by $h_i[k]$ the complex channel gain at time k between the ith antenna at the BS and the MS. We assume that $h_i[k]$ is a zero-mean, circularly symmetric complex Gaussian (CSCG) random variable (r.v) with variance $E[|h_i[k]|^2] = \Omega$. We also assume that for any i and j ($i \neq j$), and for any k_1 and k_2, $h_i[k_1]$ and $h_j[k_2]$ are uncorrelated, which can be justified for sufficient spacing between antenna elements. The vector valued channel at time k is denoted by $\mathbf{h}[k] = [h_1[k], h_2[k], \ldots, h_N[k]]^\mathsf{T}$, where T denotes transpose operation. The transmitted two-dimensional modulation symbol at time k is denoted by $s_m[k]$ which belongs to the constellation $\mathcal{S}$. The average energy of $s_m[k]$ is $E[|s_m[k]|^2] = E_s$. Let us denote by $\mathbf{w}[k] = [w_1[k], w_2[k], \ldots, w_N[k]]^\mathsf{T}$, the unit norm (i.e., $\|\mathbf{w}\|^2 = 1$) beamforming vector (b.v) at the BS at time k. Then, the received signal at the MS at time k is

$$y[k] = \mathbf{w}^{\mathsf{H}}[k]\mathbf{h}[k]s_m[k] + \eta[k], \tag{1}$$

where H is the Hermitian operator and $\eta[k]$ is a zero-mean, CSCG r.v with $E[|\eta[k]|^2] = \sigma_n^2$. The specific structure of the b.v $\mathbf{w}[k]$ at the BS depends on various design considerations and system imperfections. However, for both FDD and TDD systems on Rayleigh fading channels, the b.v $\mathbf{w}[k]$ is derived from the channel estimate, which is assumed to be jointly Gaussian with the actual channel $\mathbf{h}[k]$. Note that this assumption is well justified [20] for many practical estimation techniques such as additive channel estimation, MMSE channel estimation, and channel estimation derived from pilot-symbol assisted modulation [21]. We now dwell in detail on three popular channel feedback approaches for which we present a general framework for modeling nonideal feedback for MISO systems, and derive the DV at the input of the demodulator. These systems are: *i*) FDD with channel estimation errors and delayed feedback, *ii*) FDD with channel estimation errors, finite-rate quantization, and delayed feedback, and *iii*) TDD with channel estimation errors and channel decorrelation.

2.1 FDD System

Let us denote by $\widehat{\mathbf{h}}[k] = \left[\widehat{h}_1[k], \widehat{h}_2[k], \ldots, \widehat{h}_N[k]\right]^{\mathsf{T}}$ the channel estimate at the MS at time k. We assume that the estimate $\widehat{h}_i[k]$ on the path from the MS antenna and the ith antenna of the BS is a zero-mean, CSCG r.v with variance $E[|\widehat{h}_i[k]|^2] = \Lambda$. Similar to $\mathbf{h}[k]$, we assume that $\widehat{h}_i[k_1]$ and $\widehat{h}_j[k_2]$, for $i \neq j$, are also uncorrelated with each other. The MS simply feeds back the estimate $\widehat{\mathbf{h}}[k]$ to the BS. Assuming a feedback delay of D, the channel observed at the BS is $\widehat{\mathbf{h}}[k-D]$. The normalized delayed estimate forms the b.v at the transmitter

$$\mathbf{w}[k] = \frac{\widehat{\mathbf{h}}[k-D]}{\|\widehat{\mathbf{h}}[k-D]\|}. \tag{2}$$

Since the channel is assumed to be slowly varying, let us denote by ρ the complex correlation coefficient between $h_i[k]$ and $\widehat{h}_i[k-D]$. That is,

$$\rho = \frac{E\left[h_i[k]\widehat{h}_i^*[k-D]\right]}{\sqrt{E\left[|h_i[k]|^2\right] \times E\left[|\widehat{h}_i[k-D]|^2\right]}} = \frac{E\left[h_i[k]\widehat{h}_i^*[k-D]\right]}{\sqrt{\Omega\Lambda}}, \tag{3}$$

where $(\cdot)^*$ denotes complex conjugation. As $h_i[k]$ and $\widehat{h}_i[k-D]$ are assumed to be jointly Gaussian, we can write [22]

$$h_i[k] = \rho\sqrt{\frac{\Omega}{\Lambda}}\widehat{h}_i[k-D] + \sqrt{(1-|\rho|^2)\Omega}\varepsilon_i[k-D] \quad i = 1, \ldots, N, \tag{4}$$

where $\varepsilon_i[k]$ is zero-mean, CSCG r.v with variance $E[|\varepsilon_i[k]|^2] = 1$ and is uncorrelated with $\widehat{h}_i[k]$. Upon stacking the elements $\{h_i[k]\}$ in a column, (4) reads as

$$\mathbf{h}[k] = \rho\sqrt{\frac{\Omega}{\Lambda}}\widehat{\mathbf{h}}[k-D] + \sqrt{(1-|\rho|^2)\Omega}\varepsilon[k-D], \tag{5}$$

where $\varepsilon[k] = [\varepsilon_1[k], \varepsilon_2[k], \ldots, \varepsilon_N[k]]^{\mathsf{T}}$. Then, using (5), the equivalent received signal of (1) can be written as

$$\begin{aligned} y[k] &= \mathbf{w}^{\mathsf{H}}[k]\mathbf{h}[k]s_m[k] + \eta[k] \\ &= \mathbf{w}^{\mathsf{H}}[k]\left(\rho\sqrt{\frac{\Omega}{\Lambda}}\widehat{\mathbf{h}}[k-D] + \sqrt{(1-|\rho|^2)\Omega}\varepsilon[k-D]\right)s_m[k] + \eta[k] \\ &= \rho\sqrt{\frac{\Omega}{\Lambda}}\|\widehat{\mathbf{h}}[k-D]\|s_m[k] + \zeta[k], \end{aligned} \tag{6}$$

where $\zeta[k] = \mathbf{w}^{\mathsf{H}}[k]\sqrt{(1-|\rho|^2)\Omega}\varepsilon[k-D]s_m[k] + \eta[k]$. $\zeta[k]$ conditioned on $|s_m[k]|$, is a zero-mean, CSCG r.v with variance $\Sigma^2_{|s_m[k]|} = \sigma_n^2 + |s_m[k]|^2\Omega(1-|\rho|^2)$. We need the DV at the MS for demodulation and for SEP/BEP analysis of a transmitted modulation symbol. The MS obtains the DV by dividing $y[k]$ by $\|\widehat{\mathbf{h}}[k-D]\|$, provided that the feedback delay D is known. We remark that for constant amplitude signals (i.e., M-PSK) such a normalization is not needed as the decision regions are unchanged due to a positive scale factor. In this paper, we assume that D is known to the MS, so that the DV becomes

$$r[k] \triangleq \frac{y[k]}{\|\widehat{\mathbf{h}}[k-D]\|} = \rho\sqrt{\frac{\Omega}{\Lambda}}s_m[k] + \tilde{\zeta}[k], \tag{7}$$

where $\tilde{\zeta}[k]$, conditioned on $|s_m[k]|$ and $\|\widehat{\mathbf{h}}[k-D]\|$, is a zero-mean CSCG r.v with variance $\sigma^2_{\tilde{\zeta}} = \frac{\Sigma^2_{|s_m[k]|}}{\|\widehat{\mathbf{h}}[k-D]\|^2}$. For simplicity, let us define $\beta \triangleq \|\widehat{\mathbf{h}}[k-D]\|^2/\Lambda$. Clearly, β is a sum of N i.i.d exponential r.vs, each with unit mean. That is, β is gamma distributed with the probability density function (pdf) [22]

$$f_\beta(x) = \frac{e^{-x}x^{N-1}}{\Gamma(N)} \quad x \geq 0, \tag{8}$$

and the cumulative distribution function (cdf) [22]

$$F_\beta(x) = \text{Prob}(\beta \le x) = 1 - e^{-x}\sum_{k=0}^{N-1}\frac{x^k}{\Gamma(k+1)} \quad x \ge 0, \tag{9}$$

where $\Gamma(n) = \int_0^\infty e^{-u}u^{n-1}du$ is the standard Gamma function [23]. With this, $\sigma^2_{\tilde{\zeta}}$, conditioned on $|s_m[k]|$ and β, can be written as $\sigma^2_{\tilde{\zeta}} = \frac{\Sigma^2_{|s_m[k]|}}{\beta\Lambda}$. From (7), we notice that the effect of Imperfect Channel Estimation and feedback delay on the DV at the MS is that of scaling the transmitted symbol $s_m[k]$ by *an unknown* complex number $\rho\sqrt{\Omega/\Lambda}$ and introducing *symbol dependent non-Gaussian noise* $\tilde{\zeta}[k]$. The aforementioned framework can be directly applied to model nonideal channel feedback effects in an analog feedback system [24], where feedback is not only delayed but also noisy.

To understand the combined effects of delay and estimation errors, we now describe the structure of the combined correlation coefficient ρ and show how it relates to delay only correlation coefficient, ρ_d, and estimation error only correlation coefficient, ρ_e. With delay and no estimation errors, we have

$$h_i[k] = \rho_d h_i[k-D] + \sqrt{(1-|\rho_d|^2)\Omega}\,\tilde{\nu}_i[k], \tag{10}$$

where $\rho_d = E\left[h_i[k]h_i^*[k-D]\right]/\Omega$. On the other hand, estimation errors and no delay allows us to write

$$h_i[k] = \rho_e\sqrt{\frac{\Omega}{\Lambda}}\widehat{h}_i[k] + \sqrt{(1-|\rho_e|^2)\Omega}\,\nu_i[k], \tag{11}$$

where $\nu_i[k]$ is a zero-mean, CSCG r.v with variance $E[|\nu_i[k]|^2] = 1$ and is uncorrelated with $\widehat{h}_i[k]$. Here, ρ_e is given by $\rho_e = E\left[h_i[k]\widehat{h}_i^*[k]\right]/\sqrt{\Omega\Lambda}$. Using (10) and (11) in the definition of ρ, given in (3), we arrive at

$$\rho = \frac{E\left[\left(\rho_e\sqrt{\frac{\Omega}{\Lambda}}\widehat{h}_i[k] + \sqrt{(1-|\rho_e|^2)\Omega}\,\nu_i[k]\right)\widehat{h}_i^*[k-D]\right]}{\sqrt{\Omega\Lambda}}. \tag{12}$$

Since $\nu_i[k]$ is uncorrelated with $\widehat{h}_i[k-D]$, (12) simplifies to

$$\rho = \rho_e\frac{E\left[\widehat{h}_i[k]\widehat{h}_i^*[k-D]\right]}{\Lambda} = \rho_e\rho_d. \tag{13}$$

That is, the combined correlation coefficient ρ is the product of delay only correlation coefficient ρ_d and estimation error only correlation coefficient ρ_e. Note that in the derivation of this result no particular structure is assumed for the time-variations of the channel.

2.2 FDD with Finite Rate Feedback (FDDQ) System

As illustrated in Fig. 1, in FDDQ, the MS estimates the channel, and quantizes it into one of $C = 2^B$ code words. The index, which is represented by B bits, of the code word corresponding to the channel estimate is fed back to the BS. In this section, we assume that the feedback channel is error free, which can be justified by employing powerful error correction codes for protecting the index of the code word [14]-[18]. However, in addition to the delay in the feedback channel, channel coding introduces nonnegligible delay in decoding the code word index. For simplicity, let $\widehat{\mathbf{v}}[k-D] = \widehat{\mathbf{h}}[k-D]/\|\widehat{\mathbf{h}}[k-D]\|$, and $\tilde{\mathbf{v}}[k-D] = \mathbf{w}[k]$. Then,

$$\tilde{\mathbf{v}}[k-D] = \mathcal{Q}\left[\widehat{\mathbf{v}}[k-D]\right], \tag{14}$$

where $\mathcal{Q}$ is the quantization function. Now, the received signal with (14) is given by

$$y[k] = \tilde{\mathbf{v}}^{\mathsf{H}}[k-D]\mathbf{h}[k]s_m[k] + \eta[k]. \tag{15}$$

After substituting (5) for $\mathbf{h}[k]$, the received signal (15) can be written as

$$y[k] = \rho\sqrt{\frac{\Omega}{\Lambda}}\tilde{\mathbf{v}}^{\mathsf{H}}[k-D]\widehat{\mathbf{h}}[k-D]s_m[k] + \upsilon[k]. \tag{16}$$

Here, conditioned on $|s_m[k]|$ and $\tilde{\mathbf{v}}[k-D]$, $\upsilon[k]$ is a zero-mean, CSCG r.v with variance $\sigma_n^2 + |s_m[k]|^2(1-|\rho|^2)\Omega$. Let us define the following to simplify (16)

$$\vartheta \triangleq \langle \tilde{\mathbf{v}}[k-D], \widehat{\mathbf{v}}[k-D] \rangle, \tag{17}$$

where $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^{\mathsf{H}}\mathbf{y}$. This allows us to write $y[k]$ as

$$y[k] = \rho\sqrt{\frac{\Omega}{\Lambda}}\|\widehat{\mathbf{h}}[k-D]\|\vartheta s_m[k] + \upsilon[k]. \tag{18}$$

Since the delay D is assumed to be known to the receiver (i.e., the MS knows $\|\widehat{\mathbf{h}}[k-D]\|$ and ϑ), we form the DV as

$$r[k] \triangleq \frac{y[k]}{\|\widehat{\mathbf{h}}[k-D]\|\vartheta} = \rho\sqrt{\frac{\Omega}{\Lambda}}s_m[k] + \frac{\upsilon[k]}{\|\widehat{\mathbf{h}}[k-D]\|\vartheta} = \rho\sqrt{\frac{\Omega}{\Lambda}}s_m[k] + \tilde{\upsilon}[k], \tag{19}$$

where, conditioned on $|s_m[k]|$, β and $\tilde{\Delta}$, $\tilde{\upsilon}[k]$ is a zero-mean CSCG r.v with variance $\frac{\sigma_n^2+|s_m[k]|^2(1-|\rho|^2)\Omega}{\beta\tilde{\Delta}\Lambda}$. Here, $\tilde{\Delta} \triangleq |\vartheta|^2$. Since finding the exact pdf of $\tilde{\Delta}$ is rather difficult, [17] upper bounded $\tilde{\Delta}$ (i.e., lower bounded the average error performance) by a r.v Δ, whose pdf is given by

$$f_{\Delta}(x) = 2^B(N-1)(1-x)^{N-2}, \quad 1-\psi < x < 1, \tag{20}$$

where $\psi = 2^{-B/(N-1)}$. Independently, in [18] the authors showed that (20) is a very accurate approximation to the true pdf of $\tilde{\Delta}$. Note that when $B \to \infty$, we have $\psi \to 0$ and (20) reduces to $f_{\Delta}(x) = \delta(x-1)$ [14, 17, 18], where, δ is the dirac delta function. In what follows, we use Δ in place of $\tilde{\Delta}$.

2.3 TDD System

In a TDD system, assuming channel reciprocity, the MS sounds the channel with known symbols to facilitate the BS estimate of the channel. Assuming a single pilot symbol x_0, which is known to the BS, the received signal on the lth antenna at the BS is given by

$$p_l[k] = h_l[k]x_0 + \mu_l[k] \quad l = 1, 2, \ldots, N, \tag{21}$$

where $\mu_l[k]$ is a zero-mean, CSCG r.v with variance $E[|\mu_l[k]|^2] = \sigma_n^2$, x_0 is deterministic with power $|x_0|^2 = \sigma_x^2$, and $h_l[k]$ is the channel between the l^{th} antenna at the BS and MS. For simplicity, assuming mininum mean-square error (MMSE) channel estimation, the estimate $\widehat{h}_l[k]$ of the lth channel is given by

$$\widehat{h}_l[k] = \left(\frac{\sigma_x \Omega}{\sigma_x^2 \Omega + \sigma_n^2}\right) p_l[k] \quad l = 1, 2, \ldots, N. \tag{22}$$

Therefore, $\Lambda = E[|\widehat{h}_l[k]|^2] = \Omega\gamma_{p,\mathsf{T}}/(1+\gamma_{p,\mathsf{T}})$ and $\rho_e^2 = \gamma_{p,\mathsf{T}}/(1+\gamma_{p,\mathsf{T}})$, where $\gamma_{p,\mathsf{T}} = \Omega\sigma_x^2/\sigma_n^2$ is the average received SNR for the pilot. For FDD systems, a simple approach to channel estimation is the use of orthogonal pilot transmissions. That is, with a single pilot symbol, the number of time units required to estimate the channel with an FDD system is N times higher than the TDD system. One approach for fair comparison between TDD and FDD systems is to set the total average received pilot SNR the same. Then, the aforementioned Λ and ρ_e^2 can be used with $\gamma_{p,\mathsf{F}} = \gamma_{p,\mathsf{T}}/N$.

We note that the above formulation can easily be generalized to multiple pilot symbols, time varying channel conditions, and for various practical channel estimation schemes. We also account for a delay D between the time of channel estimation, and the time of its actual use. Note that this delay might not be as severe as that of the delay in FDD schemes. Then, the transmit b.v is given by (2) and the received signal $y[k]$ is exactly the same as (6) of the previous FDD approach, which is reproduced here as

$$y[k] = \rho\sqrt{\frac{\Omega}{\Lambda}}\|\widehat{\mathbf{h}}[k-D]\|s_m[k] + \zeta[k]. \tag{23}$$

For the demodulation of $s_m[k]$, the receiver needs the knowledge of $\|\widehat{\mathbf{h}}[k-D]\|$, which has to come from a feedback channel between the BS and the MS. This feedback requirement in TDD system is counterintuitive to the traditional argument that feedback is not required for TDD systems. Assuming ideal channel knowledge of $\|\widehat{\mathbf{h}}[k-D]\|$ at the receiver, the DV is again given by (7).

3 Error Probability Analysis

In this section, we analyze the average SEP and BEP performances of M-PSK and rectangular M-QAM modulation with Gray code symbol mapping. Upon observing (7) and (19), the DV at the demodulation input can be expressed in a parametric form as

$$r[k] = \kappa s_m[k] + \xi[k] = r_I[k] + jr_Q[k], \tag{24}$$

where $\kappa = \rho\sqrt{\Omega/\Lambda} \triangleq \mu_I + j\mu_Q$, and $\xi[k]$, conditioned on $|s_m[k]|$, β and Δ, is a CSCG r.v with variance $\mathcal{F}(|s_m[k]|)/(\beta\Delta)$, where $\mathcal{F}(|s_m[k]|) = (\sigma_n^2 + (1-|\rho|^2)|s_m[k]|^2\Omega)/\Lambda$. Note that for FDD and TDD schemes, we can set $\Delta = 1$ (i.e., $f_\Delta(x) = \delta(x-1)$). On the other hand, for FDDQ the pdf of Δ is given by (20). It is important to note that, due to the presence of signal dependant noise together with the *unknown constant* κ, it is not possible to borrow the existing error probability expressions that are available in the literature for PSK and rectangular QAM constellations and extend them to the present case of ICE, delay and finite-rate quantization. This motivates us to derive the error probability expressions *ab initio* using the DV given by (19).

3.1 M-ary PSK Constellation

Since for M-PSK, $|s_m[k]|$ is not a function of the index m, we then define $\mho \triangleq \mathcal{F}(|s_m[k]|) = (\sigma_n^2 + (1-|\rho|^2)E_s\Omega)/\Lambda$. With M-PSK modulation, the DV of interest is the phase angle Θ of the received signal $r[k]$. Conditioned on β and Δ, the cdf of Θ can be obtained as a special case of the results presented by Pawula et al. in [29], wherein the authors derived a general expression for the cdf of the phase angle between two vectors corrupted by Gaussian noise. Upon using [29], the conditional cdf of Θ, conditioned on β and Δ, when $\theta_m = 2m\pi/M$ is the transmitted phase, can be expressed as

$$
\begin{aligned}
&\text{Prob}(\omega_1 \le \Theta \le \omega_2; \lambda|\beta\Delta) \qquad (25)\\
&= \begin{cases} F_{\Theta|\beta\Delta}(\omega_2 - \theta_m - \phi_\rho; \lambda) - F_{\Theta|\beta\Delta}(\omega_1 - \theta_m - \phi_\rho; \lambda) + 1 & \text{if } \omega_1 < \theta_m + \phi_\rho < \omega_2 \\ F_{\Theta|\beta\Delta}(\omega_2 - \theta_m - \phi_\rho; \lambda) - F_{\Theta|\beta\Delta}(\omega_1 - \theta_m - \phi_\rho; \lambda) & \text{if } \omega_1 > \theta_m + \phi_\rho \text{ or } \omega_2 < \theta_m + \phi_\rho, \end{cases}
\end{aligned}
$$

where $\omega_1 < \omega_2$, ϕ_ρ is the phase angle of ρ, and

$$\lambda = \frac{|\kappa|^2 E_s}{\mho} = \frac{|\rho|^2\gamma}{1 + (1 - |\rho|^2)\gamma}, \qquad (26)$$

where $\gamma = \Omega E_s/\sigma_n^2$, is the average received SNR per symbol with ICE. In (25)

$$F_{\Phi|\beta\Delta}(\theta; \lambda) = -\frac{\text{sgn}(\theta)}{2\pi} \int_0^{\pi-|\theta|} \exp\left(-\lambda\beta\Delta \frac{\sin^2\theta}{\sin^2 x}\right) dx \qquad -\pi < \theta < \pi, \qquad (27)$$

which is also referred to as Pawula's F function [26]. In (27) $\text{sgn}(x) = 1$ for $x \ge 0$ and is equal to -1 otherwise. Due to the discontinuity of $F_{\Phi|\beta\Delta}(\theta; \cdot)$ of (27) at $\theta = 0$, for evaluating (25) either at $\omega_1 = 0$ or $\omega_2 = 0$ we have to use $F_{\Phi|\beta\Delta}(\omega_1 = 0; \cdot) = -1/2$ and $F_{\Phi|\beta\Delta}(\omega_2 = 0; \cdot) = 1/2$. For details please refer to [29].

3.1.1 Average Symbol Error Probability

In this section we derive the expressions for average SEP of M-PSK. Upon using the conditional cdf (25) of Θ with $\omega_1 = \theta_m - \pi/M$ and $\omega_2 = \theta_m + \pi/M$ and subtracting the result from unity, we can obtain the average SEP of M-PSK modulation, conditioned on β and Δ, as

$$P_{s,PSK}(\beta\Delta) = \frac{\text{sgn}(\frac{\pi}{M} - \phi_\rho)}{2\pi} \int_0^{\pi-|\frac{\pi}{M}-\phi_\rho|} e^{-\frac{\lambda\beta\Delta\sin^2(\frac{\pi}{M}-\phi_\rho)}{\sin^2\theta}} d\theta + \frac{\text{sgn}(\frac{\pi}{M} + \phi_\rho)}{2\pi} \int_0^{\pi-|\frac{\pi}{M}+\phi_\rho|} e^{-\frac{\lambda\beta\Delta\sin^2(\frac{\pi}{M}+\phi_\rho)}{\sin^2\theta}} d\theta, \qquad (28)$$

which is valid for $|\phi_\rho| < \pi/M$[1]. To arrive at the average SEP performance, we need to average (28) over β and Δ. To reduce the analytical complications, we note that it is important to take the average of (28) first over β then over Δ. Proceeding further, by averaging (28) over the pdf of β, as given in (8), we arrive at

$$
\begin{aligned}
\tilde{P}_{s,PSK}(\Delta) &= E\left[P_{s,PSK}(\beta\Delta)\right] = \frac{\text{sgn}(\frac{\pi}{M} - \phi_\rho)}{2\pi} \int_0^{\pi-|\frac{\pi}{M}-\phi_\rho|} \left(\frac{\sin^2\theta}{\sin^2\theta + \lambda\Delta\sin^2(\frac{\pi}{M} - \phi_\rho)}\right)^N d\theta + \\
&\quad \frac{\text{sgn}(\frac{\pi}{M} + \phi_\rho)}{2\pi} \int_0^{\pi-|\frac{\pi}{M}+\phi_\rho|} \left(\frac{\sin^2\theta}{\sin^2\theta + \lambda\Delta\sin^2(\frac{\pi}{M} + \phi_\rho)}\right)^N d\theta. \qquad (29)
\end{aligned}
$$

[1] When $|\phi_\rho| \ge \pi/M$, the result of (28) with $|\phi_\rho| < \pi/M$ should be subtracted from unity.

For FDD and TDD schemes, we have $\Delta = 1$. Therefore, the average SEP is given by

$$\overline{P}_{s,PSK,FDD/TDD} = \tilde{P}_{s,PSK}(1). \tag{30}$$

With FDDQ, the average SEP can be obtained by averaging (29) over the pdf of Δ which is given (20). In Appendix, this averaging is performed in closed-form, and the final expression for the average SEP with FDDQ is

$$\begin{aligned}\overline{P}_{s,PSK,FDDQ} &= \frac{\mathrm{sgn}(\frac{\pi}{M}-\phi_\rho)}{2\pi}\overline{\mathcal{G}}\left(\pi - |\frac{\pi}{M}-\phi_\rho|, \sqrt{\lambda \sin^2(\frac{\pi}{M}-\phi_\rho)}, N, B, \psi\right) \\ &+ \frac{\mathrm{sgn}(\frac{\pi}{M}+\phi_\rho)}{2\pi}\overline{\mathcal{G}}\left(\pi - |\frac{\pi}{M}+\phi_\rho|, \sqrt{\lambda \sin^2(\frac{\pi}{M}+\phi_\rho)}, N, B, \psi\right). \end{aligned} \tag{31}$$

where $\overline{\mathcal{G}}(\cdot,\cdot,\cdot,\cdot,\cdot)$ is given by (74) in Appendix. With $\rho = 1$ the average SEP, $\overline{P}_{s,PSK,FDDQ}$, simplifies to the results presented in [17].

3.1.2 Average Bit Error Probability

Since average BEP is also an important performance measure on fading channels, we now derive expressions for the average BEP of M-PSK modulation with Gray code labeling. Our approach to average BEP analysis is essentially motivated by [30]. Similar to [30], we define $\mathcal{P}(k;\beta\Delta)$ as the probability of the received signal phase, Θ, falling in a wedge of width $2\pi/M$ centered around the kth symbol point $k = 1,\ldots,M-1$, conditioned on β and Δ, when $S_0 = \sqrt{E_s}$ is the transmitted signal. With the help of (25) and $\theta_m = 0$, $\mathcal{P}(k;\beta\Delta)$ can be expressed as

$$\mathcal{P}(k;\beta\Delta) = \mathrm{Prob}\left(\theta_k - \frac{\pi}{M} \leq \Theta \leq \theta_k + \frac{\pi}{M}; \lambda|\beta\Delta\right). \tag{32}$$

To proceed further, let $|\phi_\rho - \theta_k| > \pi/M$. This allows us to simplify (32), using (25) and (27), as

$$\mathcal{P}(k;\beta\Delta) = \frac{\mathrm{sgn}(\theta_k + \frac{\pi}{M} - \phi_\rho)}{2\pi}\int_0^{\pi - |\theta_k + \frac{\pi}{M} - \phi_\rho|} e^{-\frac{\lambda\beta\Delta\sin^2(\theta_k + \frac{\pi}{M}-\phi_\rho)}{\sin^2\theta}} d\theta - \frac{\mathrm{sgn}(\theta_k - \frac{\pi}{M} - \phi_\rho)}{2\pi}\int_0^{\pi - |\theta_k - \frac{\pi}{M} - \phi_\rho|} e^{-\frac{\lambda\beta\Delta\sin^2(\theta_k - \frac{\pi}{M}-\phi_\rho)}{\sin^2\theta}} d\theta. \tag{33}$$

Note that when $|\phi_\rho - \theta_k| \leq \pi/M$, expressions analogous to (33) can be derived in a similar manner. Following the steps of (29), $\tilde{\mathcal{P}}(k;\Delta) = E[\mathcal{P}(k,\beta\Delta)]$ (i.e., averaging $\mathcal{P}(k;\beta\Delta)$ over β) is

$$\begin{aligned}\tilde{\mathcal{P}}(k;\Delta) &= \frac{\mathrm{sgn}(\theta_k + \frac{\pi}{M} - \phi_\rho)}{2\pi}\int_0^{\pi - |\theta_k + \frac{\pi}{M} - \phi_\rho|}\left(\frac{\sin^2\theta}{\sin^2\theta + \lambda\Delta\sin^2(\theta_k + \frac{\pi}{M} - \phi_\rho)}\right)^N d\theta - \\ &\quad \frac{\mathrm{sgn}(\theta_k - \frac{\pi}{M} - \phi_\rho)}{2\pi}\int_0^{\pi - |\theta_k - \frac{\pi}{M} - \phi_\rho|}\left(\frac{\sin^2\theta}{\sin^2\theta + \lambda\Delta\sin^2(\theta_k - \frac{\pi}{M} - \phi_\rho)}\right)^N d\theta. \end{aligned} \tag{34}$$

Using (74) of Appendix, the average $\overline{\mathcal{P}}(k) = E[\tilde{\mathcal{P}}(k;\Delta)]$ of (34) over Δ can be expressed as

$$\begin{aligned}\overline{\mathcal{P}}(k) &= \frac{\text{sgn}(\theta_k + \frac{\pi}{M} - \phi_\rho)}{2\pi}\overline{\mathcal{G}}\left(\pi - |\theta_k + \frac{\pi}{M} - \phi_\rho|, \sqrt{\lambda \sin^2(\theta_k + \frac{\pi}{M} - \phi_\rho)}, N, B, \psi\right) \\ &- \frac{\text{sgn}(\theta_k - \frac{\pi}{M} - \phi_\rho)}{2\pi}\overline{\mathcal{G}}\left(\pi - |\theta_k - \frac{\pi}{M} - \phi_\rho|, \sqrt{\lambda \sin^2(\theta_k - \frac{\pi}{M} - \phi_\rho)}, N, B, \psi\right), \quad (35)\end{aligned}$$

which is applicable to FDDQ only. For FDD and TDD systems, $\overline{\mathcal{P}}(k) = \tilde{\mathcal{P}}(k;1)$. Using (35), the average BEP for Gray coded M-PSK signal set with finite-rate, imperfect feedback is

$$\overline{P}_{\text{b},PSK} = \frac{1}{\log_2(M)} \sum_{k=1}^{M-1} d(k)\overline{\mathcal{P}}(k), \quad (36)$$

where $d(k)$ is the weight spectrum of Gray code, derived in [30], which is reproduced here as

$$d(k) = 2\left|\frac{k}{M} - \left\lfloor \frac{k}{M} \right\rceil\right| + 2\sum_{i=2}^{\log_2(M)} \left|\frac{k}{2^i} - \left\lfloor \frac{k}{2^i}\right\rceil\right|, \quad (37)$$

where $\lfloor x \rceil$ rounds x to the closest integer. With $M = 2$, $\Delta = 1$, $\phi_\rho = 0$ and $\rho_e = 1$ (i.e., $\rho = \rho_d$ for a delayed feedback case), (36), with the help of (34), coincides with the results presented in [8]. We also note that the average BEP expressions in [8] can be derived in a very simple way using the methodology presented here.

3.2 M-ary Rectangular QAM Constellation

Let us denote $s_m[k] = s_{m,x}[k] + js_{m,y}[k]$, $m = 0, 1, \ldots, M-1$, $x = 0, 1, \ldots, M_1-1$, $y = 0, 1, \ldots, M_2-1$, where the M-QAM constellation is of size $M = M_1M_2$. Here $s_{m,x}[k] = a_{m,x}[k]d$, and $s_{m,y}[k] = a_{m,y}[k]d$, where $a_{m,x}[k] = -(M_1 - 1) + 2x$ (i.e., $a_{m,x}[k]d$ is the in-phase M_1-PAM constellation symbol) and $a_{m,y}[k] = -(M_2-1)+2y$ (i.e., $a_{m,y}[k]d$ is the quadrature-phase M_2- PAM constellation symbol). The minimum distance of the constellation is $2d$. For simplicity, we define, for $x = 0, 1, \ldots, M_1 - 1$ and $y = 0, 1, \ldots, M_2 - 1$, the parameter $\gamma_{x,y}$ as

$$\gamma_{x,y} \triangleq \frac{2d^2}{\mathcal{F}(|s_m[k]|)} = \frac{2d^2\Lambda}{\sigma_n^2 + \Omega(1 - |\rho|^2)|s_m[k]|^2} \quad (38)$$

3.2.1 Average Symbol Error Probability

Let us denote by $\mathcal{P}_{C,x,y}(\beta\Delta)$ the probability of correctly receiving $s_{m,x}[k] + js_{m,y}[k]$, conditioned on β and Δ. For $x = 1, 2 \ldots, M_1 - 2$, $y = 1, 2, \ldots, M_2 - 2$, $\mathcal{P}_{C,x,y}(\beta\Delta)$ can be expressed as

$$\mathcal{P}_{C,x,y}(\beta\Delta) = \text{Prob}(s_{m,x}[k] - d \le r_I[k] < s_{m,x}[k] + d|\beta\Delta) \times \quad (39)$$

$$\text{Prob}(s_{m,y}[k] - d \le r_Q[k] < s_{m,y}[k] + d|\beta\Delta) =$$
$$\left\{Q(\mathsf{t}_1(x,y)\sqrt{\beta\Delta}) - Q(\mathsf{t}_2(x,y)\sqrt{\beta\Delta})\right\} \times \left\{Q(\mathsf{t}_3(x,y)\sqrt{\beta\Delta}) - Q(\mathsf{t}_4(x,y)\sqrt{\beta\Delta})\right\},$$

where, $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-u^2/2)du$, and

$$\mathsf{t}_1(x,y) = (a_{m,x}[k] - 1 - a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q)\sqrt{\gamma_{x,y}} \quad (40)$$
$$\mathsf{t}_2(x,y) = (a_{m,x}[k] + 1 - a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q)\sqrt{\gamma_{x,y}} \quad (41)$$
$$\mathsf{t}_3(x,y) = (a_{m,y}[k] - 1 - a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I)\sqrt{\gamma_{x,y}} \quad (42)$$
$$\mathsf{t}_4(x,y) = (a_{m,y}[k] + 1 - a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I)\sqrt{\gamma_{x,y}}. \quad (43)$$

For ease of referencing, expressions for $\mathcal{P}_{C,x,y}(\beta\Delta)$ for other values of x and y are given in Table 1. Let us define by $\overline{\mathcal{P}}_{C,x,y} \triangleq E[\mathcal{P}_{C,x,y}(\beta\Delta)]$ the probability of correct reception of $s_{m,x}[k] + js_{m,y}[k]$, averaged over β and Δ. Notice that, each of the $\mathcal{P}_{C,x,y}(\beta\Delta)$ expressions in Table 1 can be expressed as linear combinations of $Q(a\sqrt{\beta\Delta}) \times Q(b\sqrt{\beta\Delta})$ for real values of a and b. To derive $\overline{\mathcal{P}}_{C,x,y}$, we must determine $E[Q(a\sqrt{\beta\Delta}) \times Q(b\sqrt{\beta\Delta})]$, averaged over the distributions of β and Δ. Similar to the PSK analysis of Section 3.1.1 we notice that keeping the order of integration, first over β and then over Δ, results in efficient evaluation of the expressions. To this end, we define the following functions:

$$\mathcal{H}_1(a,b,\Delta,N) \triangleq E[Q(a\sqrt{\beta\Delta})Q(b\sqrt{\beta\Delta})] \quad (44)$$
$$= \begin{cases} \mathcal{J}_1(|a|,|b|,\Delta,N) & \text{if } a \ge 0, b \ge 0 \\ \mathcal{K}_1(|a|,\Delta,N) - \mathcal{J}_1(|a|,|b|,\Delta,N) & \text{if } a \ge 0, b < 0 \\ \mathcal{K}_1(|b|,\Delta,N) - \mathcal{J}_1(|a|,|b|,\Delta,N) & \text{if } a < 0, b \ge 0 \\ 1 - \mathcal{K}_1(|a|,\Delta,N) - \mathcal{K}_1(|b|,\Delta,N) + \mathcal{J}_1(|a|,|b|,\Delta,N) & \text{if } a < 0, b < 0 \end{cases},$$
$$\mathcal{J}_1(a,b,y,N) = E[Q(a\sqrt{y\beta})Q(b\sqrt{y\beta})] \quad \text{for } a,b,y \ge 0, \quad (45)$$
$$\mathcal{K}_1(a,y,N) = E[Q(a\sqrt{y\beta})] \quad \text{for } a,y \ge 0, \quad (46)$$
$$\mathcal{H}(a,b,N,B,\psi) = E[\mathcal{H}_1(a,b,\Delta,N)] \quad \text{for } a,\ b \text{ real}, \quad (47)$$
$$\mathcal{J}(a,b,N,B,\psi) = E[\mathcal{J}_1(a,b,\Delta,N)] \quad \text{for } a,b \ge 0, \quad (48)$$
$$\mathcal{K}(a,N,B,\psi) = E[\mathcal{K}_1(a,\Delta,N)] \quad \text{for } a \ge 0, \quad (49)$$
$$\mathcal{R}_1(a,y,N) \triangleq E[Q(a\sqrt{y\beta})] = \begin{cases} \mathcal{K}_1(|a|,y,N) & \text{if } a,y \ge 0 \\ 1 - \mathcal{K}_1(|a|,y,N) & \text{if } a < 0, y \ge 0, \end{cases} \quad (50)$$
$$\text{and} \quad \mathcal{R}(a,N,B,\psi) = E[\mathcal{R}_1(a,\Delta,N)] = \begin{cases} \mathcal{K}(|a|,N,B,\psi) & \text{if } a \ge 0 \\ 1 - \mathcal{K}(|a|,N,B,\psi) & \text{if } a < 0, \end{cases} \quad (51)$$

where, in Appendix, expressions for (45), (46), (48) and (49) are given in (59), (73), (66) and (76), respectively. Using (44) and (40)-(43), each row in Table 1 can be averaged over β to arrive at $\tilde{\mathcal{P}}_{C,x,y}(\Delta)$, $x = 0, 1, \ldots, M_1 - 1$, $y = 0, 1, \ldots, M_2 - 1$. Upon further averaging $\tilde{\mathcal{P}}_{C,x,y}(\Delta)$, with the help of (47), over Δ of (20), we arrive at $\overline{\mathcal{P}}_{C,x,y}$ for FDDQ. For FDD and TDD schemes, note that $\overline{\mathcal{P}}_{C,x,y} = \tilde{\mathcal{P}}_{C,x,y}(1)$. For convenience, $\overline{\mathcal{P}}_{C,x,y}$ are tabulated in Table 2 for FDDQ. Using them, the average SEP can be written as

$$\overline{P}_{s,QAM} = \frac{1}{M}\sum_{x=0}^{M_1-1}\sum_{y=0}^{M_2-1}(1 - \overline{\mathcal{P}}_{C,x,y}) = 1 - \frac{1}{M}\sum_{x=0}^{M_1-1}\sum_{y=0}^{M_2-1}\overline{\mathcal{P}}_{C,x,y}. \quad (52)$$

With $\kappa = 1$ and $\rho = 1$ (i.e., with PCE), it is easy to show that (52) coincides with the results presented in [17].

3.2.2 Average Bit Error Probability

In this section, we present average BEP analysis of Gray coded $M_1 \times M_2$-QAM constellation. We follow the same Gray code labeling approach as considered in [27], which is briefly described as follows. We define $k_1 = \log_2(M_1)$, $k_2 = \log_2(M_2)$, and the sets $\mathcal{X} = \{0, 1, \ldots, M_1 - 1\}$ and $\mathcal{Y} = \{0, 1, \ldots, M_2 - 1\}$. The vector $(a_{k_1-1}, a_{k_1-2}, \ldots, a_0)$ is the Gray code mapping for the in-phase signal $s_{m,x}[k]$, and $(b_{k_2-1}, b_{k_2-2}, \ldots, b_0)$ is the Gray code mapping for the quadrature-phase signal $s_{m,y}[k]$. For $i = 0, \ldots, k_1 - 1$, let us define the following sets: $X_1(i) = \{x : (x \bmod 2^{i+2}) = 2^i + l, l = 0, \ldots, 2^i - 1\} \cup \{x : (x \bmod 2^{i+2}) = 2^{i+1} + l, l = 0, \ldots, 2^i - 1\}$ and $X_0(i) = \{x : (x \bmod 2^{i+2}) = l, l = 0, \ldots, 2^i - 1\} \cup \{x : (x \bmod 2^{i+2}) = 3 \times 2^i + l, l = 0, \ldots, 2^i - 1\}$. In a similar manner, for $j = 0, \ldots, k_2 - 1$, we can define the sets $Y_1(j)$ and $Y_0(j)$. Using these sets, the decision statistic for each bit a_i, $i = 0, \ldots, k_1 - 1$, is given by the disjoint union of intervals on the x-axis [27], and is expressed as

$$\widehat{a}_i = \begin{cases} 1 & \text{if } r_I[k] \in \cup_{x \in X_1(i)}\Big[-\infty.\mathbf{1}_{\{x=0\}} + (s_{m,x}[k] - d), (s_{m,x}[k] + d) + \infty.\mathbf{1}_{\{x=M_1-1\}}\Big) \\ 0 & \text{otherwise,} \end{cases} \quad (53)$$

where $\mathbf{1}_{\{x\}} = 1$ if 'x' is true and $\mathbf{1}_{\{x\}} = 0$ if 'x' is false. In a similar manner, for bit b_j, $j = 0, \ldots, k_2 - 1$, it is given by the following disjoint union of intervals on the y-axis

$$\widehat{b}_j = \begin{cases} 1 & \text{if } r_Q[k] \in \cup_{y \in Y_1(j)}\Big[-\infty.\mathbf{1}_{\{y=0\}} + (s_{m,y}[k] - d), (s_{m,y}[k] + d) + \infty.\mathbf{1}_{\{y=M_2-1\}}\Big) \\ 0 & \text{otherwise.} \end{cases} \quad (54)$$

Following the approach presented in [25], it is straightforward to show that the average BEP for bit a_j, $j = 0, \ldots, k_1 - 1$, with FDDQ is given by

$$\begin{aligned} P_b(a_j) &= \frac{1}{M} \sum_{x_0 \in X_0(j)} \sum_{x_1 \in X_1(j)} \sum_{y \in \mathcal{Y}} \\ &\Big\{ \mathcal{R}\left(\left[-\infty.\mathbf{1}_{\{x_1=0\}} + a_{m,x_1}[k] - 1 - a_{m,x_0}[k]\mu_I + a_{m,y}[k]\mu_Q \right] \sqrt{\gamma_{x_0,y}}, N, B, \psi \right) - \\ &\quad \mathcal{R}\left(\left[\infty.\mathbf{1}_{\{x_1=M_1-1\}} + a_{m,x_1}[k] + 1 - a_{m,x_0}[k]\mu_I + a_{m,y}[k]\mu_Q \right] \sqrt{\gamma_{x_0,y}}, N, B, \psi \right) \Big\} \\ &+ \frac{1}{M} \sum_{x_1 \in X_1(j)} \sum_{x_0 \in X_0(j)} \sum_{y \in \mathcal{Y}} \\ &\Big\{ \mathcal{R}\left(\left[-\infty.\mathbf{1}_{\{x_0=0\}} + a_{m,x_0}[k] - 1 - a_{m,x_1}[k]\mu_I + a_{m,y}[k]\mu_Q \right] \sqrt{\gamma_{x_1,y}}, N, B, \psi \right) - \\ &\quad \mathcal{R}\left(\left[\infty.\mathbf{1}_{\{x_0=M_1-1\}} + a_{m,x_0}[k] + 1 - a_{m,x_1}[k]\mu_I + a_{m,y}[k]\mu_Q \right] \sqrt{\gamma_{x_1,y}}, N, B, \psi \right) \Big\}, \end{aligned} \tag{55}$$

where $\mathcal{R}(a, N, B, \psi)$ is defined in (51). In (55), the first triple summation captures the probability of transmitting $s_{m,x_0}[k] + js_{m,y}[k]$ and demodulating $s_{m,x_1}[k] + js_{m,y}[k]$, whereas the second triple summation captures the probability of transmitting $s_{m,x_1}[k] + js_{m,y}[k]$ and demodulating $s_{m,x_0}[k] + js_{m,y}[k]$. Similarly, the average BEP for b_j, $j = 0, \ldots, k_2 - 1$, with FDDQ is

$$\begin{aligned} P_b(b_j) &= \frac{1}{M} \sum_{y_0 \in Y_0(j)} \sum_{y_1 \in Y_1(j)} \sum_{x \in \mathcal{X}} \\ &\Big\{ \mathcal{R}\left(\left[-\infty.\mathbf{1}_{\{y_1=0\}} + a_{m,y_1}[k] - 1 - a_{m,x}[k]\mu_Q - a_{m,y_0}[k]\mu_I \right] \sqrt{\gamma_{x,y_0}}, N, B, \psi \right) - \\ &\quad \mathcal{R}\left(\left[\infty.\mathbf{1}_{\{y_1=M_2-1\}} + a_{m,y_1}[k] + 1 - a_{m,x}[k]\mu_Q - a_{m,y_0}[k]\mu_I \right] \sqrt{\gamma_{x,y_0}}, N, B, \psi \right) \Big\} \\ &+ \frac{1}{M} \sum_{y_1 \in Y_1(j)} \sum_{y_0 \in Y_0(j)} \sum_{x \in \mathcal{X}} \\ &\Big\{ \mathcal{R}\left(\left[-\infty.\mathbf{1}_{\{y_0=0\}} + a_{m,y_0}[k] - 1 - a_{m,x}[k]\mu_Q - a_{m,y_1}[k]\mu_I \right] \sqrt{\gamma_{x,y_1}}, N, B, \psi \right) - \\ &\quad \mathcal{R}\left(\left[\infty.\mathbf{1}_{\{y_0=M_2-1\}} + a_{m,y_0}[k] + 1 - a_{m,x}[k]\mu_Q - a_{m,y_1}[k]\mu_I \right] \sqrt{\gamma_{x,y_1}}, N, B, \psi \right) \Big\}. \end{aligned} \tag{56}$$

For FDD and TDD schemes, the average BEP can be obtained by replacing $\mathcal{R}(a, N, B, \psi)$ by $\mathcal{R}_1(a, 1, N)$ of (50). Finally, the average BEP can be obtained as

$$\overline{P}_{\mathrm{b},QAM} = \frac{1}{\log_2(M)} \left\{ \sum_{j=0}^{k_1-1} P_b(a_j) + \sum_{j=0}^{k_2-1} P_b(b_j) \right\}. \tag{57}$$

4 Results and Discussion

In this section, we present numerical (obtained through the presented analysis) and simulation results quantifying the combined effects of ICE, feedback delay and channel quantization. In all the plots in this section, we employ the well-known Jakes model [19] for the time correlation of the fading process. That is, we set $\rho_d = J_0(2\pi f_d D)$, where $J_0(x)$ is the zeroth order Bessel function [23], f_d is the maximum Doppler frequency which is related to the carrier frequency f_c and the terminal velocity v as $f_d = \mathsf{v} f_c/c$, and D is the feedback delay. We set $f_c = 2$ GHz, $\mathsf{v} = 34.5$ Kmph, and $D = 0.5$ msec, so that $\rho_d = 0.99$. Although our results can equally be applied for both FDD and TDD systems, for expository purpose, we assume an FDD system with MMSE channel estimation (as detailed in Section 2.3) with pilot SNR $\gamma_p = \gamma_{p,\mathsf{F}} \in \{15, 30\}$ dB. Then, from (13), the combined correlation coefficient is $\rho = \rho_d \times \sqrt{\gamma_p/(1+\gamma_p)}$.

Fig. 2 shows the average BEP performance of Gray-coded QPSK modulation with $N = 2$ and 3 antennas, $B = 2$ bits, and $\gamma_p = 30$ dB. For comparison, we also plot the ideal performances (i.e., with PCE, no feedback delay, and unquantized feedback). The simulation results in Fig. 2 match accurately with the numerical results, thus validating the presented analytical framework. Average SEP performance of QPSK modulation with $N = 2$ and 3 antennas, and with $B = 2$ and 4 bits is presented in Fig. 3. Along with the simulation results corroborating the analysis, Fig. 3 also shows the ideal SEP curves, and the performance with PCE and delayless finite-rate feedback. Fig. 3 shows that ICE and feedback delay cause more degradation to the error performance compared to channel quantization alone. For example, with $N = 2$ antennas and $B = 2$ bits at an SNR of 20 dB, the SEP with ICE and feedback delay is an order of magnitude worse than the SEP with PCE and no feedback delay. Fig. 3 also shows that this performance gap increases when the number of antennas is increased by one, and the number of feedback bits is increased by two. With Gray-coded 16-QAM modulation, Figs. 4 and 5, respectively, show the average BEP and SEP performances. Similar to Fig. 2, in Fig. 4, we use $N = 2$ and 3 antennas with $B = 2$ bits, whereas, similar to Fig. 3, in Fig. 5, we employ $N = 2$ and 3 antennas with $B = 2$ and 4 bits. From Figs. 4 and 5, we conclude that imperfect channel feedback degrades the average SEP performance more compared to the average BEP performance. Figs. 4 and 5 also show that the performance degradation of 16-QAM with imperfect channel feedback is qualitatively similar to that of QPSK in Figs. 2 and 3.

The effects of pilot SNR and the feedback quantization bits on the average BEP performances of 8-PSK and 64-QAM are investigated in Figs. 6 and 7, respectively. In Figs. 6 and 7, we fix the number of antennas at 3, choose $B \in \{2, 4, 8\}$ and $\gamma_p \in \{15, 30\}$ dB. From Figs. 6 and 7 we observe that at high SNR, for a given combination of B and γ_p, increasing the pilot SNR is more beneficial to improving the average BEP performance than increasing the number of feedback bits. This can be explained by the fact that ICE introduces error floor at high SNR, which can only be reduced by increasing the accuracy of channel estimation quality. For example, in Fig. 6, with $(B, \gamma_p) = (2, 15)$, increasing B to 8 while keeping γ_p at 15 dB improves the BEP by a factor of two, whereas even by keeping B at 2 and increasing γ_p to 30 dB improves the BEP by a factor of four. From Fig. 7, a similar observation can be made *which emphasizes the importance of accurate channel quality estimation compared to the feedback quality improvement with more bits.* For a fixed value of $B = 8$ bits, Fig. 8 plots the average SEP of 32-QAM, $M \in (4 \times 8)$, by varying $N \in \{2, 3, 4\}$ and $\gamma_p \in \{15, 30\}$ dB. From Fig. 8, we notice that, at high SNR, for a fixed (N, γ_p) increasing the pilot SNR has a more direct effect in reducing the error floor than increasing the number of antennas. This is due to the fact that, at high SNR, error floor makes having an additional antenna less attractive from diversity perspective. Finally, by fixing γ_p at 30 dB, in Fig. 9 we study the average SEP of 8-PSK by varying $N \in \{3, 4\}$ and $B \in \{2, 4, 8\}$. As expected, Fig. 9 shows that for a given number of antennas increasing the number of feedback bits monotonically improves the error performance. However, Fig. 9 also suggests that it is more beneficial to have a high feedback bit budget for a smaller number of antennas than to have more antennas with a smaller bit budget. Although extensive set of simulations are performed to confirm the analysis, for clarity, in Figs. 6, 7, 8 and 9 we presented only the analytical results.

5 Conclusion

In this paper, we have considered transmit beamforming for MISO systems on Rayleigh fading channels with imperfect channel feedback. We characterized the feedback imperfections in terms of noisy channel estimation, feedback delay, and finite rate channel quantization. A general framework, valid for any two-dimensional constellation and FDD, FDDQ and TDD schemes, was presented to account for the feedback imperfections. We demonstrated, through numerical and simulation results, that channel estimation inaccuracy and feedback delay are more detremental to the error performance compared to the effects of finite-rate channel quantization.

Appendix

In this appendix, we derive expressions for (45), (46), (48) and (49). We begin with (48),

$$\mathcal{J}(a,b,N,B,\psi) \triangleq E\left[Q\left(a\sqrt{\beta\Delta}\right)Q\left(b\sqrt{\beta\Delta}\right)\right] = \int\limits_{y} f_{\Delta}(y)dy \int\limits_{x=0}^{\infty} Q(a\sqrt{xy})Q(b\sqrt{xy})f_{\beta}(x)dx, \quad (58)$$

where, in (58), $a, b \geq 0$, the pdf of β is given by (8), the pdf of Δ is given by (20), N is the number of transmit antennas, and B is the number of feedback bits. Let us first focus on the inner integral of (58), which can be simplified as follows: Let $a_1 = a\sqrt{y}$ and $b_1 = b\sqrt{y}$, and using integration by-parts, we have

$$\begin{aligned}
\mathcal{J}_1(a,b,y,N) &\triangleq \int\limits_{x=0}^{\infty} Q(a_1\sqrt{x})Q(b_1\sqrt{x})f_{\beta}(x)dx = \left[Q(a\sqrt{x})Q(b\sqrt{x})F_{\beta}(x)\right]_{x=0}^{\infty} + \\
&\quad \frac{b_1}{2\sqrt{2\pi}}\int\limits_{x=0}^{\infty} F_{\beta}(x)Q(a_1\sqrt{x})e^{-\frac{b_1^2 x}{2}}x^{-\frac{1}{2}}dx + \frac{a_1}{2\sqrt{2\pi}}\int\limits_{x=0}^{\infty} F_{\beta}(x)Q(b_1\sqrt{x})e^{-\frac{a_1^2 x}{2}}x^{-\frac{1}{2}}dx \\
&= \frac{b_1}{2\sqrt{2\pi}}\int\limits_{x=0}^{\infty} F_{\beta}(x)Q(a_1\sqrt{x})e^{-\frac{b_1^2 x}{2}}x^{-\frac{1}{2}}dx + \frac{a_1}{2\sqrt{2\pi}}\int\limits_{x=0}^{\infty} F_{\beta}(x)Q(b_1\sqrt{x})e^{-\frac{a_1^2 x}{2}}x^{-\frac{1}{2}}dx \\
&= \mathcal{M}(a,b,y,N) + \mathcal{M}(b,a,y,N), \qquad (59)
\end{aligned}$$

where

$$\mathcal{M}(a,b,y,N) \triangleq \frac{b_1}{2\sqrt{2\pi}}\int\limits_{x=0}^{\infty} F_{\beta}(x)Q(a_1\sqrt{x})e^{-\frac{b_1^2 x}{2}}x^{-\frac{1}{2}}dx. \quad (60)$$

Upon using $Q(x) = (1/\pi)\int\limits_{\theta=0}^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$ for $x \geq 0$ [26], and $F_{\beta}(x)$ of (9), $\mathcal{M}(a,b,y,N)$ of (60) can be simplified as

$$\begin{aligned}
\mathcal{M}(a,b,y,N) &= \frac{b_1}{2\sqrt{2\pi}}\int\limits_{x=0}^{\infty} Q(a_1\sqrt{x})e^{-\frac{b_1^2 x}{2}}x^{-\frac{1}{2}}dx - \frac{b_1}{2\sqrt{2\pi}}\sum_{j=0}^{N-1}\frac{1}{j!}\int\limits_{x=0}^{\infty} Q(a_1\sqrt{x})e^{-(\frac{b_1^2}{2}+1)x}x^{j-\frac{1}{2}}dx \\
&= \frac{b_1}{2\pi\sqrt{2\pi}}\int\limits_{0}^{\pi/2} d\theta \int\limits_{x=0}^{\infty} e^{-\left(\frac{b_1^2}{2}+\frac{a_1^2}{2\sin^2\theta}\right)x}x^{-\frac{1}{2}}dx - \frac{b_1}{2\pi\sqrt{2\pi}}\sum_{j=0}^{N-1}\frac{1}{j!}\int\limits_{0}^{\pi/2} d\theta \int\limits_{x=0}^{\infty} e^{-\left(\frac{b_1^2}{2}+\frac{a_1^2}{2\sin^2\theta}+1\right)x}x^{j-\frac{1}{2}}dx \\
&= \frac{1}{2\pi}\int\limits_{0}^{\pi/2} d\theta\sqrt{\frac{b_1^2\sin^2\theta}{b_1^2\sin^2\theta + a_1^2}} - \sum_{j=0}^{N-1}\frac{1}{j!}\frac{\Gamma\left(j+\frac{1}{2}\right)2^{j+\frac{1}{2}}}{2\pi\sqrt{2\pi}}\int\limits_{0}^{\pi/2} d\theta b_1\left(\frac{\sin^2\theta}{b_1^2\sin^2\theta + 2\sin^2\theta + a_1^2}\right)^{j+\frac{1}{2}}
\end{aligned}$$

$$= \frac{1}{2\pi} \int_0^{\pi/2} d\theta \sqrt{\frac{b^2 \sin^2\theta}{b^2\sin^2\theta + a^2}} - \sum_{j=0}^{N-1} \frac{1}{j!} \frac{b\Gamma\left(j+\frac{1}{2}\right) 2^{j+\frac{1}{2}}}{2\pi\sqrt{2\pi}} \times$$
$$\int_0^{\pi/2} d\theta \left(\frac{\sin^2\theta}{a^2 + b^2\sin^2\theta}\right)^{j+\frac{1}{2}} \sqrt{y} \left[y + \frac{2\sin^2\theta}{a^2+b^2\sin^2\theta}\right]^{-\left(j+\frac{1}{2}\right)}. \quad (61)$$

Upon using (59), (60) and (61) in (58), we have

$$\mathcal{J}(a,b,N,B,\psi) = \int_y f_\Delta(y)\mathcal{M}(a,b,y,N)dy + \int_y f_\Delta(y)\mathcal{M}(b,a,y,N)dy. \quad (62)$$

In the absence of feedback quantization (i.e., as $B \to \infty$ and $\psi \to 0$), recall that we have $f_\Delta(y) = \delta(y-1)$. Then, (62) reduces to

$$\mathcal{J}(a,b,N,\infty,0) = \mathcal{M}(a,b,1,N) + \mathcal{M}(b,a,1,N), \quad (63)$$

which is also equal to $\mathcal{J}_1(a,b,1,N)$ of (45). For finite B, using (20) for the pdf of Δ, we have

$$\mathcal{J}(a,b,N,B,\psi) = 2^B(N-1) \int_{1-\psi}^{1} (1-y)^{N-2} \left\{\mathcal{M}(a,b,y,N) + \mathcal{M}(b,a,y,N)\right\} dy. \quad (64)$$

To further simplify (64), let us define

$$\mathcal{M}_1(a,b,\psi,N) \triangleq \int_{1-\psi}^{1} (1-y)^{N-2}\mathcal{M}(a,b,y,N)dy \quad (65)$$

so that (64) is

$$\mathcal{J}(a,b,N,B,\psi) = 2^B(N-1)\left\{\mathcal{M}_1(a,b,\psi,N) + \mathcal{M}_1(b,a,\psi,N)\right\}. \quad (66)$$

Using (61) in (65), we obtain

$$\mathcal{M}_1(a,b,\psi,N) = \frac{1}{2\pi}\int_{1-\psi}^{1}(1-y)^{N-2}\int_0^{\pi/2}\sqrt{\frac{b^2\sin^2\theta}{b^2\sin^2\theta+a^2}}dyd\theta - \sum_{j=0}^{N-1}\frac{1}{j!}\frac{b\Gamma\left(j+\frac{1}{2}\right)2^{j+\frac{1}{2}}}{2\pi\sqrt{2\pi}} \times$$
$$\int_0^{\pi/2} d\theta\left(\frac{\sin^2\theta}{a^2\sin^2\theta+b^2}\right)^{j+\frac{1}{2}} \int_{1-\psi}^{1} dy(1-y)^{N-2}\sqrt{y}\left[y+\frac{2\sin^2\theta}{a^2\sin^2\theta+b^2}\right]^{-\left(j+\frac{1}{2}\right)} \quad (67)$$

Owing to the fact that $\int_{1-\psi}^{1} f_\Delta(y)dy = 1$, the first double integral of (67) reduces to

$$\int_{1-\psi}^{1}(1-y)^{N-2}\int_0^{\pi/2}\sqrt{\frac{b^2\sin^2\theta}{b^2\sin^2\theta+a^2}}dyd\theta = \frac{1}{2^B(N-1)}\int_0^{\pi/2}\sqrt{\frac{b^2\sin^2\theta}{b^2\sin^2\theta+a^2}}d\theta$$
$$= \frac{1}{2^B(N-1)}\frac{ab}{(a^2+b^2)}\,{}_2F_1\left(1,1;\frac{3}{2};\frac{b^2}{a^2+b^2}\right), \quad (68)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the hypergeometric function [23], and we have used [26, Eqn. (5.17)] to simplify the integral in (68). On the other hand, to simplify the inner integral of (67) let us consider the following result [28] for positive integer values of m:

$$\mathcal{D}_1(\psi,\alpha,m,n) \triangleq \int_{1-\psi}^{1} \sqrt{y}(1-y)^m(y+\alpha)^{-n}dy = \sum_{l=0}^{m}(-1)^l\binom{m}{l}\int_{1-\psi}^{1} y^{l+\frac{1}{2}}(y+\alpha)^{-n}dy$$
$$= \sum_{l=0}^{m}\frac{(-1)^l\binom{m}{l}\alpha^{-n}}{(l+\frac{3}{2})}\Big\{ {}_2F_1\left(l+\frac{3}{2},n;l+\frac{5}{2};\frac{-1}{\alpha}\right) - (1-\psi)^{l+\frac{3}{2}}\ {}_2F_1\left(l+\frac{3}{2},n;l+\frac{5}{2};\frac{\psi-1}{\alpha}\right)\Big\}. \quad (69)$$

Using (69), the inner integral of (67) simplifies to

$$\int_{1-\psi}^{1} dy(1-y)^{N-2}\sqrt{y}\left[y+\frac{2\sin^2\theta}{a^2\sin^2\theta+b^2}\right]^{-\left(j+\frac{1}{2}\right)} = \mathcal{D}_1\left(\psi,\frac{2\sin^2\theta}{a^2\sin^2\theta+b^2},N-2,j+\frac{1}{2}\right). \quad (70)$$

Using (68) and (70), (67) simplifies to

$$\begin{aligned}\mathcal{M}_1(a,b,\psi,N) \;=\; & \frac{1}{\pi 2^{B+1}(N-1)}\frac{ab}{(a^2+b^2)}\,{}_2F_1\left(1,1;\frac{3}{2};\frac{b^2}{a^2+b^2}\right) - \sum_{j=0}^{N-1}\frac{1}{j!}\frac{b\Gamma\left(j+\frac{1}{2}\right)2^{j+\frac{1}{2}}}{2\pi\sqrt{2\pi}} \times \\ & \int_0^{\pi/2} d\theta\left(\frac{\sin^2\theta}{a^2\sin^2\theta+b^2}\right)^{j+\frac{1}{2}} \times \mathcal{D}_1\left(\psi,\frac{2\sin^2\theta}{a^2\sin^2\theta+b^2},N-2,j+\frac{1}{2}\right). \quad (71)\end{aligned}$$

Finally, using (71) in (66), we arrive at a single-integral based formula for $\mathcal{J}(a,b,N,B,\psi)$.

To simplify (46) and (49), now consider the following random variable

$$\mathcal{G}(\Delta,\phi,b,N) = \int_{\theta=0}^{\phi}\left(\frac{\sin^2\theta}{\sin^2\theta+b^2\Delta}\right)^N d\theta \quad (72)$$

with the pdf of Δ as defined in (20). Note that the function $\mathcal{K}_1(a,y,N)$ of (46) for $a,y\geq 0$ can be expressed in terms of (72) as

$$\mathcal{K}_1(a,y,N) = E[Q(a\sqrt{y\beta})] = \frac{1}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{\sin^2\theta}{\sin^2\theta+ya^2/2}\right)^N d\theta = \frac{1}{\pi}\mathcal{G}\left(y,\frac{\pi}{2},\frac{a}{\sqrt{2}},N\right). \quad (73)$$

The average of (72) over Δ can be performed as follows

$$\begin{aligned}\overline{\mathcal{G}}(\phi,b,N,B,\psi) \;\triangleq\; & E\left[\mathcal{G}(\Delta,\phi,b,N)\right] = 2^B(N-1)\int_{y=1-\psi}^{1}(1-y)^{N-2}\mathcal{G}(y,\phi,b,N)dy \\ =\; & 2^B(N-1)\int_{\theta=0}^{\phi}\left(\frac{\sin^2\theta}{b^2}\right)^N\int_{y=1-\psi}^{1}(1-y)^{N-2}\left(y+\frac{\sin^2\theta}{b^2}\right)^{-N}dyd\theta\end{aligned}$$

$$= \quad 2^B(N-1)\int\limits_{\theta=0}^{\phi}\left(\frac{\sin^2\theta}{b^2}\right)^N \mathcal{D}_2\left(\psi,\frac{\sin^2\theta}{b^2},N-2,N\right)d\theta, \tag{74}$$

$$\text{where } \mathcal{D}_2(\psi,\alpha,m,n) \triangleq \int\limits_{1-\psi}^{1}(1-y)^m(y+\alpha)^{-n}dy$$

$$= \frac{-\alpha^n}{1+m}\,{}_2F_1\left(1,n;2+m;-\frac{1}{\alpha}\right)-\frac{\alpha^{1-n}(1+\alpha)^m}{n-1}\,{}_2F_1\left(1-n,-m;2-n;\frac{\alpha}{\alpha+1}\right)$$
$$+\frac{(1+\alpha)^m(1+\alpha-\psi)^{1-n}}{n-1}\,{}_2F_1\left(1-n,-m;2-n;\frac{1+\alpha-\psi}{\alpha+1}\right), \tag{75}$$

and the simplification is due to [28]. As a result, we have, for $a>0$,

$$\mathcal{K}(a,N,B,\psi)=E\left[\mathcal{K}_1(a,\Delta,N)\right]=\frac{1}{\pi}E\left[\mathcal{G}\left(\Delta,\frac{\pi}{2},\frac{a}{\sqrt{2}},N\right)\right]=\frac{1}{\pi}\overline{\mathcal{G}}\left(\frac{\pi}{2},\frac{a}{\sqrt{2}},N,B,\psi\right) \tag{76}$$

which is (49).

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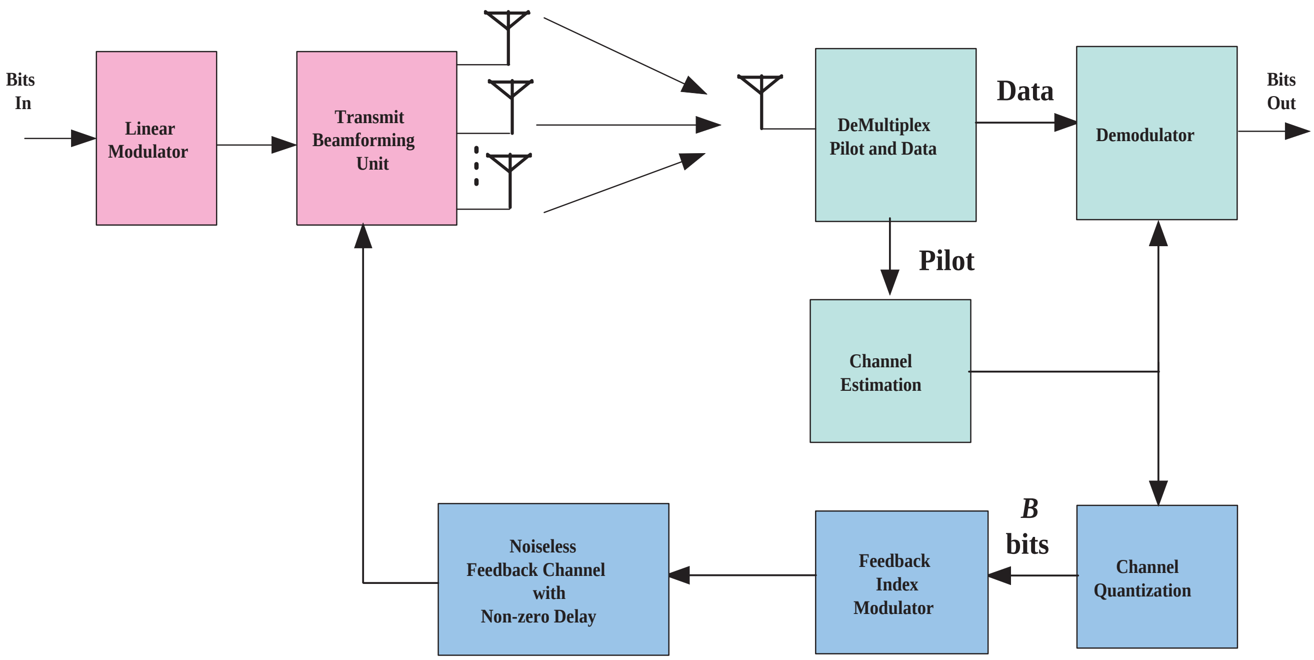

Figure 1: Block Diagram of a transmit beamforming MISO system with imperfect feedback. Feedback imperfections include, inaccurate channel estimation, channel quantization and feedback delay.

x	y	$P_{C,x,y}(\beta\Delta) = \text{Prob}\left(s_{m,x}[k] + js_{m,y}[k] \text{ received successfully} \mid \beta\Delta\right)$
$\{1,\ldots,M_1-2\}$	$\{1,\ldots,M_2-2\}$	$\text{Prob}(s_{m,x}[k]-d \le r_I[k] < s_{m,x}[k]+d \mid \beta\Delta) \times \text{Prob}(s_{m,y}[k]-d \le r_Q[k] < s_{m,y}[k]+d \mid \beta\Delta) =$ $\left\{Q\left([a_{m,x}[k]-1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) - Q\left([a_{m,x}[k]+1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right)\right\} \times$ $\left\{Q\left([a_{m,y}[k]-1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right) - Q\left([a_{m,y}[k]+1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right)\right\}$
$\{1,\ldots,M_1-2\}$	$\{0\}$	$\text{Prob}(s_{m,x}[k]-d \le r_I[k] < s_{m,x}[k]+d \mid \beta\Delta) \times \text{Prob}(-\infty < r_Q[k] < s_{m,y}[k]+d \mid \beta\Delta) =$ $Q\left(-[a_{m,y}[k]+1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right) \times$ $\left\{Q\left([a_{m,x}[k]-1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) - Q\left([a_{m,x}[k]+1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right)\right\}$
$\{1,\ldots,M_1-2\}$	$\{M_2-1\}$	$\text{Prob}(s_{m,x}[k]-d \le r_I[k] < s_{m,x}[k]+d \mid \beta\Delta) \times \text{Prob}(s_{m,y}[k]-d \le r_Q[k] < \infty \mid \beta\Delta) =$ $Q\left([a_{m,y}[k]-1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right) \times$ $\left\{Q\left([a_{m,x}[k]-1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) - Q\left([a_{m,x}[k]+1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right)\right\}$
$\{0\}$	$\{1,\ldots,M_2-2\}$	$\text{Prob}(-\infty < r_I[k] < s_{m,x}[k]+d \mid \beta\Delta) \times \text{Prob}(s_{m,y}[k]-d \le r_Q[k] < s_{m,y}[k]+d \mid \beta\Delta) =$ $Q\left(-[a_{m,x}[k]+1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) \times$ $\left\{Q\left([a_{m,y}[k]-1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right) - Q\left([a_{m,y}[k]+1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right)\right\}$
$\{M_1-1\}$	$\{1,\ldots,M_2-2\}$	$\text{Prob}(s_{m,x}[k]-d \le r_I[k] < \infty \mid \beta\Delta) \times \text{Prob}(s_{m,y}[k]-d \le r_Q[k] < s_{m,y}[k]+d \mid \beta\Delta) =$ $Q\left([a_{m,x}[k]-1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) \times$ $\left\{Q\left([a_{m,y}[k]-1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right) - Q\left([a_{m,y}[k]+1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right)\right\}$
$\{0\}$	$\{0\}$	$\text{Prob}(-\infty < r_I[k] < s_{m,x}[k]+d \mid \beta\Delta) \times \text{Prob}(-\infty < r_Q[k] < s_{m,y}[k]+d \mid \beta\Delta) =$ $Q\left(-[a_{m,x}[k]+1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) Q\left(-[a_{m,y}[k]+1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right)$
$\{M_1-1\}$	$\{0\}$	$\text{Prob}(s_{m,x}[k]-d \le r_I[k] < \infty \mid \beta\Delta) \times \text{Prob}(-\infty < r_Q[k] < s_{m,y}[k]+d \mid \beta\Delta) =$ $Q\left([a_{m,x}[k]-1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) Q\left(-[a_{m,y}[k]+1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right)$
$\{0\}$	$\{M_2-1\}$	$\text{Prob}(-\infty < r_I[k] < s_{m,x}[k]+d \mid \beta\Delta) \times \text{Prob}(s_{m,y}[k]-d \le r_Q[k] < \infty \mid \beta\Delta) =$ $Q\left(-[a_{m,x}[k]+1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) Q\left([a_{m,y}[k]-1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right)$
$\{M_1-1\}$	$\{M_2-1\}$	$\text{Prob}(s_{m,x}[k]-d \le r_I[k] < \infty \mid \beta\Delta) \times \text{Prob}(s_{m,y}[k]-d \le r_Q[k] < \infty \mid \beta\Delta) =$ $Q\left([a_{m,x}[k]-1-a_{m,x}[k]\mu_I + a_{m,y}[k]\mu_Q]\sqrt{\gamma_{x,y}\beta\Delta}\right) Q\left([a_{m,y}[k]-1-a_{m,x}[k]\mu_Q - a_{m,y}[k]\mu_I]\sqrt{\gamma_{x,y}\beta\Delta}\right)$

Table 1: For each $x \in \{0,1,\ldots,M_1-1\}$ and $y \in \{0,1,\ldots,M_2-1\}$, conditioned on β and Δ, the probability of correct reception of the symbol $s_m[k] = s_{m,x}[k] + js_{m,y}[k]$ is tabulated on the third column for an $M_1 \times M_2$ rectangular QAM constellation.

x	y	$\overline{\mathcal{P}}_{C,x,y} = E\left[\mathcal{P}_{C,x,y}(\beta\Delta)\right]$
$\{1,2,\ldots,M_1-2\}$	$\{1,2,\ldots,M_2-2\}$	$\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)-\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)-$ $\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)+\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)$
$\{1,2,\ldots,M_1-2\}$	$\{0\}$	$\mathcal{R}\left(\mathsf{t}_1(x,y),N,B,\psi\right)-\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)-$ $\mathcal{R}\left(\mathsf{t}_2(x,y),N,B,\psi\right)+\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)$
$\{1,2,\ldots,M_1-2\}$	$\{M_2-1\}$	$\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)-\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)$
$\{0\}$	$\{1,2,\ldots,M_2-2\}$	$\mathcal{R}\left(\mathsf{t}_3(x,y),N,B,\psi\right)-\mathcal{R}\left(\mathsf{t}_4(x,y),N,B,\psi\right)-$ $\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)+\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)$
$\{M_1-1\}$	$\{1,2,\ldots,M_2-2\}$	$\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)-\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)$
$\{0\}$	$\{0\}$	$1-\mathcal{R}\left(\mathsf{t}_4(x,y),N,B,\psi\right)-$ $\mathcal{R}\left(\mathsf{t}_2(x,y),N,B,\psi\right)+\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)$
$\{M_1-1\}$	$\{0\}$	$\mathcal{R}\left(\mathsf{t}_1(x,y),N,B,\psi\right)-\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_4(x,y),N,B,\psi\right)$
$\{0\}$	$\{M_2-1\}$	$\mathcal{R}\left(\mathsf{t}_3(x,y),N,B,\psi\right)-\mathcal{H}\left(\mathsf{t}_2(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)$
$\{M_1-1\}$	$\{M_2-1\}$	$\mathcal{H}\left(\mathsf{t}_1(x,y),\mathsf{t}_3(x,y),N,B,\psi\right)$

Table 2: For each $x \in \{0,1,\ldots,M_1-1\}$ and $y \in \{0,1,\ldots,M_2-1\}$ the average probability of correct reception of the symbol $s_m[k] = s_{m,x}[k] + js_{m,y}[k]$ is tabulated on the third column for an $M_1 \times M_2$ rectangular QAM constellation with FDDQ scheme. The functions $\mathsf{t}_1(x,y), \mathsf{t}_2(x,y), \mathsf{t}_3(x,y), \mathsf{t}_4(x,y)$ are defined in (40)-(43), respectively. The function $\mathcal{H}(a,b,N,B,\psi)$ is defined in (47), whereas the function $\mathcal{R}(a,N,B,\psi)$ is defined in (51). Expressions for $\overline{\mathcal{P}}_{C,x,y}$ for FDD and TDD schemes can easily be obtained by replacing $\mathcal{H}(a,b,N,B,\psi)$ by $\mathcal{H}_1(a,b,1,N)$ of (44) and $\mathcal{R}(a,N,B,\psi)$ by $\mathcal{R}_1(a,1,N)$ of (50).

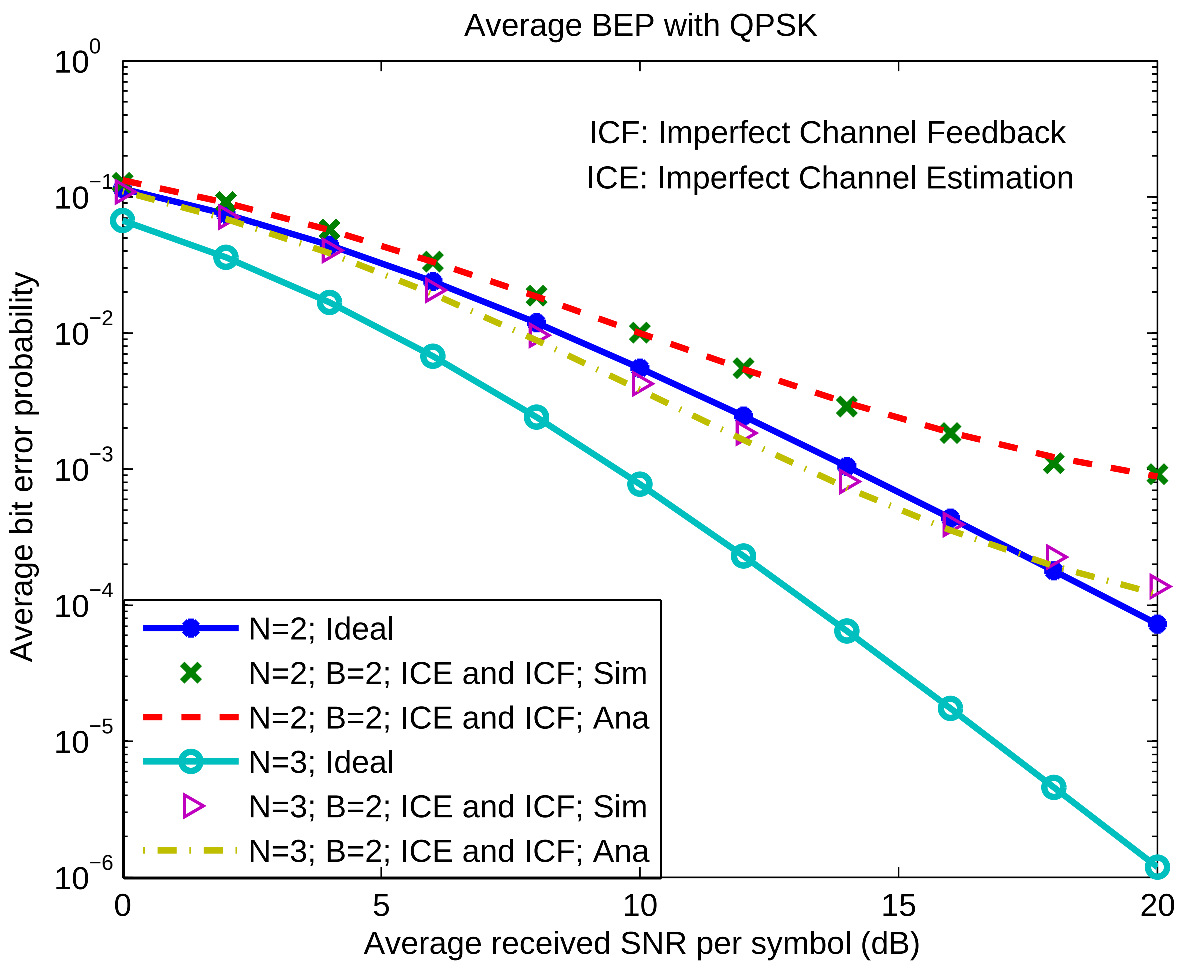

Figure 2: Average BEP performance of QPSK modulation with imperfect channel estimation, feedback delay, and feedback channel quantization. Here, we assume $N = 2$ and 3 antennas, with $B = 2$ feedback bits, $\rho_d = 0.99$ and average received SNR of the pilot channel $\gamma_p = 30$ dB. Both analytical as well as simulation results are shown.

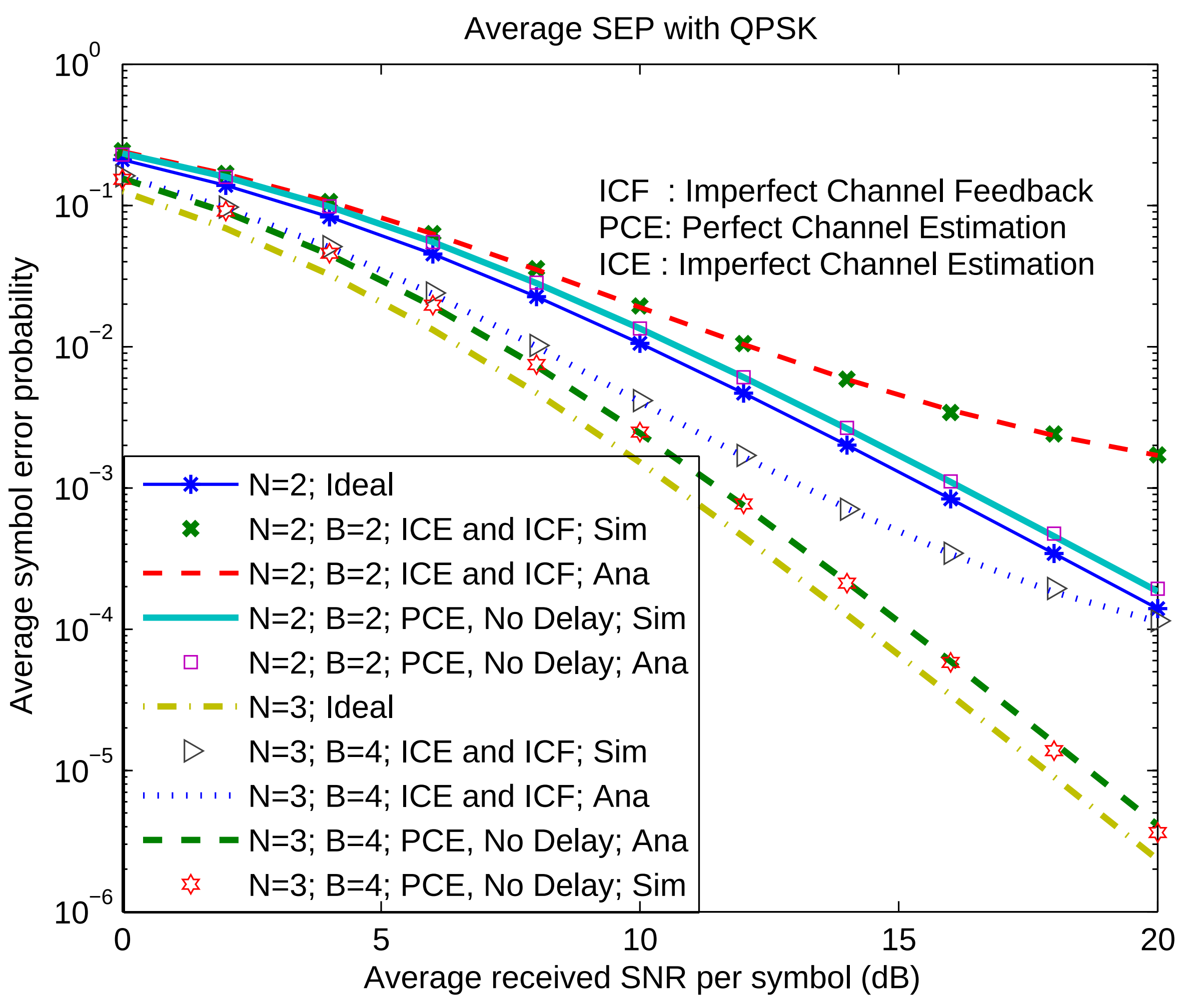

Figure 3: Average SEP performance of QPSK modulation with imperfect channel estimation, feedback delay, and feedback channel quantization. Here, we assume $N = 2$ and 3 antennas, with $B \in \{2, 4\}$ feedback bits, $\rho_d = 0.99$ and average received SNR of the pilot channel $\gamma_p = 30$ dB. For comparison, SEP performance with channel quantization alone is also plotted. For all the cases, both analytical as well as simulation results are shown.

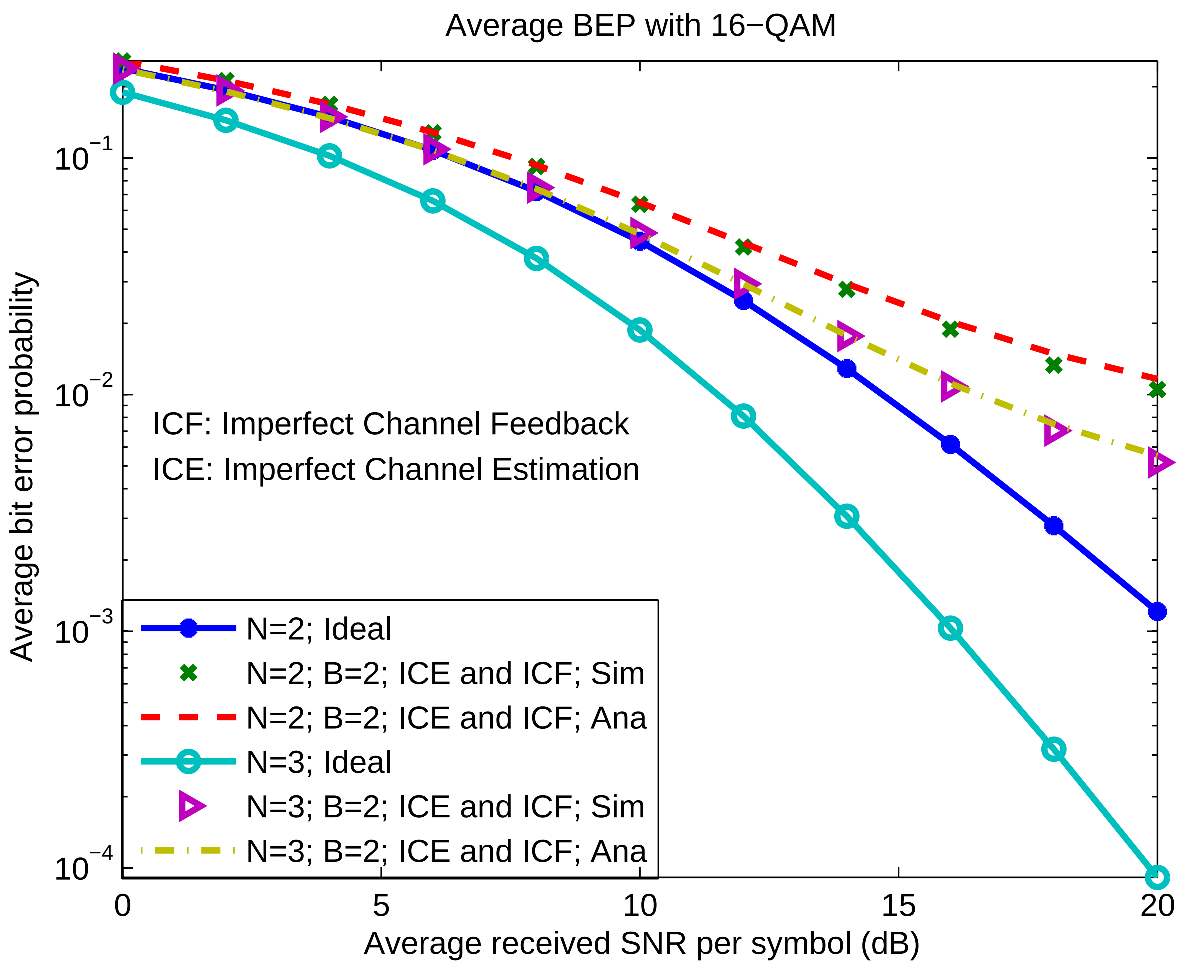

Figure 4: Average BEP performance of Gray coded 16-QAM modulation with imperfect channel estimation, feedback delay, and channel quantization. Here, we assume $N = 2$ and 3 antennas, with $B = 2$ feedback bits, $\rho_d = 0.99$ and average received SNR of the pilot channel $\gamma_p = 30$ dB. Both analytical as well as simulation results are shown.

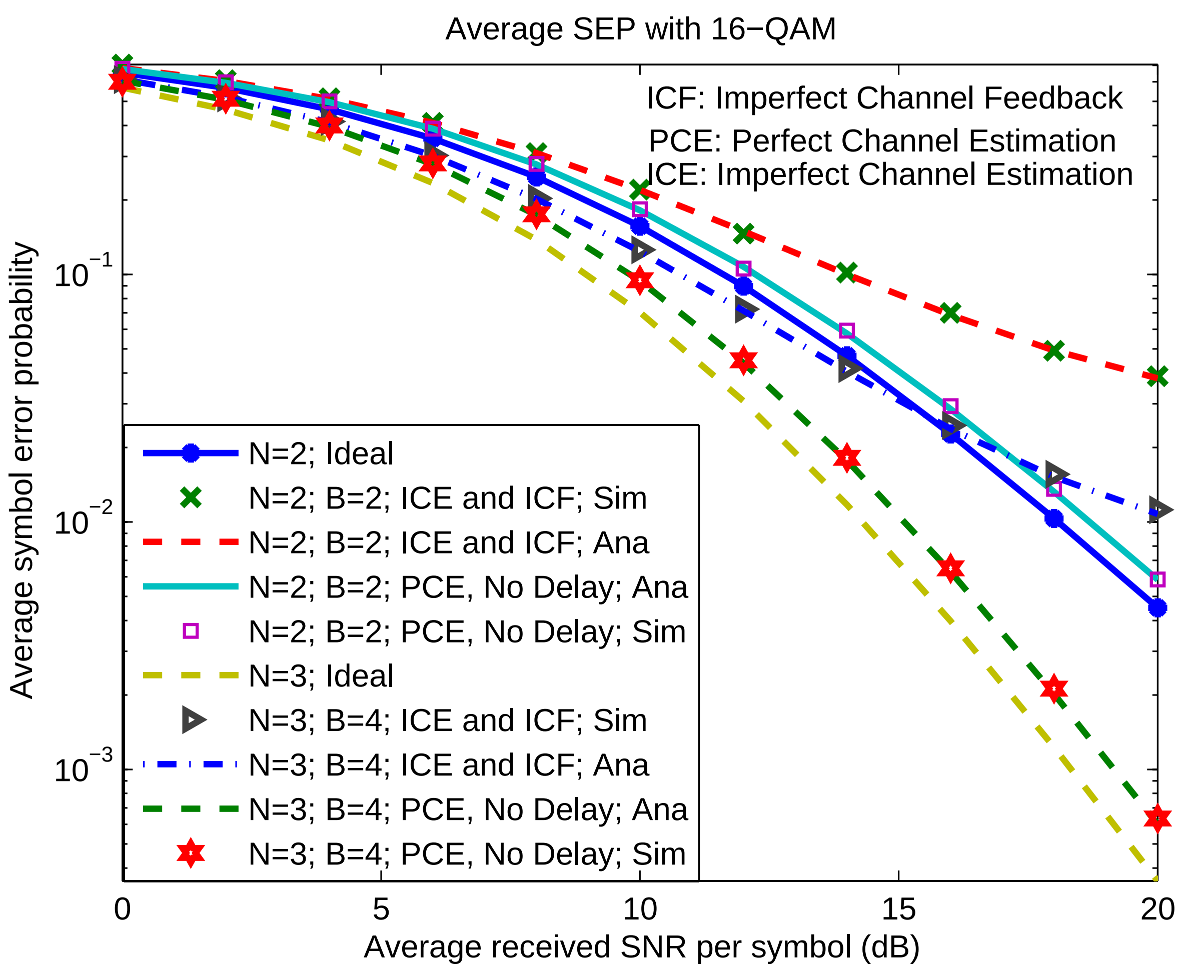

Figure 5: Average SEP performance of 16-QAM modulation with imperfect channel estimation, feedback delay, and channel quantization. Here, we assume $N = 2$ and 3 antennas, with $B \in \{2, 4\}$ feedback bits, $\rho_d = 0.99$ and average received SNR of the pilot channel $\gamma_p = 30$ dB. For comparison, SEP performance with channel quantization alone is also plotted. For all the cases, both analytical as well as simulation results are shown.

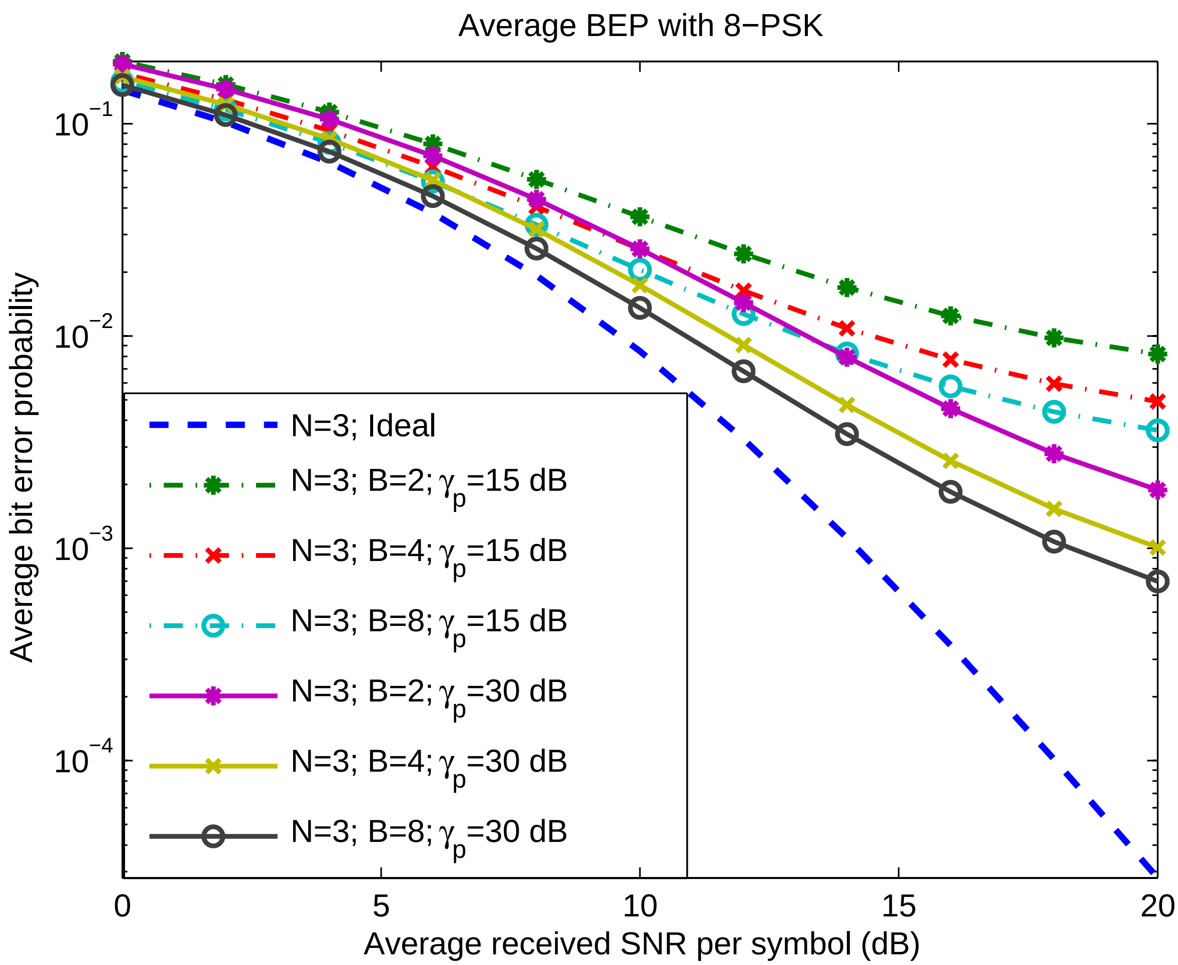

Figure 6: Average BEP performance of Gray coded 8-PSK modulation as a function of the number of feedback bits $B \in \{2, 4, 8\}$ and the average pilot SNR $\gamma_p \in \{15, 30\}$ dB, for a fixed feedback delay with $\rho_d = 0.99$. Here, we set $N = 3$ antennas.

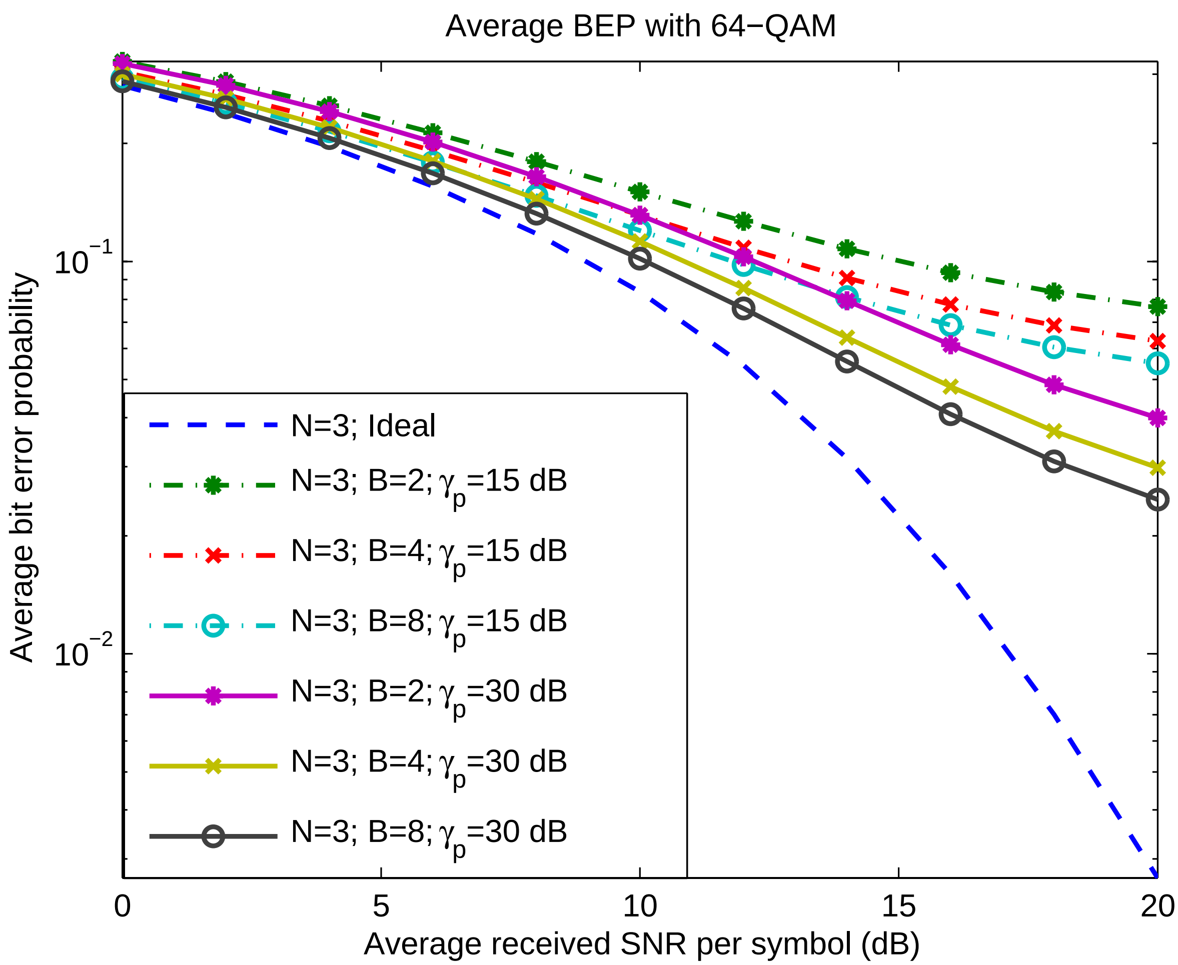

Figure 7: Average BEP performance of Gray coded 64-QAM modulation as a function of the number of feedback bits $B \in \{2, 4, 8\}$ and the average pilot SNR $\gamma_p \in \{15, 30\}$ dB, for a fixed feedback delay with $\rho_d = 0.99$. Here, we set $N = 3$ antennas.

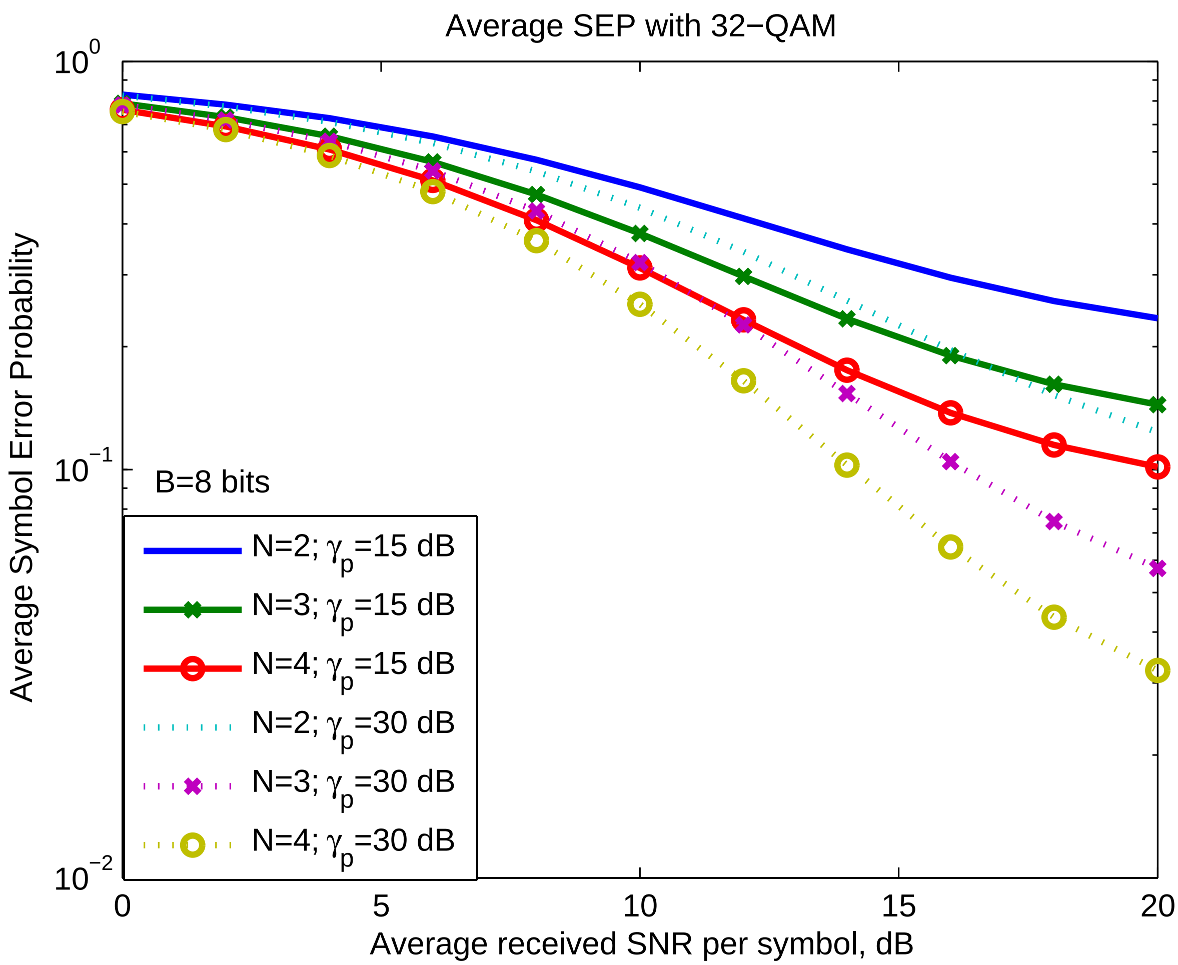

Figure 8: Average SEP performance of 32-QAM modulation as a function of the number of antennas $N \in \{2, 3, 4\}$ and the average pilot SNR $\gamma_p \in \{15, 30\}$ dB, for a fixed feedback delay with $\rho_d = 0.99$. Here, we set $B = 8$ bits.

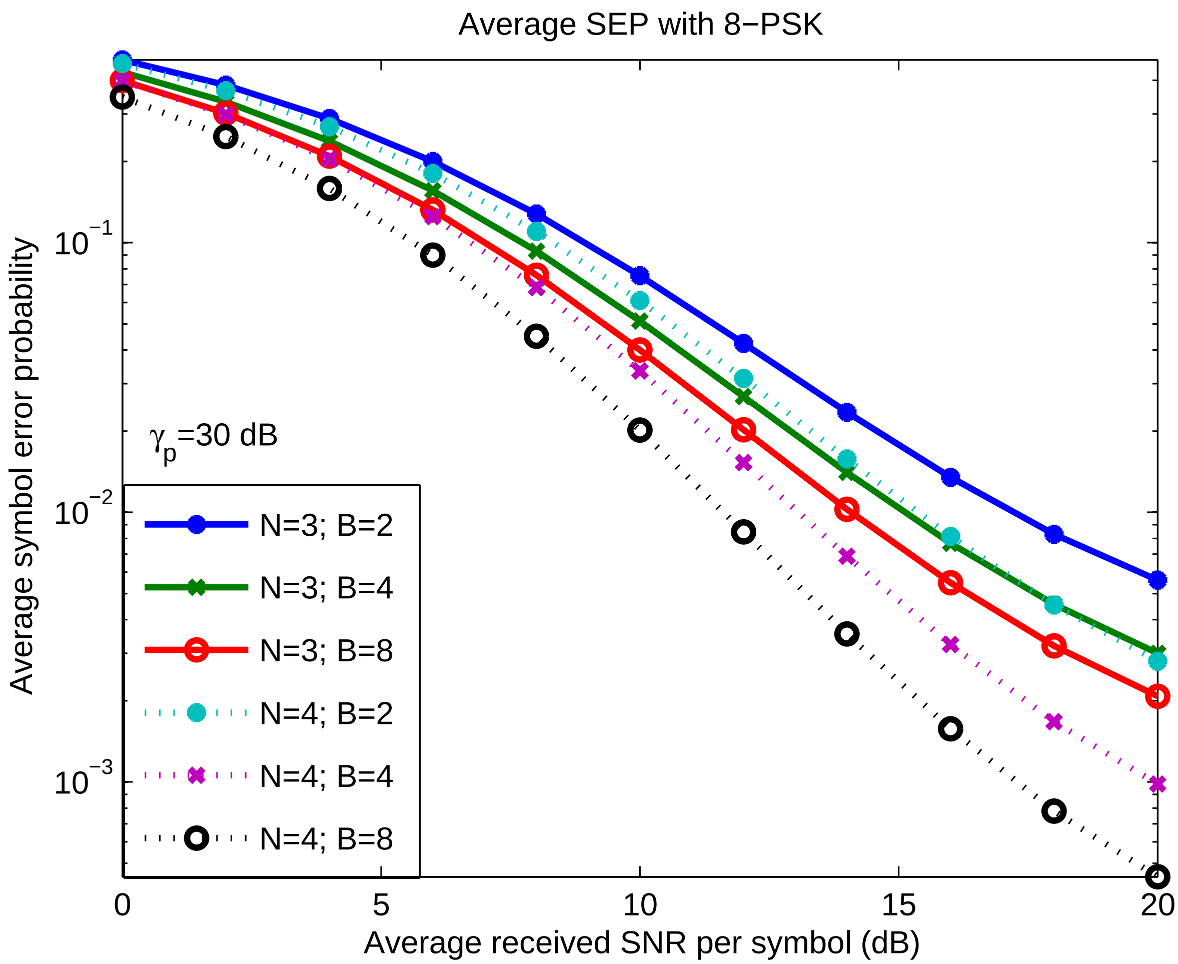

Figure 9: Average SEP performance of 8-PSK modulation as a function of the number of antennas $N \in \{3, 4\}$ and the number of feedback bits $B \in \{2, 4, 8\}$. Here, we set $\gamma_p = 30$ dB.