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Enclosure 1

REGULARIZED CHANNEL DISTRIBUTION INVERSION (RCDI) AND PARAMETERIZATION IN THE MIMO BROADCAST CHANNEL

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ABSTRACT

Linear precoding (beamforming) techniques exist that maximize the sum rate in multi-antenna broadcast channels. Such algorithms require accurate channel state information (CSI) at the transmitter and are performance sensitive to erroneous or outdated CSI. The current work focuses on adapting these algorithms to work on the statistics of the channel, rather than channel state, in order to provide more stable performance. This stable beamforming method is shown to require the full spatial correlation matrix thus suffering from high complexity in the feedback channel. Parameterization of the spatial correlation using popular channel models is also considered to make the proposed beamforming practical.

Index Terms— MIMO systems, Broadcast channels, Array signal processing, Time-varying channels

I. INTRODUCTION

The multi-user multiple-input multiple-output (MIMO) wireless broadcast channel sum capacity is achieved using nonlinear dirty-paper coding (DPC) based on accurate channel state information (CSI) at the transmitter (CSIT) and receiver (CSIR). A good overview of DPC and multi-user MIMO capacity limits can be found in [1]. A linear precoding technique that maximizes the sum rate in a multiple-input single-output (MISO) broadcast channel was shown in [2] and extended for use in the MIMO channel in [3]. This rate-maximizing beamforming algorithm is referred to in this work as the standard regularized channel inversion (RCI) beamformer. We have shown [4] that both DPC and RCI are sensitive to delayed channel updates which result in sum rate loss. We further showed in [4] a heuristic beamformer that uses channel distribution information (CDI), in the form of spatial correlation matrices, in order to provide stable performance in the MIMO broadcast channel. Unlike DPC or RCI, this algorithm is able to use either CSI or CDI equally depending on the available information at the transmitter. In this current work we adapt the RCI algorithm from [2] for use with either CSI or CDI at the transmitter (CDIT) or receiver (CDIR) in the MIMO broadcast channel.

As will be shown, to develop this beamforming algorithm that is stable in the time-varying channel will require a form of the full spatial correlation matrix. For the simplified scenario of N transmit and receive antennas, the number of matrix elements grows in size as N^4 versus only N^2 for the channel transfer matrix required for CSI-based schemes. Consequently, it becomes imperative to develop a method of parsing the amount of feedback in such a way that information throughput is preserved. This is accomplished in this paper by using channel models in which the correlation matrix can be parameterized resulting in more manageable amounts of feedback. The models under consideration are the classic Kronecker model [5] as well as the more recent Weichselberger model [6].

II. SYSTEM MODEL

The received signal for user j in the N_u user broadcast channel can be written as

$$\mathbf{y}_j = \mathbf{H}_j \mathbf{b}_j x_j + \sum_{i \neq j}^{N_u} \mathbf{H}_j \mathbf{b}_i x_i + \boldsymbol{\eta}_j \tag{1}$$

where $\mathbf{H}_j$ is the $N_r \times N_t$ channel transfer matrix, $\mathbf{b}_j$ is the $N_t \times 1$ transmit beamformer, x_j is a unit variance Gaussian random variable representing the input signal, and $\boldsymbol{\eta}_j$ is additive white Gaussian noise (AWGN). Additionally, a beamforming algorithm will generally produce a receive beamforming vector $\mathbf{w}_j$ used for decoding purposes. Power is constrained such that $\sum_j \mathbf{b}_j^H \mathbf{b}_j = P$ where $\{\cdot\}^H$ is the matrix conjugate transpose. Given transmit and receive beamforming vectors, the sum rate of the broadcast channel with linear processing can be written as

$$C = \sum_{j=1}^{K} \log\left(1 + \rho_j\right) \tag{2}$$

where

$$\rho_j = \frac{|\mathbf{w}_j^H \mathbf{H}_j \mathbf{b}_j|^2}{1 + \sum_{i \neq j} |\mathbf{w}_j^H \mathbf{H}_j \mathbf{b}_i|^2} \tag{3}$$

assuming unit variance noise, normalized received beamformers, and phase synchronization on the output signal. The sum rate in Eq. (2) is maximized using the RCI algorithm from [2], [3] with perfect CSI at the transmitter and receiver.

In [4] an approximation on the *average* sum rate was used for maximization in order to provide stable performance

$$\bar{C} = \sum_{j=1}^{K} \log\left(1 + \frac{\bar{n}_j}{\bar{d}_j}\right) \quad (4)$$

where $\bar{n}_j = E[\text{num}(\rho_j)]$, $\bar{d}_j = E[\text{den}(\rho_j)]$, and $\text{num}(\cdot)$ and $\text{den}(\cdot)$ return the numerator and denominator of the argument, respectively.

III. CHANNEL MODEL AND ESTIMATION ERROR

Modeling spatial correlation in the multi-user MIMO broadcast channel is facilitated by measurements taken by Brigham Young University [7]. For broadcast channel measurements, the receiver testbed is placed at a specific location in a given environment while the channel is sampled as the receiver traverses a fixed path. The receiver measurement equipment is then moved to a another location and the measurement process is repeated. In this manner, realizations of the multiple user, multiple antenna, time-varying channel can be created. Post-processing on the dataset is then performed to estimate the full spatial correlation matrix for the j^{th} user

$$\mathbf{R}_j = \frac{1}{M}\sum_{m=1}^{M} \text{vec}\left(\mathbf{H}_j(m)\right)\text{vec}\left(\mathbf{H}_j(m)\right)^H \quad (5)$$

where the integer index m represents samples into the measured data.

Once $\mathbf{R}_j$ has been estimated for each user, channels can be realized using a random matrix model with the full correlation matrix

$$\mathbf{H}_j = \text{mat}\left\{\sqrt{\mathbf{R}_j}\text{vec}(\mathbf{H}_w)\right\} \quad (6)$$

where $\mathbf{H}_w$ is an $N_r \times N_t$ matrix with unit-variance complex Gaussian entries, $\text{vec}(\cdot)$ is the matrix column stacking operator while $\text{mat}(\cdot)$ is its inverse (e.g. $\text{mat}(\text{vec}(\mathbf{A})) = \mathbf{A}$). This full correlation model (6) coupled with statistically measured samples from (5) allow for the use of realistic correlation values while also providing a simplified method of analysis. However, with such analysis, the full correlation model can be prohibitively complex ($N_t^2 N_r^2$ complex values) for feedback in a practical system motivating the use of parameterized models as shown in this work. Regardless of the complexity, the full correlation is used as a universal benchmark for algorithm performance and also a general framework for channel generation.

In addition to spatial correlation inherent in wireless systems, due to mobility, estimation errors from limited training, and/or quantization effects, the channel samples used at the transmitter and receiver will never reflect perfectly the current channel that the signal propagates through. In order to model this behavior of inaccurate CSI, channel estimates will be corrupted by [3]

$$\hat{\mathbf{H}}_j = \mathbf{H}_j + \mathbf{N}_j \quad (7)$$

where $\mathbf{N}_j$ is an $N_r \times N_t$ random matrix with Gaussian entries and time indices have been dropped for convenience. The amount and severity of estimation error will be quantized by the value $\sigma_e^2 = \frac{E[||\mathbf{N}_j||]}{E[||\mathbf{H}_j||]}$ where $||\cdot||$ represents the matrix Frobenius norm. Thus the precoding and detection schemes that require CSI will use $\hat{\mathbf{H}}_j$ to calculate the beamforming vectors while the actual channel is $\mathbf{H}_j$.

IV. AVERAGE SUM RATE MAXIMIZATION

In order to have stability against errors in channel estimation and feedback, beamforming vectors will be found that maximize the average sum rate approximation from Eq. (4) without using CSI

$$\max_{\mathbf{w}_j, \mathbf{b}_j} \bar{C} \quad (8)$$

with power constraints on the input beamforming vectors. For the maximization process the input parameters are created with nonlinear permutations of the spatial correlation matrix [8]

$$\begin{aligned} \mathbf{S}_{t,j} &= E[\mathbf{H}_j^T \otimes \mathbf{H}_j^H] \\ \mathbf{S}_{r,j} &= E[\mathbf{H}_j^* \otimes \mathbf{H}_j] \end{aligned} \quad (9)$$

where $\{\cdot\}^T$ is the matrix transpose, $\{\cdot\}^*$ is the element-wise matrix conjugate, and $\otimes$ is the Kronecker product. Given these input quantities, the beamformer which maximizes the average rate approximation is written as

$$\mathbf{B} = \left(\frac{\text{tr}(\mathbf{D})}{P}\mathbf{I} + \sum_{i=1}^{i=K}\mathbf{D}_{i,i}\bar{\mathbf{H}}_i\right)^{-1}\mathbf{\Lambda} \quad (10)$$

with the defined quantities

$$\begin{aligned} \bar{\mathbf{H}}_j &= \text{mat}\left(\mathbf{S}_{t,j}\text{vec}(\mathbf{w}_j\mathbf{w}_j^H)\right) \\ \mathbf{B} &= [\mathbf{b}_1, \ldots, \mathbf{b}_K] \\ \mathbf{\Lambda} &= \left[\frac{(\bar{\mathbf{H}}_1\mathbf{B})_1}{\bar{d}_1}, \ldots, \frac{(\bar{\mathbf{H}}_K\mathbf{B})_K}{\bar{d}_K}\right] \\ \mathbf{D} &= \text{diag}\left(\frac{\bar{n}_1}{\bar{d}_1(\bar{d}_1+\bar{n}_1)}, \ldots, \frac{\bar{n}_K}{\bar{d}_K(\bar{d}_K+\bar{n}_K)}\right) \end{aligned} \quad (11)$$

where $\text{diag}(\cdot)$ returns a diagonal matrix of the inputs. In order to find the beamforming weights that maximize $\bar{C}$ each partial derivative $\frac{\partial\bar{C}}{\partial B_{i,j}} = 0$ is solved individually and combined to produce the final beamforming weights. Details of the output form of the beamformer can be found in [8] and is referred to as the regularized channel distribution inversion (RCDI) beamformer.

The solution to this maximization is carried out by following the iterations suggested in [2], [3]. The iterative RCDI beamforming algorithm alternates updating the transmit weights using (10) and the receive weights as in [4]. The only external inputs needed for the algorithm are $\mathbf{S}_{t,j}$ and $\mathbf{S}_{r,j}$ which are large matrices of the size of the full spatial correlation matrix. Also, the RCDI algorithm reduces exactly to the RCI algorithm when the expectation operator

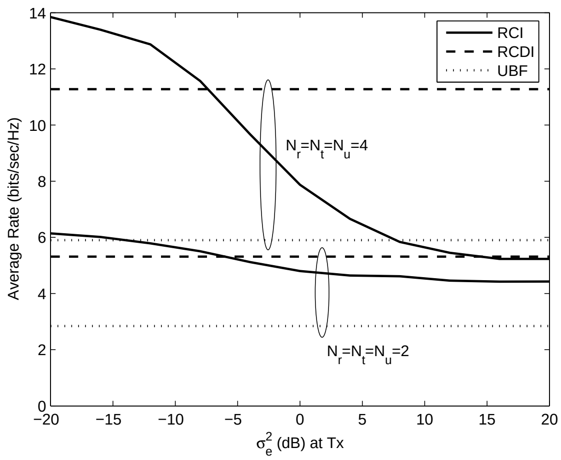

Fig. 1. Average rate loss for the spatially correlated broadcast channel with $N_r = N_t = N_u = 4$ and $P = 10$ for RCDI using either CSI or CDI. The receiver has perfect CSI while the transmitter uses CSI with an estimation error of σ_e^2.

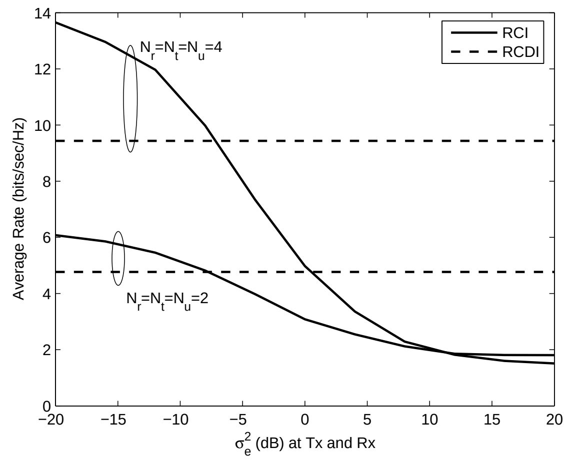

Fig. 2. Average rate loss for the spatially correlated broadcast channel with $N_r = N_t = N_u = 4$ and $P = 10$ for RCDI using either CSI or CDI. The receiver and transmitter share CSI with an estimation error of σ_e^2.

is removed (i.e. $\mathbf{S}_{t,j} = \mathbf{H}_j^T \otimes \mathbf{H}_j^H$) and for the same initial conditions.

An important dependence of the RCI and RCDI beamformers are the initial conditions $\mathbf{D}$ and $\mathbf{\Lambda}$. Due to the nonconvex nature of beamforming capacity expression, the algorithms only guarantee convergence to a local maximum and may not produce the true sum capacity of the broadcast channel with linear precoding [2], though a good starting point for RCI is the regularized pseudo-inverse of the channel. An analogous initial condition for the RCDI algorithm has not yet been derived; therefore, several starting points are tested where the initial condition producing the highest approximation on average sum rate is used to select the output weights.

Figure 1 shows average rate (2) of the RCI and RCDI algorithms when CSI is corrupted by noise at the transmitter but CDI remains error-free due to an assumption that simulations remain within the stationarity time of the channel. The simulations use a broadcast channel with $N_r = N_t = N_u = 4$ and a power of $P = 10$. The performance is considered by adding an error of σ_e^2 in the feedback channel to the transmitter CSI while the receiver has error-free CSI. The uninformed beamformer (UBF) is included for comparison purposes as a scheme that uses no CSIT. UBF simply multiplexes the data across random transmit beamformers with time-sharing used to remove multiple-access interference. Note that the RCDI beamformer uses only CDIT and is resilient to errors in the channel while RCI degrades rapidly. Figure 2 displays the same scenario with the addition that the receiver also has erroneous CSIR for the RCI algorithm and the receiver uses CDIR for the RCDI algorithm. For this case, the degradation is much more rapid and severe as the receiver is unable to provide optimal beamforming with outdated CSIR though RCDI beamforming still provides stability when no CSI is used at any nodes.

V. CHANNEL DISTRIBUTION PARAMETERIZATION

As shown in the previous section, the RCDI algorithm proposed in this work provides robustness against channel estimation errors but requires input parameters $\mathbf{S}_t$ and $\mathbf{S}_r$ that are nonlinear permutations on the large spatial correlation matrix

$$\mathbf{R} \quad = \quad E[\text{vec}(\mathbf{H})\text{vec}(\mathbf{H})^H]$$

where the user index is dropped for convenience. This section details how to use various channel models in order to parameterize $\mathbf{S}_t$ and $\mathbf{S}_r$ and limit the amount of required feedback. This work considers random matrix models; therefore, the following quantities are defined for simplicity

$$\begin{aligned}\mathbf{I}_t &= E[\mathbf{H}_w^T \otimes \mathbf{H}_w^H] \\ \mathbf{I}_r &= E[\mathbf{H}_w^* \otimes \mathbf{H}_w].\end{aligned}$$

Additionally, two properties of the Kronecker product are used: $\mathbf{AB} \otimes \mathbf{CD} = (\mathbf{A} \otimes \mathbf{C})(\mathbf{B} \otimes \mathbf{D})$ and $(\mathbf{A} \odot \mathbf{B}) \otimes (\mathbf{C} \odot \mathbf{D}) = (\mathbf{A} \otimes \mathbf{C}) \odot (\mathbf{C} \otimes \mathbf{D})$ where $\odot$ is the matrix element-by-element product operator.

V-A. Correlation Model

The correlation model uses the full spatial correlation matrix to model behavior in the channel - no parameterization is used on $\mathbf{R}$. For the correlation model, channel realizations are generated as shown in Eq. (6). Given this model, the input parameters for the RCDI algorithm can simply be written as

$$\begin{aligned} \mathbf{S}_{t,\text{Corr}} &= E[\mathbf{H}^T \otimes \mathbf{H}^H] \\ \mathbf{S}_{r,\text{Corr}} &= E[\mathbf{H}^* \otimes \mathbf{H}] \end{aligned} \quad (12)$$

which are only functions of $\mathbf{R}$. For the correlation model the number of complex valued numbers needed for feedback is $N_r^2 N_t^2$.

V-B. Kronecker Model

The Kronecker model [5] assumes a separability between transmit and receive correlation matrices. This decoupling of transmit and receive antennas allows channel realizations to be generated by

$$\mathbf{H}_{\text{Kron}} = \sqrt{\mathbf{R}_r}\mathbf{H}_w\sqrt{\mathbf{R}_t} \quad (13)$$

where the one-sided correlation matrices are calculated from $\mathbf{R}_r = E[\mathbf{H}\mathbf{H}^H]$ and $\mathbf{R}_t = E[\mathbf{H}^H\mathbf{H}]$. The input parameters to the RCDI algorithm assuming the Kronecker model reduce to

$$\begin{aligned} \mathbf{S}_{t,\text{Kron}} &= \left(\sqrt{\mathbf{R}_t}^T \otimes \sqrt{\mathbf{R}_t}^H\right)\mathbf{I}_t\left(\sqrt{\mathbf{R}_r}^T \otimes \sqrt{\mathbf{R}_r}^H\right) \\ \mathbf{S}_{r,\text{Kron}} &= \left(\sqrt{\mathbf{R}_r}^* \otimes \sqrt{\mathbf{R}_r}\right)\mathbf{I}_r\left(\sqrt{\mathbf{R}_t}^* \otimes \sqrt{\mathbf{R}_t}\right). \end{aligned} \quad (14)$$

The complexity in the feedback channel is $N_r^2 + N_t^2$ complex numbers.

V-C. Kronecker Model with Rank-1 Approximation

For this work, we will also use the Kronecker model where we first decompose the full correlation matrix using a Rank-1 approximation to the Kronecker product [9] producing estimates of the one-sided correlations by minimizing $||\mathbf{R} - \hat{\mathbf{R}}_t \otimes \hat{\mathbf{R}}_r||^2$. The estimates $\hat{\mathbf{R}}_r$, $\hat{\mathbf{R}}_t$ are then fed back to the transmitter which can reproduce the necessary matrix estimates in Eq. (14) to perform RCDI beamforming. The complexity in the feedback channel remains $N_r^2 + N_t^2$ complex numbers.

V-D. Weichselberger Model

The Weichselberger model [6] introduces a coupling matrix used to model the "cross"-correlations between transmit and receive antennas. For this model, channel realizations are generated with

$$\mathbf{H}_{\text{Weichs}} = \mathbf{U}_r\left(\tilde{\mathbf{\Omega}} \odot \mathbf{H}_w\right)\mathbf{U}_t^T \quad (15)$$

where $\tilde{\mathbf{A}}$ is the element-wise square root on the matrix $\mathbf{A}$, and the matrices $\mathbf{U}_r$ and $\mathbf{U}_t$ contain the eigenvectors of $\mathbf{R}_r$ and $\mathbf{R}_t$ from the Kronecker model, respectively. Using properties of the Kronecker product the RCDI input parameters become

$$\begin{aligned} \mathbf{S}_{t,\text{Weichs}} &= (\mathbf{U}_t \otimes \mathbf{U}_t^*)\left\{(\tilde{\mathbf{\Omega}}^T \otimes \tilde{\mathbf{\Omega}}) \odot \mathbf{I}_t\right\}\left(\mathbf{U}_r^T \otimes \mathbf{U}_r^H\right) \\ \mathbf{S}_{r,\text{Weichs}} &= (\mathbf{U}_r^* \otimes \mathbf{U}_r)\left\{(\tilde{\mathbf{\Omega}}^* \otimes \tilde{\mathbf{\Omega}}) \odot \mathbf{I}_r\right\}\left(\mathbf{U}_t^H \otimes \mathbf{U}_t^T\right) \end{aligned} \quad (16)$$

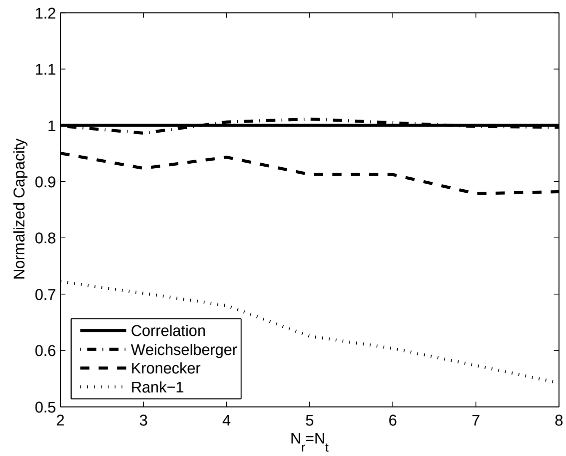

Fig. 3. Normalized channel capacity versus $N_r = N_t$ in the single-user channel. Power is constrained to $P = 10$.

The Weichselberger model requires a complexity of $N_r^2 + N_t^2 + N_r N_t$ in the feedback channel.

Prior to examining the performance of each parameterization technique, it is worthwhile to first note the resulting modeling error that each technique introduces if it were used to model channel capacity. Fig. 3 examines this modeling error by way of normalized capacity with respect to the full correlation model. For this plot, the measured data is used to estimate the various parameters necessary for each model. In other words, the full correlation matrix and its Rank-1 approximation is calculated as well as the left- and right-sided correlation matrices for the Kronecker model and the coupling matrix for the Weichselberger model. Once estimated, new channels are realized for each of these models, Eqs. (6) (13) (15), and the resulting uninformed capacity is found and normalized by the capacity found under the full correlation model. Note that the results are similar to those presented in other work [10]; namely, the Kronecker model produces higher error with more antennas while the Weichselberger model provides for a good model of the spatial correlation. The Rank-1 approximation of the Kronecker model results in a poor model of the channel capacity since the framework used constrains the one-sided correlation matrices to be conjugate symmetric but places no constraint on the matrices being positive semidefinite.

VI. RESULTS

The effects of channel distribution parameterization are examined in Fig. 4. For this plot, the power is held constant at $P = 10$ while the number of antennas and users is swept. The results show that, depending on the model used, parameterizing the channel distribution does not significantly affect the average sum rate; however, the amount of feedback savings is immense. For $N_r = N_t = 6$ the total number of complex numbers fed back per user for

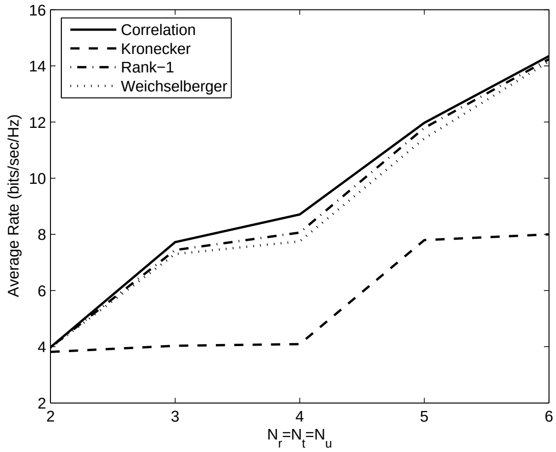

Fig. 4. Average rate versus system size for $\mathbf{S}_t$ and $\mathbf{S}_r$ generated by models: the full correlation matrix, the Weichselberger model, and the Kronecker model.

the correlation model is 1296 while only 108 and 72 for the Weichselberger and Rank-1 models, respectively - an order of magnitude difference. The Rank-1 approximation outperforms the standard Kronecker model parameterization due to the underlying assumptions made by the Kronecker model about the separability of the spatial correlations. It should be reiterated that for each curve in Fig. 4 the same RCDI algorithm is used with different estimates of $\mathbf{S}_t$ and $\mathbf{S}_r$ and the resulting weights are run through the exact same channel realizations as created with the measured data.

Conclusions that can be drawn from Fig. 4 are two-fold. First, with the correct model parameterization the majority of performance can be captured as shown by the Weichselberger and Rank-1 curves. And conversely, the majority of performance can be lost if an incorrect model is assumed for the measured channel even when such model does an adequate job in describing the channel capacity.

VII. CONCLUSION

The sum rate maximizing beamformer has been adapted to use CDI as well as CSI depending on available channel information. This beamforming algorithm is robust to temporal variations and delay in the feedback channel. Furthermore, simple parameterization of the channel correlation matrix using well known channel models provides for a significant reduction in the amount of information fed back to the transmitter resulting in only a small loss in performance.

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