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Enclosure 1

Outage probability evaluation in random wireless networks with multiple antennas

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***Abstract*—We consider a network of transmitters, each with a receiver at a fixed distance, and locations drawn according to a homogeneous Poisson point process. The transmitters and the receivers are equipped with an equal number of antennas. Under a channel model that includes Rayleigh fading and path-loss with exponent $b > 2$, we derive an upper/lower bound to the outage/success probability at the receiver, when a spatial diversity order N is available. It is shown that the main effect of spatial diversity is to provide an approximate gain $N^{-2/b}$ in *spatial contention*, recently defined in [1] as the slope of the outage probability as a function of the transmitter density, when the latter is zero. Using a well known lower bound to the mutual information of the multiple-input multiple-output (MIMO) channel by Foschini et. al., we employ the aforementioned expression to derive a lower bound to the success probability in a MIMO channel, in the presence of network interference. The usefulness of these findings is showcased in determining the number of transmitted streams that maximize the transmission capacity of the network.**

***Index Terms*—Poisson point process, MIMO, outage probability, transmission capacity**

I. INTRODUCTION

The study of random wireless networks has recently gathered a lot of momentum in the research community, e.g., see [1]–[6]. The main motivation behind this work is the use of tools from stochastic geometry in order to derive insightful analytical results on how different physical, MAC and network layer parameters affect the network performance.

The objective of this paper is to shed more light on the impact of the use of multiple antennas and, more specifically, multiple stream transmission, in random networks. We consider a well established single-hop network model where transmitters are distributed on the plane according to a homogeneous Poisson point process. Each transmitter has a corresponding receiver at a fixed distance and channel knowledge is only available at the receivers.

A. Related work

The outage probability for different spatial diversity techniques and single-stream transmission was evaluated in [5]. The main result of this work was that, in the small outage probability regime, the transmission capacity, i.e., the maximum network throughput such that a constraint on the outage probability is satisfied, scales as $N^{2/b}$, where N is the spatial diversity order and b is the path-loss exponent. The authors in [7] considered various multiple-input multiple-output (MIMO) techniques, including spatial multiplexing, as a component of a physical layer that employs frequency hopping and coding in combating interference. They arrived at similar scaling laws to [5] regarding the network throughput and the expected progress, albeit from a different analytical path. Multiple stream transmission with perfect channel knowledge at the transmitter was studied in [8] and the optimal number of spatial modes, in terms of maximizing the transmission capacity for a given density, was illustrated. More recently, *multi-user* MIMO techniques such as interference cancelation and space-division multiple-access have been considered in [9]–[11].

B. Contributions

We first consider single-stream transmission and derive an upper/lower bound to the outage/success[1] probability in a Poisson field of interferers, when a spatial diversity order N is available at the receiver. This expression is simpler and more compact than the one derived in [5]. It is shown that the effect of spatial diversity is to provide an approximate gain of $N^{-2/b}$ in terms of spatial contention, i.e., the rate of increase of the outage probability as a function of the transmitter density, when the latter is zero. This result provides an alternative interpretation to the scaling law in [5].

Using a lower bound to the mutual information of the MIMO channel, derived in [12], we then provide a lower bound to the success probability in the MIMO channel, in the presence of network interference. We show that the gain in terms of spatial contention is approximately $N_{\text{eff}}^{-2/b}$, where N_{eff} is the arithmetic mean of the spatial diversity orders of the subchannels, if detection at the receiver is performed via zero forcing and perfect successive interference cancellation.

We employ these results in order to maximize the transmission capacity over the number of transmitted streams, in the small outage probability regime. It is shown that, for $b \geq 4$, it is optimal to use all transmit antennas, while, for $b < 4$, the number of streams must be judiciously chosen such that the optimal trade-off between network interference and rate

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[1]The terms "outage" and "success" probability, since complementary, are used interchangeably throughout the paper.

increase is achieved. We also examine the case where each packet is transmitted on the same subchannel in a slot (V-BLAST) and demonstrate that transmitting a packet across the antennas (D-BLAST) yields a significant gain in terms of transmission capacity. In both cases, the transmission capacity scales linearly in the number of receive antennas.

The rest of the paper is organized as follows. In Section II, the system model is introduced. A lower bound and an approximation to the success probability are derived in Section III. The transmission capacity is then evaluated and optimized in Section IV and numerical examples are presented in Section V. Section VI concludes the paper.

We note the following regarding the notation: a zero-mean complex Gaussian random vector $\boldsymbol{x}$, with covariance matrix $\mathbf{Q} = E[\boldsymbol{x}\boldsymbol{x}^{\mathrm{H}}]$ is denoted as $\boldsymbol{x} \sim \mathcal{CN}(0, \mathbf{Q})$; a central chi-square random variable (r.v.) x with parameter $1/2$ and $l \geq 1$ degrees of freedom is denoted as $x \sim \chi_l^2$; and the $l \times l$ identity and zero matrices are denoted as $\mathbf{I}_l$, $\mathbf{O}_l$, respectively.

II. System Model

The transmitters are distributed on the plane according to a homogeneous Poisson process of density λ' and each transmitter (TX) has a corresponding receiver (RX) at distance R. Transmissions take place concurrently and in a synchronized manner, during a slot. Due to mobility and/or random access with a small probability p, we assume that the set of active TXs from slot to slot is a new realization of a Poisson point process Π with effective density $\lambda = \lambda' p$.

The channel between each TX-RX pair over a sub-band consists of constant flat Rayleigh fading with a coherence time of one slot and path-loss according to the law r^{-b}, where $b > 2$ is the propagation exponent. The power from each antenna is the same across all TXs and equal to one. Generally, there is a different number of antennas at the TX and the RX; however, for convenience, we assume that N antennas are available at both the TX and the RX.[2] Moreover, the channel coefficients are independent across different antennas. We also disregard additive noise, considering interference from concurrent transmissions as the only cause of errors in communication (interference-limited scenario).

Suppose that M antennas are employed for transmission, with $M \leq N$. The received vector at the typical RX is

$$\boldsymbol{y} = R^{-\frac{b}{2}}\mathbf{H}\boldsymbol{x} + \sum_{i\in\Pi} R_i^{-\frac{b}{2}}\mathbf{H}_i\boldsymbol{x}_i, \tag{1}$$

where $\mathbf{H}$ is the $N \times M$ channel matrix between TX and RX, with i.i.d. elements $[\mathbf{H}]_{nm} \sim \mathcal{CN}(0,1)$; $\boldsymbol{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is the $M \times 1$ symbol vector transmitted by TX; $\mathbf{H}_i$ is the channel matrix between interfering TX i (denoted as TX$_i$) and RX, with i.i.d. elements $[\mathbf{H}_i]_{nm} \sim \mathcal{CN}(0,1)$; $\boldsymbol{x}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ is the $M \times 1$ symbol vector transmitted by TX$_i$; and R_i is the distance between TX$_i$ and RX. Note that, given $\{R_i, \mathbf{H}_i\}$, it holds that $w \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q})$, where $\boldsymbol{w}$ is the interference term defined as $\boldsymbol{w} = \sum_{i\in\Pi} R_i^{-\frac{b}{2}}\mathbf{H}_i\boldsymbol{x}_i$ and $\mathbf{Q} = \sum_{i\in\Pi} R_i^{-b}\mathbf{H}_i\mathbf{H}_i^{\mathrm{H}}$ is its correlation matrix.

Due to the complicated nature of $\mathbf{Q}$, the evaluation of the mutual information of the channel in (1) and the respective outage probability appears intractable. A lower bound to the success probability, P_s', can be obtained by setting $\mathbf{Q} = M\sum_{i\in\Pi} R_i^{-b}\mathbf{I}_N$, i.e., by substituting the term $\mathbf{H}_i\mathbf{H}_i^{\mathrm{H}}$ in the original definition of $\mathbf{Q}$, with its expected value. Assuming an information rate $\mathcal{R}$ for the typical TX-RX link, we therefore have that

$$P_s' = \mathrm{P}\left(\log\det\left(\mathbf{I}_N + \frac{1}{z}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right) > \mathcal{R}\right), \tag{2}$$

where $z = M\sum_{i\in\Pi} R_i^{-b}$ is the interference power over a given slot, per RX antenna, averaged over the interferers' fading coefficients. Moreover, using the result in [12], P_s' can be further lower bounded by

$$P_s = \mathrm{P}\left(\sum_{m=1}^{M}\log\left(1 + \frac{a_m}{z}\right) > \mathcal{R}\right), \tag{3}$$

where $\{a_m\}$ are independent random variables with $a_m \sim \chi^2_{2(N-m+1)}$. The respective upper bound to the outage probability is $P_o = 1 - P_s$. As described in [12], the performance in (3) can be approached with the D-BLAST architecture.

The evaluation of (3) requires the statistics of z. However, it is well known [2], [13] that z is an α-stable r.v. with stability exponent $\alpha = 2/b$ and moment generating function

$$\Phi_z(s) = \mathrm{E}[\mathrm{e}^{-sz}] = \mathrm{e}^{-cs^{\alpha}},\ s > 0, \tag{4}$$

where $c = \lambda\pi R^2\Gamma(1-\alpha)M^{\alpha}$ and $\Gamma(x),\ x > 0$, is the gamma function. In the following section, we use (4) in order to evaluate (3).

III. Evaluation of the outage probability

A. Single-stream transmission

We first consider the transmission of a single stream, i.e., $M = 1$. Defining $\mathcal{R} = \log(1+\theta)$, where θ is an appropriate signal-to-interference-ratio (SIR) threshold, (3) becomes

$$P_s = \mathrm{P}\left(\frac{a_1}{z} > \theta\right). \tag{5}$$

Note that (5) is an approximation[3] to the success probability, when maximal-ratio-combining is employed at the RX. The evaluation of (5) is carried out in [5], in the process of studying the effect of different spatial diversity techniques on the network transmission capacity. We propose a different approach here that results in a compact expression for P_s.

For ease of exposition, denote a_1 by a. The complementary cumulative distribution function (ccdf) of $\gamma = a/z$ is

$$\bar{F}_\gamma(\gamma) = \mathrm{P}(a > \gamma z) = \int_0^{+\infty} \bar{F}_a(\gamma z) f_z(z)\mathrm{d}z \tag{6}$$

[2]This assumption is not unrealistic in an ad hoc network, where a node can be a TX and a RX interchangeably throughout time.

[3]It is an approximation, as the interferers' fading has been "averaged out" in (2). Nevertheless, for the case of single-stream transmission, the interferers' fading can be easily taken into account, as shown in [5].

where

$$\bar{F}_a(a) = \mathrm{e}^{-a} \sum_{n=0}^{N-1} \frac{a^n}{n!} \tag{7}$$

is the ccdf of a. Substituting (7) in (6), we have that

$$\bar{F}_\gamma(\gamma) = \Phi_z(\gamma) + \sum_{n=1}^{N-1} \frac{\gamma^n}{n!} \int_0^{+\infty} z^n f_z(z) \mathrm{e}^{-\gamma z} \mathrm{d}z$$

From the Laplace transform property (also employed in [5])

$$f_z(z) z^n \longleftrightarrow (-1)^n \frac{\mathrm{d}^n \Phi_z(s)}{\mathrm{d}s^n}$$

we have that

$$\bar{F}_\gamma(\gamma) = \Phi_z(\gamma) + \sum_{n=1}^{N-1} \frac{\gamma^n}{n!} (-1)^n \frac{\mathrm{d}^n \Phi_z(\gamma)}{\mathrm{d}\gamma^n} \tag{8}$$

We now employ identity 0.430.1 on p.24 of [14], in order to evaluate the n^{th} derivative of the composite function $\Phi_z(\gamma) = \mathrm{e}^{-c\gamma^\alpha}$. After considerable algebra we obtain

$$\frac{\mathrm{d}^n \Phi_z(\gamma)}{\mathrm{d}\gamma^n} = \gamma^{-n} \mathrm{e}^{-c\gamma^\alpha} \sum_{k=1}^{n} \frac{\beta_k^n}{k!} (c\gamma^\alpha)^k \tag{9}$$

where

$$\beta_k^n = \sum_{l=1}^{k} (-1)^l \binom{k}{l} (\alpha l)_n, \ k = 1, \ldots, n \tag{10}$$

and $(\alpha l)_n \triangleq \alpha l \ldots (\alpha l - n + 1)$ denotes the falling sequential product. Substituting (9) in (8) and regrouping terms results in

$$\begin{aligned} \bar{F}_\gamma(\gamma) &= \mathrm{e}^{-c\gamma^\alpha} + \mathrm{e}^{-c\gamma^\alpha} \sum_{n=1}^{N-1} \frac{1}{n!} \sum_{k=1}^{n} \frac{(-1)^n \beta_k^n}{k!} (c\gamma^\alpha)^k \\ &= \mathrm{e}^{-c\gamma^\alpha} + \mathrm{e}^{-c\gamma^\alpha} \sum_{k=1}^{N-1} \frac{(c\gamma^\alpha)^k}{k!} \sum_{n=k}^{N-1} \frac{(-1)^n \beta_k^n}{n!} \end{aligned} \tag{11}$$

Note that $\beta_1^n = -(\alpha)_n$. Since $\alpha < 1$, it follows that $(-1)^n \beta_1^n \geq 0$. This property can be shown for all $\{\beta_k^n\}$ defined in (10), i.e., $(-1)^n \beta_k^n \geq 0$ (see Lemma 1). By the definition of P_s in (5), we have that $P_s = \bar{F}_\gamma(\theta)$ or

$$P_s = \mathrm{e}^{-c\gamma^\alpha} + \mathrm{e}^{-c\gamma^\alpha} \sum_{k=1}^{N-1} \frac{(c\gamma^\alpha)^k}{k!} \sum_{n=k}^{N-1} \frac{|\beta_k^n|}{n!} \tag{12}$$

From (12), we can see that the success probability for spatial diversity order N is a product of the term $\mathrm{e}^{-c\theta^\alpha}$ (the success probability for $N = 1$) and a polynomial in $c\theta^\alpha$ of degree $N - 1$ and non-negative coefficients. Clearly, increasing the spatial diversity order N, increases the success probability as more positive terms are added to the polynomial.

We want to obtain more insight into the effect of $N > 1$ on the outage/success probability. In this direction, we evaluate the derivative

$$\eta = -\left.\frac{\partial P_s}{\partial \lambda}\right|_{\lambda=0}, \tag{13}$$

defined in [1] for single-antenna networks, as the spatial contention parameter. By its definition, it measures the slope of the outage probability in a random access network as a function of the density λ, at $\lambda = 0$. The larger η is, the sharper the increase of the outage probability as λ increases. Denoting $c' = c\theta^\alpha/\lambda$, from (12) we have that

$$\begin{aligned} \eta &= c' - c' \sum_{n=1}^{N-1} \frac{|\beta_1^n|}{n!} \\ &= c' + c' \sum_{n=1}^{N-1} \frac{(-1)^n (\alpha)_n}{n!} \\ &= c' \sum_{n=0}^{N-1} \frac{(-1)^n (\alpha)_n}{n!} \qquad | \ (\alpha)_0 \triangleq 1 \\ &= c' \frac{(-1)^{N-1}}{(N-1)!} \sum_{n=0}^{N-1} \binom{N-1}{n} (-1)_{N-1-n} (\alpha)_n \\ &= c' \frac{(-1)^{N-1}}{(N-1)!} (\alpha - 1)_{N-1} \qquad (14) \\ &= c' \frac{\Gamma(N-\alpha)}{\Gamma(N)\Gamma(1-\alpha)} \qquad (15) \end{aligned}$$

where (14) stems from the binomial identity for falling sequential products and (15) is the result of the successive application of the gamma function property $\Gamma(x + 1) = x\Gamma(x)$. As N increases, it can be shown that [7]

$$\frac{\Gamma(N-\alpha)}{\Gamma(N)} \simeq N^{-\alpha}, \tag{16}$$

therefore $\eta \simeq \pi R^2 \theta^\alpha N^{-\alpha}$. It is seen that a spatial diversity order N provides an approximate gain of $N^{-\alpha}$ in terms of spatial contention. This result provides an alternative interpretation to the scaling law derived in [5]. In simpler terms, the spatial diversity at the RX roughly decreases the SIR threshold θ by a factor N.

The following lemma provides an upper bound on P_s.

Lemma 1: For single-stream transmission and a RX spatial diversity order N, P_s is upper-bounded by

$$\begin{aligned} \bar{P}_s(N) &= \exp\left(-\frac{\Gamma(N-\alpha)}{\Gamma(N)\Gamma(1-\alpha)} c\theta^\alpha\right) \\ &= \exp\left(-\eta\lambda\right). \end{aligned} \tag{17}$$

The equality holds for $N = 1$.

Proof: Since for $N = 1$, $P_s = \mathrm{e}^{-c\theta^\alpha}$, (12) holds as an equality for $N = 1$. For $N > 1$, we take the Taylor series expansion of $\bar{P}_s$ and compare individual terms with (12). It can be seen that it suffices to prove that

$$A_1 \triangleq \sum_{n=k}^{N-1} \frac{(-1)^n \beta_k^n}{n!} \leq \left(\sum_{n=1}^{N-1} \frac{(-1)^n \beta_1^n}{n!}\right)^k \triangleq A_2, \tag{18}$$

with $k = 1, \ldots, N-1$. If $k = 1$, (18) holds as an equality. For $k > 1$, it holds that

$$A_2 = \sum_{n_1=1}^{N-1} \cdots \sum_{n_k=1}^{N-1} \delta_1^{n_1} \ldots \delta_1^{n_k} \tag{19}$$

where, for convenience, we have defined $\delta_k^n = \frac{(-1)^n \beta_k^n}{n!}$. Moreover, by (10), we can show that β_k^n is equal to the following derivative

$$\beta_k^n = \left. \frac{\mathrm{d}^n \left(1 - x^\alpha\right)^k}{\mathrm{d}x^n} \right|_{x=1}. \tag{20}$$

Using (20), the following iterative relation can be proved

$$\beta_k^n = \sum_{m_1=1}^{n} \binom{n}{m_1} \beta_1^{m_1} \beta_{k-1}^{n-m_1}. \tag{21}$$

Applying (21) successively, we obtain

$$\frac{(-1)^n \beta_k^n}{n!} = \sum_{m_1=1}^{n} \sum_{m_2=1}^{n-m_1} \cdots \sum_{m_{k-1}=1}^{n-m_{k-2}-\cdots-m_1} \delta_1^{m_1} \delta_1^{m_2} \ldots \delta_1^{m_k} \tag{22}$$

where $m_k = n - m_{k-1} - \cdots - m_1$.[4] Substituting (22) and (19) in (18), we can see that (18) is a true statement. This is due to the fact that the summation that gives A_1 is over a subset of the terms that are summed to give A_2. ■

We can verify that the bound in (17) becomes tight as $\lambda \to 0$, by letting $\lambda \to 0$ in (12). As a result, (17) can be used as an approximation to P_s in the small outage probability regime.

B. Multiple stream transmission

We now turn our attention to the case $1 < M \leq N$, and the evaluation of (3). Defining $\mathcal{R} = M \log(1 + \theta)$, we have

$$\begin{aligned} P_o &= \mathrm{P}\left(\sum_{m=1}^{M} \log\left(1 + \frac{a_m}{z}\right) < \mathcal{R} \right) \\ &= \mathrm{P}\left(\prod_{m=1}^{M} \left(1 + \frac{a_m}{z}\right)^{\frac{1}{M}} < 1 + \theta \right) \end{aligned}$$

We observe that, roughly, an outage occurs when the interference power z obtains a large value. In this case, the geometric mean of $\{1 + \frac{a_m}{z}\}_{m=1}^{M}$ is approximately equal to the arithmetic mean, thus

$$P_o \simeq \mathrm{P}\left(\frac{1}{z} \sum_{m=1}^{M} a_m < M\theta \right). \tag{23}$$

However, note that $\sum_{m=1}^{M} a_m \sim \chi^2_{2N_{\mathrm{tot}}}$, where $N_{\mathrm{tot}} = \sum_{m=1}^{M} (N - m + 1)$. For the case of multiple-stream transmission, we can therefore obtain an approximation to P_s by (12), if we set $N \to N_{\mathrm{tot}}$ and $\theta \to M\theta$. Moreover, Lemma 1, in conjunction with (16), implies that the spatial contention parameter can be approximated by $\eta \simeq \pi R^2 M^\alpha \theta^\alpha N_{\mathrm{eff}}^{-\alpha}$, where $N_{\mathrm{eff}} = \frac{1}{M} N_{\mathrm{tot}}$. Note that N_{eff} is the arithmetic mean of the diversity order of each stream/subchannel, if detection at the receiver is performed via zero forcing and perfect [5] successive interference cancelation [12], [15].

IV. Transmission Capacity

We now utilize the findings of the previous section in evaluating the transmission capacity of the network, defined as the network throughput such that a constraint $P_o = \epsilon$ is satisfied [4]. Mathematically,

$$C = \lambda_\epsilon (1 - \epsilon) \mathcal{R}, \tag{24}$$

where the *maximum contention density* λ_ϵ is determined by the constraint $P_o = \epsilon$. In the small outage probability regime, e.g., $\epsilon \leq 0.1$, we can invoke Lemma 1 to determine an approximation to λ_ϵ, i.e.,

$$\begin{aligned} \exp\left(-\frac{\Gamma(N_{\mathrm{tot}} - \alpha)}{\Gamma(N_{\mathrm{tot}})} \lambda_\epsilon \pi R^2 M^\alpha (M\theta)^\alpha \right) &\simeq 1 - \epsilon \\ \lambda_\epsilon \simeq -\frac{\log(1-\epsilon)}{\pi R^2 \theta^\alpha} \frac{\Gamma(N_{\mathrm{tot}})}{\Gamma(N_{\mathrm{tot}} - \alpha) M^{2\alpha}}&. \end{aligned} \tag{25}$$

From (25) and (24), an approximation of the transmission capacity is

$$C \simeq \log(1+\theta) \frac{(\epsilon - 1)\log(1-\epsilon)}{\pi R^2 \theta^\alpha} \cdot \frac{\Gamma(N_{\mathrm{tot}}) M^{1-2\alpha}}{\Gamma(N_{\mathrm{tot}} - \alpha)} \tag{26}$$

Since $N_{\mathrm{tot}} = \frac{2NM - M^2 + M}{2}$, (26) allows the optimization of C with respect to M.[6] In order to obtain an approximate analytical expression for the optimal number of streams, we invoke (16) and set the following derivative to zero

$$\frac{\partial}{\partial M} \left(2N - M + 1\right)^\alpha M^{1-\alpha} = 0. \tag{27}$$

After some calculations, this yields $M_o = (1-\alpha)(2N+1)$. Since the constraints $M_o \leq N$ and $M \in \mathcal{Z}^+$ must also be satisfied, a good approximation to the optimal number of streams is

$$M_o = \min\left\{ \lceil (1-\alpha)(2N+1) \rfloor, N \right\} \tag{28}$$

Note that $(1-\alpha)(2N+1) \leq N$ is only possible if $\alpha > 1/2$. As a result, if $\alpha \leq 1/2$, or $b \geq 4$, (28) indicates that transmission with all antennas maximizes the transmission capacity. Also, if we set $M = M_o$ in (26), we can verify that C scales linearly with N.

Eq. (26) and (28) are based on the definition of the success probability given in (2). As discussed in [12], [15], a performance close to (2) can be achieved with the D-BLAST architecture, in conjunction with zero-forcing (ZF)

[4] Eq. (22) is also proof that $(-1)^n \beta_k^n \geq 0$, since $\delta_1^{m_1}, \ldots, \delta_1^{m_k} \geq 0$.

[5] The term "perfect" is used in the sense that the contribution of previously detected streams is always removed.

[6] We can also optimize over the SIR threshold θ [1].

and successive interference cancelation (SIC), in order to separate the different streams. In D-BLAST, a packet is encoded, then separated into segments which are diagonally transmitted across antennas and time, such that each segment experiences a different subchannel. In contrast, in V-BLAST, each packet is transmitted on the same subchannel for the duration of a slot [15]. An outage or packet failure therefore occurs, if a packet is transmitted on a subchannel that happens to be in a deep fade. Assuming that the packets are separated with ZF-SIC at the receiver, the spatial diversity order of the "worst" stream is $N-M+1$. According to the previous discussion, given that the SIR threshold for each stream is θ, the maximum contention density is

$$\lambda_{\epsilon}^{\mathrm{vb}} \simeq -\frac{\log(1-\epsilon)}{\pi R^2\theta^{\alpha}}\frac{\Gamma(N-M+1)}{\Gamma(N-M+1-\alpha)M^{\alpha}} \tag{29}$$

and, assuming perfect SIC, the transmission capacity is

$$C^{\mathrm{vb}} \simeq \lambda_{\epsilon}^{\mathrm{vb}} \log(1+\theta)\sum_{m=1}^{M}\bar{P}_s(N-m+1), \tag{30}$$

where $\bar{P}_s(N-m+1)$ is the success probability over the m^{th} stream, as computed by Lemma 1 (note that $\bar{P}_s(N-M+1)=\epsilon$).

Finally, if the streams are separated by simple ZF, the diversity order of each stream is $N-M+1$ and the transmission capacity is

$$C^{\mathrm{zf}} \simeq \log(1+\theta)\frac{(\epsilon-1)\log(1-\epsilon)}{\pi R^2\theta^{\alpha}}\cdot\frac{\Gamma(N-M+1)M^{1-\alpha}}{\Gamma(N-M+1-\alpha)}. \tag{31}$$

Following the same procedure as in (27), in [7] it was shown that the number of streams that maximizes C^{zf} is approximately

$$M_o^{\mathrm{zf}} = \min\left\{\lceil(1-\alpha)(N+1)\rfloor, N\right\}. \tag{32}$$

Letting $M = M_o^{\mathrm{zf}}$ in (31), we can verify that C^{zf} scales linearly with N.

V. Numerical results

In this section, we evaluate the outage probability and transmission capacity for a network with default parameter values $R = 20$ m, $b = 4$, $\theta = 6$ dB and $\epsilon = 0.1$.

In Figs. 1 and 2, we plot the success probability vs. the TX density for $N = 4$, and $M = 1, 3$, respectively. We observe that the match between (12) and the simulated success probability in (2) is very good. In agreement with Lemma 1, (17) provides an upper bound to (12), which becomes tight as $\lambda \to 0$. Also, as expected, the simulated success probability of the channel in (1) upperbounds (12). This gap decreases as the number of transmitted streams M increases, as shown in Fig. 2.

Fig. 3 shows the dependence of C, C^{vb} and C^{zf} on the number of transmitted streams for $N = 4, 8$ and $b = 3$. Note that a D-BLAST-like transmission scheme results in higher

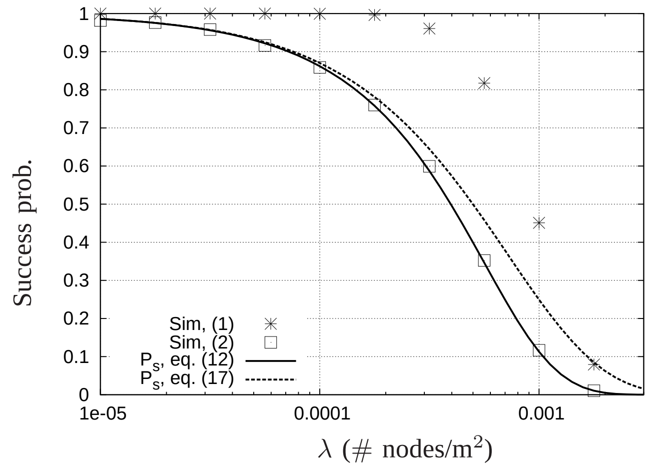

Fig. 1. Success probability vs. λ for $N = 4$ and $M = 1$ ($R = 20$ m, $b = 4$, $\theta = 6$ dB).

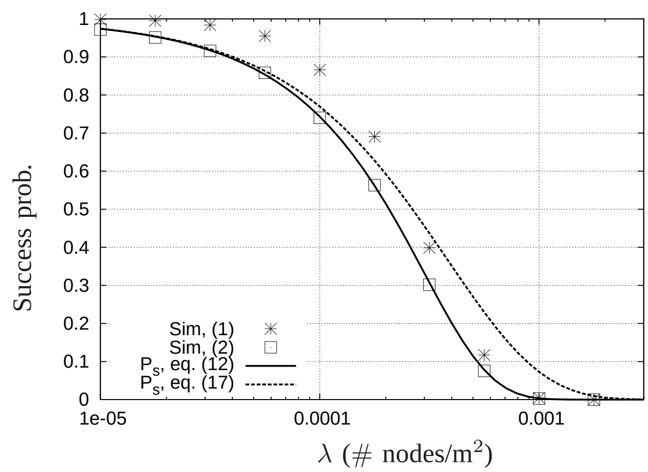

Fig. 2. Success probability vs. λ for $N = 4$ and $M = 3$ ($R = 20$ m, $b = 4$, $\theta = 6$ dB).

transmission capacity compared to V-BLAST with ZF-SIC or ZF. Moreover, the gain between ZF-SIC and simple ZF is marginal, as, with ZF-SIC, the maximum contention density is determined by the subchannel with the smallest diversity order. As predicted by (28), the optimal number of streams for $N = 4$ is $(1-\alpha)(2N+1) = 3$, and, for $N = 8$, $\lceil(1-\alpha)(2N+1)\rfloor = 6$. For ZF, the respective optimal numbers of streams given by (32) are 2 and 3, which are approximately correct (as can be seen in Fig. 3, it is slighlty better to employ only one stream for $N = 4$).

In Fig. 3, we increase the propagation exponent to $b = 4$. As discussed in Section IV, for a D-BLAST scheme, activating all the TX antennas maximizes the transmission capacity. In the case of V-BLAST, the optimal number of streams is dictated by (32). The good agreement of the theoretical curves with the simulation results confirms the validity of the approximations of Section IV.

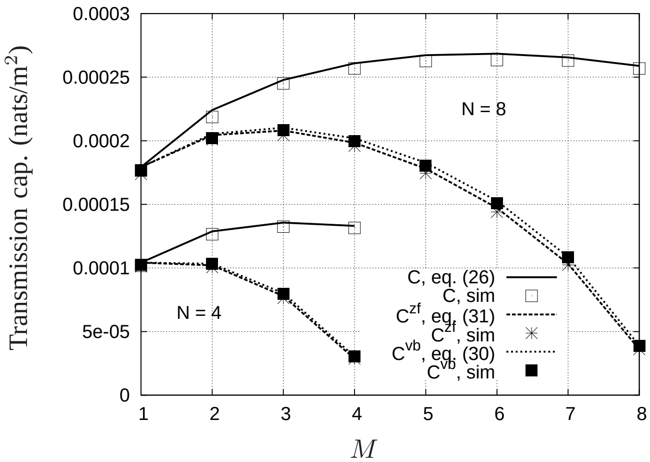

Fig. 3. Transmission capacity vs. M for $N = 4, 8$ ($R = 20$ m, $b = 3$, $\theta = 6$ dB).

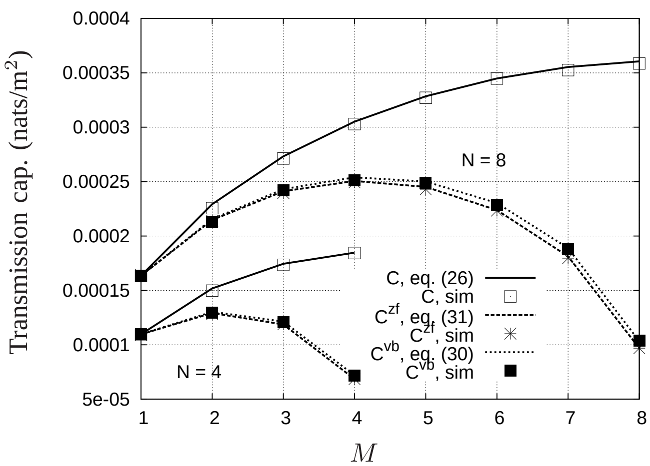

Fig. 4. Transmission capacity vs. M for $N = 4, 8$ ($R = 20$ m, $b = 4$, $\theta = 6$ dB).

VI. Concluding remarks

We conducted a study of an interference-limited, multiple-antenna network, where the locations of the transmitters are determined according to a homogeneous Poisson point process. Assuming channel knowledge at the RX only and that interference from concurrent transmissions is regarded as noise, we derived an exact lower bound to the success probability in the case of single-stream transmission. It was shown that a degree of spatial diversity N introduces an approximate gain $N^{-2/b}$ in terms of spatial contention. In the case of multiple stream transmission, we demostrated that the respective gain is $N_{\text{eff}}^{-2/b}$, where N_{eff} is the arithmetic mean of the spatial diversity orders of the subchannels, if detection at the receiver is performed via zero forcing and perfect successive interference cancellation. These approximations were used to determine the number of streams, such that the transmission capacity of the network is maximized, provided that the network is operated in a small outage probability regime (or, equivalently, if the effective density of transmitters is small).

We showed that, employing a D-BLAST-like strategy, where a transmitted packet experiences an average subchannel, results in substantial capacity gains compared to V-BLAST, where each packet is transmitted on the same subchannel during a slot. Moreover, for a propagation exponent $b \geq 4$, it was demonstrated that the maximum transmission capacity for D-BLAST is achieved when all transmit antennas are activated.

The results in this paper shed light on how MIMO techniques, which are well understood in the single-user context, affect the capacity of a random network. At the heart of the analysis and the resulting design guidelines lies an interference model that takes into account the topology of the interferering transmitters.

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References

[1] M. Haenggi, "Outage and local throughput and capacity of random wireless networks," 2009, *submitted for publication in the IEEE Trans. on Wireless Commun.*

[2] F. Baccelli, B. Błaszczyszyn, and P. Mühlethaler, "An Aloha protocol for multi-hop mobile wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, pp. 421–436, Feb. 2006.

[3] S. P. Weber, X. Yang, J. G. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Trans. Inf. Theory*, vol. 51, pp. 4091–4102, Dec. 2005.

[4] S. P. Weber, J. G. Andrews, and N. Jindal, "The effect of fading, channel inversion and threshold scheduling on ad hoc networks," *IEEE Trans. Inf. Theory*, vol. 53, pp. 4127–4149, Nov. 2007.

[5] A. M. Hunter, J. G. Andrews, and S. P. Weber, "Capacity scaling of ad hoc networks with spatial diversity," 2008, *accepted for publication in the IEEE Trans. on Wireless Commun.*

[6] K. Stamatiou and J. G. Proakis, "Assessing the impact of physical layer techniques on ad hoc network performance," *Elsevier Physical Communication*, vol. 1, pp. 84–91, 2008.

[7] K. Stamatiou, J. G. Proakis, and J. R. Zeidler, "Evaluation of MIMO techniques in FH-MA ad hoc networks," in *Proc. IEEE GLOBECOM*, Washington D.C., Nov. 2007.

[8] A. M. Hunter and J. G. Andrews, "Adaptive rate control over multiple spatial channels in ad hoc networks," in *Workshop on Spatial Stochastic Models for Wireless Networks (SPASWIN)*, Berlin, Apr. 2008.

[9] K. Huang, J. G. Andrews, R. W. H. Jr., D. Guo, and R. Berry, "Spatial interference cancelation for multi-antenna mobile ad hoc networks," 2009, *submitted for publication in the IEEE Trans. on Information Theory*.

[10] N. Jindal, J. G. Andrews, and S. W. Weber, "Rethinking MIMO for wireless networks: linear throughput increases with multiple receive antennas," 2009, *to appear in IEEE ICC*.

[11] M. Kountouris and J. G. Andrews, "Capacity scaling of SDMA in wireless ad hoc networks," 2009, *submitted to IEEE ISIT*.

[12] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications, Kluwer Academic Publishers*, vol. 6, pp. 311–335, 1998.

[13] E. S. Sousa and J. A. Silvester, "Optimum transmission ranges in a direct-sequence spread-spectrum multi-hop packet radio network," *IEEE J. Sel. Areas Commun.*, vol. 8, pp. 762–771, Jun. 1990.

[14] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series and products*, 4th ed. Academic Press, 1994.

[15] D. N. C. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, 1st ed. Cambridge University Press, 2005.