

# On the Statistics and MAC Implications of Channel Estimation Errors in MIMO Ad Hoc Networks

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**Abstract**—In this paper, we propose an analytical technique to evaluate the statistics of the channel estimation error in a simple multi-user ad hoc networking scenario. This problem is very relevant in situations where advanced PHY techniques are used (e.g., MIMO or interference cancellation) and channel state information may be needed. The presence of several simultaneous and non-orthogonal signals makes the problems significantly more complicated than in traditional channel estimation. In particular, there is direct dependence of the channel estimation error on the instantaneous channel matrix. The proposed model makes it possible to quickly evaluate the performance of channel estimation schemes as a function of the system parameters. In this light, we include the effect of channel estimation errors in an ad hoc networking protocol simulator and thoroughly evaluate their impact. Our results show that there exists a significant interplay between the performance of MAC protocols for MIMO networks and the accuracy of channel estimation. Moreover, we show that relevant tradeoffs arise between MAC- and PHY-level parameters which lead to the definition of design guidelines.

## I. INTRODUCTION

MIMO communications and channel estimation have enjoyed great interest in recent years. The performance gains of MIMO systems (as well as other systems where advanced PHY technology is adopted) depend on the ability to accurately estimate the channel at the receiver. Channel estimation, as well as the effects of channel estimation errors on the overall system performance, are an important area of research that has received significant attention [1]–[6]. While this is a well-studied subject for point-to-point communications, comparatively less work has been done for networking scenarios, where signals coming from different transmitters are non-orthogonal and asynchronous, and multiuser interference may lead to significant estimation errors.

Our goal in this paper is to provide an analytical approach to study the effects of channel estimation errors on the performance of a MIMO ad hoc network scenario, where terminals are equipped with multiple antennas and use a low-complexity estimation technique based on training sequence correlation. The main difference with respect to previous work is that we explicitly consider a multiuser scenario, whose key implications are that the incoming signals from different antennas may have different average powers and cannot be assumed synchronous in general. More specifically, only signals coming from antennas of the same node have the same average power (which depends on the physical distance and on macroscopic propagation effects) and can be assumed to be symbol-synchronous (as the path length differences between pairs of antennas of a given transmitter/receiver pair are negligible compared to the symbol duration). Conversely,

signals coming from antennas at different nodes have different average powers in general and are randomly displaced in time with respect to each other. A direct consequence of these facts is that separation of orthogonal training sequences for channel estimation is only possible among signals transmitted by the antennas of the same node, whereas training sequences belonging to different nodes cannot be completely separated (not even in a flat fading environment) because of the impossibility of guaranteeing zero cross-correlation in this case, due to the lack of synchronization. As a result, the channel estimation error does not only depend on the noise power and on the length of the training sequence, as is usually the case in this kind of analysis, but also on the channel matrix itself.

In this paper, we provide some analytical developments that lead to precise expressions for the channel estimation error matrix in a multiuser scenario. Also, in order to precisely evaluate the impact of imperfect channel estimation on MAC-level protocols, we consider the approach proposed in [7] and thoroughly analyze its performance by means of simulation. We also provide relevant tradeoff curves that can be used to tune MAC behavior depending on system-level constraints such as throughput, efficiency, and so on.

## II. CHANNEL ESTIMATION ERROR ANALYSIS

Assume that each node has  $N_A$  antennas. To receive, all antennas are used, whereas in transmission in general a node  $i$  may use  $n_i \leq N_A$  antennas. We focus on a single receiving node, that hears signals from all transmitting antennas at the various nodes  $i = 1, \dots, M$ . The channel matrix (of size  $N_A \times \sum_{i=1}^M n_i$ ) is made up by all channel coefficients between all transmitting antennas at all nodes and all the  $N_A$  antennas at the receiving node. We assume flat fading, iid across the different antenna pairs. That is, the channel between antenna  $j$  of transmitting node  $i$  and antenna  $\ell$  at the receiver is a complex scalar  $h_{j\ell}^{(i)}$ , with zero-mean complex Gaussian statistics and variance  $\sigma_i^2$  (the same for all antennas  $j, \ell$  since it only depends on the distance between the transmitting and receiving nodes).

For channel estimation purposes, antenna  $j$  of user  $i$  sends a training sequence  $s_{ij}(t)$  of  $N$  real binary symbols in  $\{-1, +1\}$ , each of duration  $T$ . During the training phase, the signal received at antenna  $\ell$  is given by<sup>1</sup>

$$r_\ell(t) = \sum_{i=1}^M \sum_{j=1}^{n_i} h_{j\ell}^{(i)} s_{ij}(t - \tau_i) + z_\ell(t) \quad (1)$$

<sup>1</sup>Note that in this model we implicitly assume that all sequences  $s_{ij}$  are transmitted with equal power. In the presence of an overall per-node power constraint, the power assigned to each transmitted sequence is inversely proportional to the number of antennas used,  $n_i$ , and the following analysis can be readily extended to cover this case as well.

where  $z_\ell(t)$  is the thermal noise at the receiver, modeled as white circular complex Gaussian with zero mean and power spectral density  $N_0/2$  (per dimension), and  $\tau_i \sim U[0, T]$  is the propagation delay of the signals transmitted by the antennas at user  $i$  (note that this propagation delay does not depend on  $j, \ell$  but only on  $i$ , as the path length differences between pairs of antennas of a given transmitter/receiver pair are negligible compared to the symbol duration  $T$ ). With no loss in generality, we focus on the estimation of the channel of user  $m$ , and assume  $\tau_m = 0$ . Specifically, in order to estimate the channel coefficient between antenna  $k$  of user  $m$  and antenna  $\ell$  of the receiver, the following quantity is computed:

$$\begin{aligned}\hat{h}_{k\ell}^{(m)} &= \frac{1}{NT} \int_0^{NT} r_\ell(t) s_{mk}(t) dt \\ &= h_{k\ell}^{(m)} + \sum_{i \neq m} \sum_{j=1}^{n_i} h_{j\ell}^{(i)} J_{jk}^{(i,m)} + Z_\ell\end{aligned}\quad (2)$$

where

$$Z_\ell = \frac{1}{NT} \int_0^{NT} z_\ell(t) s_{mk}(t) dt \quad (3)$$

is the filtered noise term and

$$J_{jk}^{(i,m)} = \frac{1}{NT} \int_0^{NT} s_{ij}(t - \tau_i) s_{mk}(t) dt \quad (4)$$

is the matched filter output corresponding to the signal of antenna  $j$  of user  $i$ . Note that in Equation (2) we have used the fact that  $J_{jk}^{(m,m)} = 1$  for  $j = k$ , and 0 otherwise, as the antennas of the same user can use orthogonal sequences with zero cross-correlation as they are synchronously received, whereas sequences coming from different users have random time displacement.

The statistics of  $J_{jk}^{(i,m)}$  can be evaluated as follows. The product  $S(t) = s_{ij}(t - \tau_i) s_{mk}(t)$  in the time interval  $[0, NT]$  can be represented as a sequence of  $N$  intervals  $\Delta_p^{(\alpha)}$ ,  $p = 1, \dots, N$ , of width  $\tau_i$ , in which  $S(t) = \alpha_p$ , each followed by an interval  $\Delta_p^{(\beta)}$ , of width  $T - \tau_i$ , in which  $S(t) = \beta_p$ . Each  $\alpha_p$  and  $\beta_p$  is the product of different pairs of symbols of  $s_{ij}$  and  $s_{mk}$ , and therefore  $\alpha_p, \beta_p, p = 1, \dots, N$ , are iid binary random variables with  $P[-1] = P[+1] = 0.5$ . We then have

$$J_{jk}^{(i,m)} = \frac{1}{NT} \sum_{p=1}^N (\alpha_p \tau_i + \beta_p (T - \tau_i)) \quad (5)$$

so that  $E[J_{jk}^{(i,m)}] = 0$  and

$$\text{Var}[J_{jk}^{(i,m)} | \tau_i] = \frac{1}{(NT)^2} \sum_{p=1}^N [\tau_i^2 + (T - \tau_i)^2] \quad (6)$$

$$\text{Var}[J_{jk}^{(i,m)}] = \frac{1}{(NT)^2} 2NE[\tau_i^2] = \frac{1}{6N} \quad (7)$$

For the filtered noise term we have  $E[Z_\ell] = 0$  and

$$E[|Z_\ell|^2] = \frac{1}{(NT)^2} \int_0^{NT} N_0 s_{mk}^2(t) dt = \frac{N_0}{NT} \quad (8)$$

so that  $Z_\ell$  has circular complex Gaussian distribution with zero mean and variance  $\frac{N_0}{NT}$ . Finally, the output of the matched filter is

$$\hat{h}_{k\ell}^{(m)} = h_{k\ell}^{(m)} + \Delta h_{k\ell}^{(m)} \quad (9)$$

where

$$\Delta h_{k\ell}^{(m)} = \sum_{i \neq m} \sum_{j=1}^{n_i} h_{j\ell}^{(i)} J_{jk}^{(i,m)} + Z_\ell \quad (10)$$

is the channel estimation error.

Consider now the joint statistics of  $J_{j_1, k_1}^{(i_1, m)}$  and  $J_{j_2, k_2}^{(i_2, m)}$ . In particular, we have that

$$\begin{aligned}& E[J_{j_1, k_1}^{(i_1, m)} J_{j_2, k_2}^{(i_2, m)}] \\ &= E\left[\frac{1}{(NT)^2} \int_0^{NT} dt_1 \int_0^{NT} dt_2 s_{i_1 j_1}(t_1 - \tau_{i_1}) \times \right. \\ &\quad \left. s_{m k_1}(t_1) s_{i_2 j_2}(t_2 - \tau_{i_2}) s_{m k_2}(t_2)\right] \\ &= \frac{1}{(NT)^2} \int_0^{NT} dt_1 \int_0^{NT} dt_2 E[s_{i_1 j_1}(t_1 - \tau_{i_1}) \times \\ &\quad s_{m k_1}(t_1) s_{i_2 j_2}(t_2 - \tau_{i_2}) s_{m k_2}(t_2)]\end{aligned}\quad (11)$$

Consider the case  $i_1, i_2 \neq m$ . (Note that  $i \neq m$  is the only case of interest in (2), and in any event  $J_{jk}^{(i,m)}$  for  $i = m$  is deterministic so that the computation is trivial anyway.) In this case we have

$$\begin{aligned}& E[s_{i_1 j_1}(t_1 - \tau_{i_1}) s_{m k_1}(t_1) s_{i_2 j_2}(t_2 - \tau_{i_2}) s_{m k_2}(t_2)] \\ &= E[s_{i_1 j_1}(t_1 - \tau_{i_1}) s_{i_2 j_2}(t_2 - \tau_{i_2})] E[s_{m k_1}(t_1) s_{m k_2}(t_2)]\end{aligned}\quad (12)$$

which is clearly zero if  $i_1 \neq i_2$  or  $j_1 \neq j_2$  or  $k_1 \neq k_2$  (modeling the used sequences as independent random binary sequences). Similarly, it is possible to show that  $E[J_{j_1, k_1}^{(i_1, m_1)} J_{j_2, k_2}^{(i_2, m_2)}] = 0$ ,  $\forall i_1, i_2, j_1, j_2, k_1, k_2$ , if  $m_1 \neq m_2$ , because in this case  $E[s_{m_1 k_1}(t_1) s_{m_2 k_2}(t_2)] = 0$ .

Therefore, we can conclude that the variables  $J_{jk}^{(i,m)}$  can be modeled as a set of iid random variables, each with approximately Gaussian statistics (by virtue of their being sums of several binary independent random variables, see (5)) of zero mean and variance  $1/(6N)$ .

Let  $\mathbf{H}^{(i)}$  be the channel matrix between user  $i$  and the receiver, defined as

$$\mathbf{H}^{(i)} = \begin{pmatrix} h_{11}^{(i)} & \dots & h_{n_i 1}^{(i)} \\ h_{12}^{(i)} & \dots & h_{n_i 2}^{(i)} \\ \dots & \dots & \dots \\ h_{1N_A}^{(i)} & \dots & h_{n_i N_A}^{(i)} \end{pmatrix} \quad (13)$$

and let the overall channel matrix from all transmitting antennas to the antennas of the receiving node be

$$\mathbf{H} = [\mathbf{H}^{(1)}, \mathbf{H}^{(2)}, \dots, \mathbf{H}^{(M)}] \quad (14)$$

Define now a column vector that collects all variables  $J_{jk}^{(i,m)}$  for all the antennas used by the transmitting users as

$$\begin{aligned}\mathbf{J}_k^{(m)} &= [J_{1k}^{(1,m)}, J_{2k}^{(1,m)}, \dots, J_{n_1 k}^{(1,m)}, \\ &\quad J_{1k}^{(2,m)}, \dots, J_{n_2 k}^{(2,m)}, \dots, J_{1k}^{(M,m)}, \\ &\quad \dots, J_{(n_M-1)k}^{(M,m)}, J_{n_M k}^{(M,m)}]^T\end{aligned}\quad (15)$$

in which all entries  $J_{j,k}^{(m,m)} \forall j$ , have been set to zero, and collect  $\mathbf{J}_k^{(m)}$ ,  $k = 1, \dots, n_m$ , to obtain the matrix

$$\mathbf{J}^{(m)} = [\mathbf{J}_1^{(m)}, \mathbf{J}_2^{(m)}, \dots, \mathbf{J}_{n_m}^{(m)}] \quad (16)$$

With this notation, and defining the column vectors

$$\mathbf{h}_k^{(m)} = [h_{k1}^{(m)}, h_{k2}^{(m)}, \dots, h_{kN_A}^{(m)}]^T \quad (17)$$

$$\hat{\mathbf{h}}_k^{(m)} = [\hat{h}_{k1}^{(m)}, \hat{h}_{k2}^{(m)}, \dots, \hat{h}_{kN_A}^{(m)}]^T \quad (18)$$

$$\mathbf{Z} = [Z_1, Z_2, \dots, Z_{N_A}]^T \quad (19)$$

we can rewrite Equations (9) and (10) in vector form as

$$\hat{\mathbf{h}}_k^{(m)} = \mathbf{h}_k^{(m)} + \mathbf{H}\mathbf{J}_k^{(m)} + \mathbf{Z} \quad (20)$$

which shows that the channel estimation error vector  $\mathbf{H}\mathbf{J}_k^{(m)} + \mathbf{Z}$  can be generated by multiplying the channel matrix  $\mathbf{H}$  by the training sequence correlation vector  $\mathbf{J}_k^{(m)}$  (whose elements can be drawn randomly and independently  $\sim N(0, \frac{1}{6N})$ ), and adding the noise vector. By grouping in a matrix all the above quantities with subscript  $k = 1, \dots, n_m$ , we obtain

$$\hat{\mathbf{H}}^{(m)} = \mathbf{H}^{(m)} + \mathbf{H}\mathbf{J}^{(m)} + \mathbf{Z}' \quad (21)$$

and by further grouping all these results for all nodes  $m = 1, \dots, M$  into a bigger matrix we have

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{H}\mathbf{J} + \mathbf{Z}'' \quad (22)$$

where

$$\mathbf{J} = [\mathbf{J}^{(1)}, \mathbf{J}^{(2)}, \dots, \mathbf{J}^{(M)}] \quad (23)$$

and  $\mathbf{Z}', \mathbf{Z}''$  are matrices of iid elements  $\sim N(0, \frac{N_0}{NT})$ .

Equation (22) clearly shows that the channel estimation error matrix has two components, one proportional to the channel matrix ( $\mathbf{H}\mathbf{J}$ ), and one independent of it ( $\mathbf{Z}''$ ).

This model allows to analytically compute various statistical metrics related to the channel estimation errors. Also, it can be easily used to simulate the effect of channel estimation errors in an ad hoc network, and to study their impact on the network performance.

### A. Case Study

The above analysis can be used to derive the statistics of the estimation error in a multiuser ad hoc environment. As an example, consider a network with three nodes, i.e., two transmitters and one receiver, each equipped with two antennas. The transmitting nodes use both their antennas. The receiver wants to estimate the eight flat fading channel coefficients between each of the transmit antennas and its own receive antennas. The channel matrix can be written as  $\mathbf{H} = (h_{kl}^{(m)})$ , with  $k, l, m = 1, 2$ , where  $h_{kl}^{(m)}$  is the channel coefficient between the  $k$ -th antenna of the  $m$ -th transmitter and the  $l$ -th antenna of the receiver. According to Eq. (22), the estimation error depends on the instantaneous responses of all channels. For a given realization of the channel conditions, we are interested in computing the mean and variance of the estimation error,  $\Delta h_{kl}^{(m)}$ ,  $m, k, l = 1, 2$ , which is given by Equation (10). We have

$$E[\Delta h_{kl}^{(m)}] = 0 \quad (24)$$

and

$$E[|\Delta h_{kl}^{(m)}|^2] = \sum_{i \neq m} \sum_{j=1}^{n_i} |h_{jl}^{(i)}|^2 \frac{1}{6N} + \frac{N_0}{NT} \quad (25)$$

With a slight abuse of notation, we define the matrices  $|\mathbf{H}|^2 = (|h_{kl}^{(m)}|^2)$  and  $E[|\Delta\mathbf{H}|^2] = (E[|\Delta h_{kl}^{(m)}|^2])$  with  $k, l, m = 1, 2$ .

The following equations give an example of results obtained for two independent and randomly generated channel realizations. The transmitters are at the same distance from the receiver, noise is neglected, and  $N = 16$ .<sup>2</sup> First realization:

$$|\mathbf{H}|^2 = \begin{pmatrix} 0.0663 & 0.00435 & 0.554 & 0.00132 \\ 0.164 & 0.00156 & 2.12 & 0.645 \end{pmatrix} \quad (26)$$

$$E[|\Delta\mathbf{H}|^2] = \begin{pmatrix} 5.78e-3 & 5.78e-3 & 7.35e-4 & 7.35e-4 \\ 0.0288 & 0.0288 & 1.72e-3 & 1.72e-3 \end{pmatrix} \quad (27)$$

Second realization:

$$|\mathbf{H}|^2 = \begin{pmatrix} 3.04e-3 & 0.038 & 5.33e-5 & 1.58e-3 \\ 0.467 & 0.0308 & 5.33 & 16.0 \end{pmatrix} \quad (28)$$

$$E[|\Delta\mathbf{H}|^2] = \begin{pmatrix} 1.7e-5 & 1.7e-5 & 4.27e-4 & 4.27e-4 \\ 0.223 & 0.223 & 5.19e-3 & 5.19e-3 \end{pmatrix} \quad (29)$$

These results confirm that  $E[|\Delta h_{kl}^{(m)}|^2]$  does not depend on  $k$ , as expected. Also, they show that the ratio of the error variance to the channel strength greatly depends on the channel matrix, which poses significant challenges for accurate channel estimation in a multi-user scenario.

## III. NETWORK SIMULATION

In order to evaluate the impact of imperfect channel estimation on MAC-level protocols, we consider the same scenario as in [7]. We assume that a number of nodes must communicate within a certain network area by using MIMO techniques. More specifically, we advocate the use of V-BLAST [8] both to enhance the transmit bit rate and to mitigate multiple-access interference through successive interference cancellation. We recall that V-BLAST is based on spatial multiplexing, i.e., on the superposition of multiple transmit flows, which are transmitted simultaneously through different antennas. This can potentially give rise to harsh interference, especially in a multi-user scenario, where many transmitters are expected to access the channel at the same time. While V-BLAST provides multiuser detection capabilities in the form of successive interference cancellation, it is also very prone to errors in high interference scenarios. For example if a number of superimposed signals have to be detected, and the first bears a low signal to interference and noise ratio, its detection is likely to fail, so that its cancellation would increase the amount of interference affecting subsequent detections.

The need to regulate multiple access in MIMO ad hoc networks has led to the definition of protocols that balance between two concurring objectives, namely throughput and protection from interference. In other words, a "good" protocol for MIMO ad hoc networks would encourage multiple access to enhance the aggregate network throughput, but without allowing too many simultaneous transmissions, that would cause excessive interference, and ultimately lead to severe data loss. One such protocol is presented in [7] and is based on frames. Each frame is divided into an RTS, a CTS, a data and an ACK phase. RTSs are used by transmitters to issue communication requests, which specify, for each intended receiver, how many PDUs are to be transmitted. These signaling packets have the only purpose of advertising transmission requests, and do

<sup>2</sup>These results can be easily scaled to other values of  $N$ , as the variance of the channel estimation is inversely proportional to  $N$  in this case.

not block any neighbor, unlike in 802.11 [9]. The protocol in [7] includes a CTS policy called Follow Traffic (FT), which is used to compose CTSs in response to received RTSs. After CTS reception, data is sent according to the information reported in the CTS; ACKs are finally sent to confirm correct packet reception. All transmissions (e.g., all RTSs) take place simultaneously during the corresponding phase.

We highlight that FT interacts with the underlying PHY layer in order to grant at least one transmission request directed to the receiving node, trying to ensure that any granted transmission is sufficiently protected from interference. This protection is achieved by limiting grants based on the perception of neighboring network activity, which is obtained, in turn, through RTS overhearing. Furthermore, the aforementioned protocol and policy can be augmented with backoff strategies that are applied every time a CTS is not received in response to an RTS, in order to limit channel access persistence and thus reduce congestion. Two backoff policies have been considered in [7] and thoroughly evaluated in [10]: node-lock (NL) which blocks any transmission, and dest-lock (DL), which blocks further transmissions toward the node that did not send the CTS, and thus leaves the transmitter free to communicate with other neighbors. In either case, the duration of the backoff interval is equal to a number of frames randomly chosen within the interval  $[1, B_{max}]$ , where  $B_{max} = W \cdot 2^{N_u - 1}$ ,  $N_u$  is the number of consecutive unanswered RTSs, and  $W$  is a window length parameter.

Previous studies on the protocol described above [5], [7], [10], [11] have been carried out assuming perfect channel state information at the receiver (CSIR). However, estimation inaccuracies are expected to have a significant impact on PHY- and thus MAC-level performance. In order to evaluate this impact, we assume that a training sequence of  $N$  bits precedes any transmitted packet.<sup>3</sup> The objective of Section III-B is to provide an in-depth study of PHY and MAC behavior in the presence of imperfect channel estimation, for varying estimation accuracy.

### A. Network Setting

For the results in the forthcoming subsection, we have considered a network of 25 nodes deployed in a grid within a  $100 \text{ m} \times 100 \text{ m}$  area, so that the distance between nearest neighbors is 25 m. All nodes have  $N_A = 8$  antennas. While all antennas are used during reception phases, only as many antennas as allowed according to the CTS policy are used for transmission (node  $i$  uses exactly  $n_i$  antennas, see also Section II). We assume that packets are generated according to a Poisson process of rate  $\lambda$  packets per second per node. Each new packet is formed by a random number of 1000 bits-long<sup>4</sup> packet data units (PDUs), which are suitable to be transmitted using one antenna. The number of PDUs in a packet is uniformly chosen between 1 and 4. Signaling packets are 200 bits long.<sup>4</sup> PHY-level details are accounted for in our simulator by means of the analytical approach devised in [11]. As regards the window parameter chosen for the backoff policies, we employ  $W = 1$  for NL and  $W = 16$  for DL, in order to achieve the maximum throughput [10].

<sup>3</sup>Under the assumption that the channel does not vary significantly within a frame, ACKs do not require training, as they can be transmitted with the same antenna previously used for the CTS.

<sup>4</sup>These values do not account for the length of the training sequence.

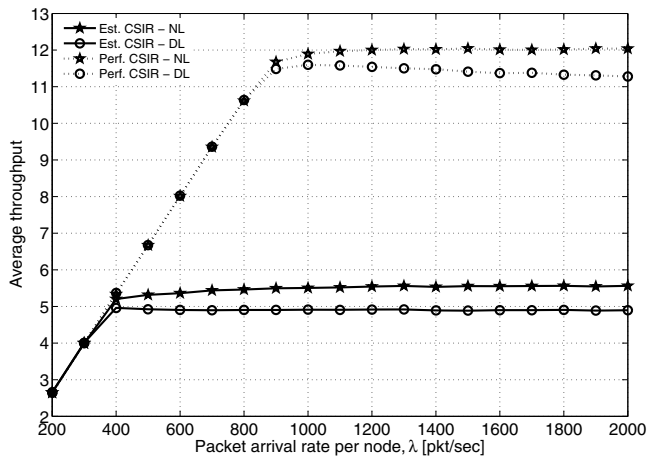


Figure 1. Average throughput as a function of  $\lambda$ ,  $N = 32$ .

### B. Results

The first set of results (Figures 1 and 2) we present in the sequel quantify the impact of imperfect channel estimation as a function of traffic. The length of the training sequences is taken to be  $N = 32$ . Figure 1 depicts throughput as a function of  $\lambda$  for NL and DL, with both perfect and imperfect (estimated) CSIR. Throughput is defined as the average number of PDUs that correctly reach their destination per frame. This figure shows the performance degradation incurred with imperfect CSIR, which translates into a lower maximum throughput (50% less than with perfect CSIR) for both DL and NL. In fact, imperfect CSIR makes the V-BLAST detection algorithm much more prone to excess interference, causing a greater probability of error and thus lower throughput. However, note that the throughput does not drop to zero, but is instead maintained at a constant level thanks to MAC level backoff, which limits channel access persistence. As expected, DL undergoes a worse throughput decrease because of its greater aggressiveness (recall that, unlike NL, DL defers transmissions only toward the node that did not send a CTS). We highlight that the effective throughput should be defined by removing all overhead (see also Figures 3 and 5). This means that a larger  $N$  increases the duration of the frame, thus improving channel estimation and detection performance at the price of a longer frame completion delay.

Results similar to those in Figure 1 can be seen in Figure 2, which shows the correct transmission delay (i.e., the average number of frames elapsed from the generation of a packet to its correct delivery) as a function of traffic. Before describing this picture, we stress that there are two negative effects to the detection performance of V-BLAST. First, a wrong signal detection and cancellation doubles the interference of the current signal on following detections; this effect is present even with perfect channel estimation. Second, a wrong channel estimation leads to the cancellation of an incorrect contribution from the received signal, hence subsequent detections are interfered by previous cancellations even in the case of correct detection. This means that the last signals in the V-BLAST detection order are the most affected, which actually limits the maximum achievable parallelism. In this light, both NL and DL experience worse performance in the presence of imperfect CSIR. However, while NL limits parallelism and thus reduces

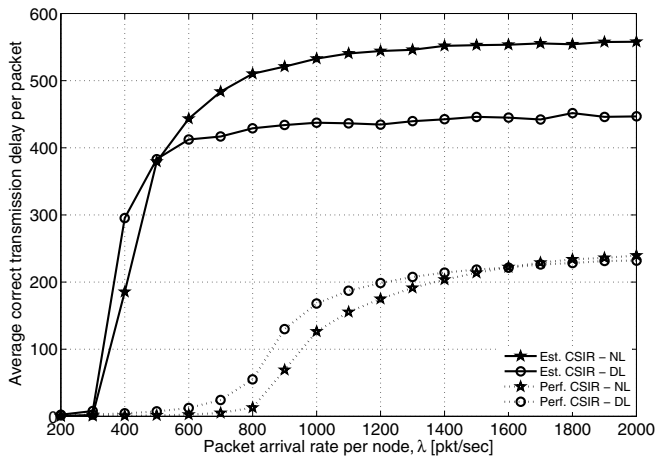


Figure 2. Average delay before a correct transmission as a function of  $\lambda$ ,  $N = 32$ .

the aforementioned effects, the greater aggressiveness of DL pays off from the point of view of delay, as DL eventually outperforms NL. Interestingly, the crossing point is at around  $\lambda = 1600$  for perfect CSIR while for imperfect CSIR it decreases to around  $\lambda = 500$ . The reason behind this fact is the same explained before for throughput: namely, imperfect CSIR increases the number of errors that affect packet detections, hence causing the NL strategy to induce progressively longer backoffs and thus longer transmission delays.

While in general a longer training sequence improves estimation performance, see (7), it also increases overhead, decreasing the protocol efficiency. We define efficiency here as the ratio of correctly received information bits over all sent bits. This definition allows to balance between the greater estimation accuracy (thus better probability of success) and the greater overhead incurred by increasing the length of the training sequence,  $N$ . This tradeoff is reported in Figure 3, that compares throughput and efficiency for varying  $N$  at  $\lambda = 1200$ . Note that, unlike in Figure 1, here throughput is defined as the average number of correct bits that are received per second, and hence accounts for the longer frame duration induced by greater values of  $N$ . Both DL and NL are considered. First, we observe that the maximum efficiency obtained for  $N = 1$  is representative of a situation where there are but very few transmissions. In fact, for  $N = 1$ , the quality of channel estimation is very poor, and the first messages to fail being delivered are in fact RTSs (see also the discussion regarding Figure 5). This prevents successful handshakes, so that very few nodes actually transmit. This directly translates into a high efficiency, as the duration of the frame is very short and the few senders are almost always successful (see also Figure 4). However, we look for an operating point where more transmissions take place and yet efficiency is preserved. To this end,  $N = 32$  ( $N = 64$ ) yields the best efficiency for NL (DL). We also observe that this efficiency is comparable to the case  $N = 1$ , while throughput is more acceptable. Moreover, efficiency curves exhibit a smooth slope from the right, hence efficiency does not significantly decrease when choosing, e.g.,  $N = 128$ , which yields better throughput for both NL and DL.

There exists an interesting low-level interplay between channel estimation accuracy, throughput and efficiency. This

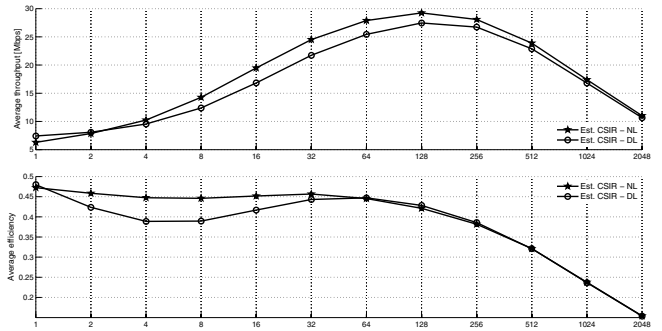


Figure 3. Comparison between throughput in Mbps and efficiency as a function of  $N$ ,  $\lambda = 1200$ .

can be inferred by looking at Figure 4, that shows the average transmission success ratios for RTSs and PDUs as a function of traffic. As expected, both the RTS and the PDU success ratios decrease in the presence of imperfect CSIR. The performance loss is greater for PDUs, due to the greater length of the packet (1000 bits instead of 200), and in addition DL experiences worse losses than NL, as expected due to its greater aggressiveness. We observe that the success ratio curves tend to stabilize to an almost constant value after a certain value of  $\lambda$ , corresponding to the value where throughput stabilizes as well. However, it should be noted that not only the PDU success ratio curves present this behavior: RTS curves follow entirely analogous trends. This means that the throughput loss due to imperfect channel estimation is not simply due to more frequent PDU losses, but also to signaling packet losses. This last effect plays a very important role in our network. In fact, as traffic increases, RTSs are sent more frequently to handle the larger amount of generated packets. However, imperfect CSIR tends to corrupt some of them: therefore, their senders do not get any CTS in response and back off as a consequence. In turn, this reduces the number of RTSs sent, increasing the chance that transmitted ones get through. This behavior has the net effect to stabilize throughput and success ratio, and also explains why NL performs better than DL: NL's milder access persistence inherently reduces the number of sent RTSs, thus relieving the congestion effect described above. While this behavior originates predominantly at the PHY layer, it has important consequences on MAC performance, and shows that the protocol adapts to imperfect CSIR by reducing channel accesses. Furthermore, this interplay between PHY and MAC could be used as a rough yet very effective way of tuning the network behavior.

To delve more into this subject, let us allow the signaling packets and PDUs to use training sequences of different lengths ( $N_{sig}$  and  $N_{data}$ , respectively) for channel estimation. In Figure 5, we plot contour curves of efficiency and throughput obtained by varying these two parameters. The explicit consideration of the length of the training sequence for signaling and data makes it possible to study in more detail the effects of channel estimation on the overall MAC performance. As observed before, a greater  $N_{sig}$  protects RTSs better, so that more of them can be received correctly, and more links can be established. Conversely, a greater  $N_{data}$  protects PDUs, but only those that have been in fact sent. Therefore, a greater  $N_{data}$  can increase throughput significantly only if

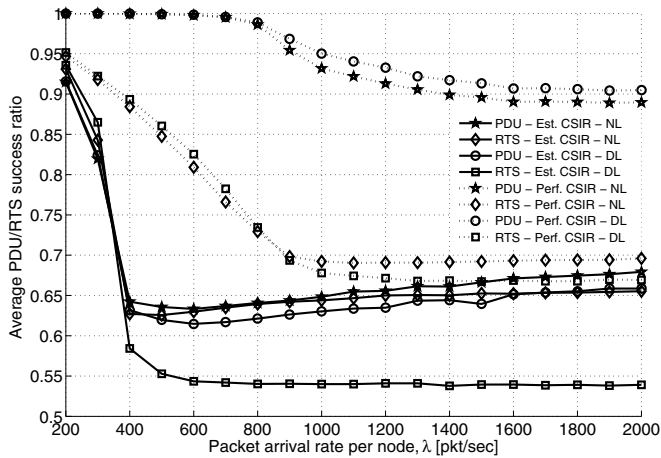


Figure 4. Average RTS and PDU success ratio as a function of  $\lambda$ ,  $N = 32$ .

enough links are currently active, i.e., only for high  $N_{sig}$ . For example, take  $N_{sig} = 32$ : increasing  $N_{data}$  effectively increases throughput only up to roughly 30 Mbps, after which no further improvements are possible (the number of active links is limited by  $N_{sig}$ ) and choosing a greater  $N_{data}$  only increases the duration of the frame, thereby reducing throughput. In other words, increasing  $N_{data}$  uselessly provides a better channel estimation and success ratio to sent PDUs, but cannot increase the number of active links per frame, and eventually introduces too much overhead. Conversely, take  $N_{data} = 32$ : while increasing  $N_{sig}$  initially improves throughput, after  $N_{sig} = 128$  throughput is decreased. The reason is that a greater  $N_{sig}$  allows the setup of more links, and the low  $N_{data}$  cannot grant a sufficiently accurate channel estimation to enable correct detection. This effect adds to the increase of the frame duration due to the greater  $N_{sig}$ , which decreases throughput even further. Finally, note that Figure 5 can be used as a quick reference to tune the working point on the efficiency-throughput tradeoff by varying  $N_{sig}$  and  $N_{data}$ . For example, assume that we wish to ensure an efficiency of at least 0.5 and a throughput of at least 20 Mbps: we can choose any pair of  $N_{sig}$  and  $N_{data}$  within the intersection of the curves representing this efficiency and throughput. For example, the choice  $N_{sig} = 16$ ,  $N_{data} = 64$  satisfies both constraints. Conversely, if the constraints are too strict (e.g., an efficiency of 0.55 and a throughput of 30 Mbps) the curves do not intersect, meaning that the requirements are not compatible.

#### IV. CONCLUSIONS

In this paper, we have described an analytical model for the computation of the statistics of the channel estimation error in a multi-user MIMO ad hoc network. This analysis highlights that in such scenario the channel estimation error directly depends on the channel matrix, and makes it possible to express in compact form this dependence relationship. As an example of application, we have computed results for a simple three-node scenario. It is to be noted that the technique scales well, and the three-node scenario was merely chosen for ease of presentation of the results.

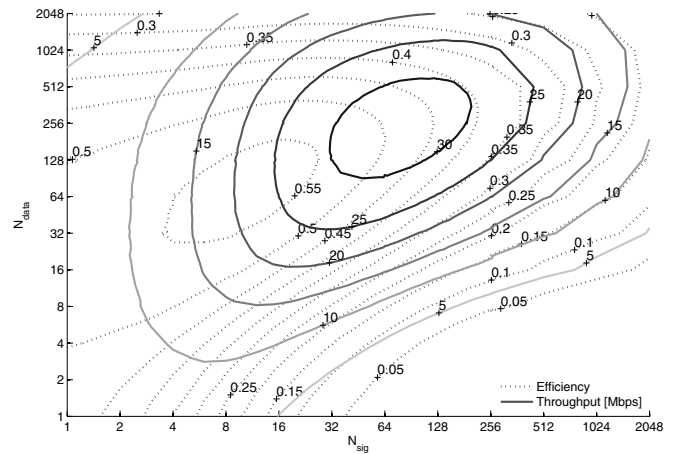


Figure 5. Contour curves of average efficiency and throughput in Mbps for NL as a function of the length of training sequences used for signaling messages ( $N_{sig}$ ) and PDUs ( $N_{data}$ ). Each curve in the Figure is obtained as the intersection of the efficiency or throughput surfaces as a function of  $N_{data}$  and  $N_{sig}$  with a horizontal plane corresponding to the efficiency or throughput value indicated on the curve.

We have applied this analysis to the evaluation of the impact of channel estimation errors on ad hoc networking protocols (such as [5], [7]), thereby evaluating their robustness against estimation inaccuracies. To this end, we have presented an in-depth evaluation of a MAC protocol for MIMO ad hoc networks, highlighting the interplay between PHY and MAC and the relevant tradeoffs that arise.

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